University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Mechanics (Physics 212A) Fall 2015
Assignment 7

Due 12:30pm Wednesday, November 18, 2015

1. **Shape-invariant potentials.** [Commins]

   A particle of unit mass moves in a certain one-dimensional potential $V_-(x)$. The boundstates $u_0, u_1 ... u_n$ have energies $E_0, E_1 ... E_n$ respectively, in order of increasing energy. The groundstate wavefunction is

   $$u_0 \propto (\text{sech}\beta x)^p$$

   where $\beta, p > 0$ are real parameters.

   (a) Find the superpotential $W(x)$.

   (b) Find V_- and its partner potential V_+.

   (c) Obtain a general formula for the E_n. What restriction applies to p so that there exist n boundstates?

   (d) Find u_1, up to normalization.

   (e) Use the result of part 1c to discuss the potential $V = -V_0 \text{sech}^2 \beta x$ where $V_0 > 0$, and show the boundstate eigenvalues for $m = \hbar = 1$ are given by

   $$E_n = -\frac{\beta^2}{8}\left(-(1+2n) + \sqrt{1 + 8V_0/\beta^2}\right)^2 .$$

2. **WKB vs exact solution.**

   Consider again the problem of a particle in a linear confining potential $V(x) = |x|$.

   Compare the numerical values of the first three energy levels to the WKB approximation answers.

   Consider the case of a quantum ball bouncing on the floor. Approximate the floor as a hard wall.

   How can you obtain the eigenfunctions for this case from the previous case?

   Draw the phase space orbits for this case.

3. **Whittaker functions!** [optional]

   In deriving the connection formulae which relate WKB solutions to the Schrödinger equation in neighboring allowed and forbidden regions, we assumed that the potential had nonzero slope F_0 at the turning point. What should you do if this assumption fails, and the potential looks like this: