University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Mechanics (Physics 212A) Fall 2015
Assignment 5

Due 12:30pm Wednesday, November 4, 2015

1. **Electrons and Stern-Gerlach.** Show that it's impossible to measure the electron magnetic moment with a Stern-Gerlach apparatus.

   Hints: (1) The electron is a charged particle. (2) This problem is intended as an application of the uncertainty principle.

2. **Galilean invariance of the Schrödinger equation.**

   Show that the free-particle Schrödinger equation maps to itself under the Galilean boost acting by
   $$\vec{x} \to \vec{x}' \equiv \vec{x} + \vec{v}t, \quad t \to t' \equiv t, \quad \psi \to \psi'$$
   where the boosted position-space wavefunction is (prime is not derivative here)
   $$\psi'(x', t') \equiv e^{\mathbf{i}\varphi(x',t')}\psi(x, t)$$
   and
   $$\varphi(x, t) \equiv av^2 t + b\vec{v} \cdot \vec{x}.$$
   (You must determine a and b.)

3. **Toy model of the hydrogen molecular ion.** [from Commins]

   A particle of mass m is in a potential
   $$V(x) = W_0 \left(\delta(x - a) + \delta(x + a)\right)$$
   with $W_0 < 0$.

   (a) Find the expression that determines the bound-state energy for even-parity states, and determine graphically how many even-parity boundstates exist.

   (b) What is the wave function for the lowest even-parity bound state? Sketch this function for large, intermediate, and small a (compared to what?) ?

   (c) Repeat the previous parts for odd parity. For what values of W_0 is there at least one such boundstate?

   (d) Find the even- and odd-parity binding energies for $m|W_0|a \gg \hbar^2$. Explain why these energies move closer together as $a \to \infty$.

(e) Suppose that the two potential centers at a experience some (repulsive) Coulomb interaction, $U_c(a) = \frac{\mu}{a}$ for some constant μ. Including this effect and the energy of the electron in its groundstate what can you say about the equilibrium value of a?

4. **Numerical quantum mechanics.** [optional bonus projects]

(a) **Time evolution.** Write some code to time-evolve a 1d wavefunction for a particle in a potential $V(x)$. That is: choose an initial wavefunction $\psi(x, 0)$; choose a potential $V(x)$, find numerically $\psi(x, n\delta t)$ for some set of n, and some choice of δt. Draw pictures.

A good strategy is to use the *split-step method* which approximates the time-evolution operator as

$$\mathbf{U}(\delta t) = e^{\mathbf{i}\delta t\mathbf{H}} = e^{\mathbf{i}\delta t\left(\frac{\mathbf{p}^2}{2m} + V(\mathbf{x})\right)} = e^{\mathbf{i}\delta t\frac{\mathbf{p}^2}{2m}} e^{\mathbf{i}\delta t V(\mathbf{x})} e^{\mathcal{O}(\delta t)^2}.$$

Ignoring the $\mathcal{O}(\delta t)^2$ correction, time evolution occurs by action of

$$\mathbf{U}(t = n\delta t) = \mathbf{U}(\delta t)^n \approx e^{\mathbf{i}\delta t\frac{\mathbf{p}^2}{2m}} e^{\mathbf{i}\delta t V(\mathbf{x})} e^{\mathbf{i}\delta t\frac{\mathbf{p}^2}{2m}} e^{\mathbf{i}\delta t V(\mathbf{x})} e^{\mathbf{i}\delta t\frac{\mathbf{p}^2}{2m}} e^{\mathbf{i}\delta t V(\mathbf{x})} \cdots e^{\mathbf{i}\delta t\frac{\mathbf{p}^2}{2m}} e^{\mathbf{i}\delta t V(\mathbf{x})}.$$

Each step is diagonal in either position space or momentum space, so we just need to fourier transform back and forth after each step.

(b) **Eigenvalue problem.**

Use the method of shooting to solve numerically for the groundstate wavefunction of a potential which grows at large $|x|$.