University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2016
Assignment 5

Due 11am Tuesday, November 1, 2016

1. **Scalar Yukawa amplitudes.**

   Consider again the scalar Yukawa theory of a complex scalar Φ and a real scalar ϕ. In the following, assume all particles are in momentum eigenstates. Use artisanal methods.

   (a) Compute the amplitude for the annihilation of a Φ particle and a $\Phi^\star$ particle into a ϕ particle, at leading order in the coupling g.

   (b) Compute the amplitude for $\Phi + \phi \to \Phi + \phi$ scattering to the leading order in the coupling at which it is nonzero.

2. **Wick example.**

   For a real scalar field, verify by hand Wick's prediction for the difference

   $$\mathcal{T}\left(\phi(x_1)\phi(x_2)\phi(x_3)\right) - :\phi(x_1)\phi(x_2)\phi(x_3):$$

3. **Fields and forces.** [from Banks] Consider a real free relativistic scalar field of mass m $S[\phi] = \int d^{d+1}x \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right)$.

   (a) Calculate the vacuum expectation value

   $$\langle 0|\,\mathcal{T}\left(e^{\mathbf{i}\int d^{d+1}x\ \phi(x)J(x)}\right)|0\rangle \equiv e^{\mathbf{i}W[J]}$$

   where J is a fixed, external source. Make a series expansion in powers of J and draw some diagrams. To understand the structure of the series, recall the formula on a previous homework for $\left\langle e^{K\cdot q}\right\rangle$ in any gaussian theory.

   (b) Now specialize to the case where the source is static and is present for a time $2T$:

   $$J(x) = J_{\text{static}} \equiv \theta(T-t)\theta(t+T)\left(\delta^{d+1}(x) - \delta^{d+1}(x-R)\right)$$

   with $T \gg R \gg 1/m$. You should find an answer of the form

   $$W\left[J_{\text{static}}(x)\right] = TV(R)$$

   where $V(R)$ is the Yukawa potential.

   (c) Chant the following incantation:

   Static sources experience a force due to exchange of virtual particles.