University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2017
Assignment 7

Due 11am Thursday, November 16, 2017

1. **Brain warmer.** Mandelstam variables are Lorentz-invariant combinations of the kinematic variables in $2 \leftarrow 2$ scattering

$$s = (p_1 + p_2)^2, t = (p_1 - p_3)^2, u = (p_1 - p_4)^2$$

where p_i^μ are on-shell four-vectors $p_i^2 = m_i^2$ satisfying overall momentum conservation $p_1 + p_2 = p_3 + p_4$. Show that they are not independent but rather satisfy the relation

$$s + t + u = \sum_i m_i^2$$

where i runs over the four external particles.

2. **Particle production by a source, continued.**

Consider again the Hamiltonian

$$H = H_0 + \int d^3x \left(-j(t, \vec{x})\phi(x)\right)$$

where H_0 is the free Klein-Gordon Hamiltonian, ϕ is the Klein-Gordon field, and j is a c-number scalar function.

Show that the probability of producing n particles is given by the Poisson distribution,

$$P(n) = \frac{1}{n!}\lambda^n e^{-\lambda}.$$

Conclude that $\langle n \rangle = \lambda$.

3. **Decay of a scalar particle.**

Peskin, problem 4.2. Consider the following Lagrangian, involving two real scalar fields Φ and ϕ:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2 + \partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) - \mu\Phi\phi^2.$$

The last term is an interaction that allows a Φ particle to decay into two ϕs, if the kinematics allow it. Calculate the lifetime of the Φ particle to lowest order in μ. What is the condition on the masses for a finite lifetime?