University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2017
Assignment 10

Due 11am Thursday, December 14, 2017

1. **Brain-warmer: Electron-positron scattering.**

   Draw and evaluate the two diagrams which contribute to $e^+e^- \to e^+e^-$ (Bhabha) scattering at tree level in QED. Be careful about the relative sign of their contributions.

   Compare with the case of $e^+e^- \to \mu^+\mu^-$ and with $e^-e^- \to e^-e^-$.

2. **Meson scattering.**

   Consider the Yukawa theory with fermions again, with $\mathcal{L}_{\text{int}} = g\bar{\Psi}\Psi\phi$.

   (a) Draw the Feynman diagram which gives the leading contribution to the process $\phi\phi \to \phi\phi$.

   (b) Derive the correct sign of the amplitude by considering the relevant matrix elements of powers of the interaction hamiltonian.

   (c) Evaluate the diagram in terms of a spinor trace and a momentum integral. Do not do the momentum integral. Suppose that the integral is cutoff at large k by some cutoff Λ. Estimate the dependence on Λ.

3. **Electron-photon scattering at low energy.**

   Consider the process $e\gamma \to e\gamma$ in QED at leading order.

   (a) Draw and evaluate the two diagrams.

   (b) Find $\frac{1}{4}\sum_{\text{spins/polarizations}} |\mathcal{M}|^2$.

   (c) Construct the two-body final-state phase space measure in the limit where the photon frequency is $\omega \ll m$ (the electron mass), in the rest frame of the electron. I suggest the following kinematical variables:

   $$p_1 = (\omega, 0, 0, \omega), p_2 = (m, 0, 0, 0), p_4 = (\omega', \omega' \sin\theta, 0, \omega' \cos\theta), p_3 = p_1 + p_2 - p_4 = (E', p')$$

   for the incoming photon, incoming electron, outgoing photon and outgoing electron respectively.

(d) Find the differential cross section $\frac{d\sigma}{d\cos\theta}$ as a function of ω, θ, m. (The expression can be prettified by using the on-shell condition $p_3^2 = m^2$ to relate ω' to ω, θ. It is named after Klein and Nishina.)

(e) Show that the limit $E \ll m$ gives the (Thomson) scattering cross section for classical electromagnetic radiation from a free electron.

4. **Non-relativistic interactions from QFT.**

   (a) **Coulomb potential.**

   Derive from QED that the force between electrons is a repulsive $1/r^2$ force law!

   (b) **Pseudoscalar Yukawa theory.**

   Consider the theory of a massive Dirac fermion Ψ and a massive pseudoscalar φ interacting via the term

$$V_5 \equiv g_5 \bar{\Psi} \gamma^5 \Psi \varphi.$$

   Convince yourself that this theory is parity invariant.

   List the Feynman rules.

   Draw and evaluate the diagrams contributing to $\Psi\Psi \to \Psi\Psi$ scattering at leading order in g_5.

   Consider the non-relativistic limit, $m \gg |\vec{p}|$ and find the effective interaction hamiltonian. If you happen to find zero for the leading term, then it's not the leading term.

5. **Ward identity in scalar QED.** We noted in lecture that scalar QED is different from the usual spinor QED in that the coupling to the gauge field is not just $j^\mu A_\mu$ where j^μ is a current independent of A. In this problem we'll see how this changes the proof of the Ward identity.

   (a) Consider a Green's function in scalar QED of the form $G \equiv \langle 0|\mathcal{T}\mathcal{O}_1 \cdots \mathcal{O}_n|0\rangle$ where $\mathcal{O}_i \equiv \mathcal{O}_i(x_i)$ has charge Q_i under the transformation

$$\Phi(x) \to e^{\mathbf{i}\alpha(x)}\Phi(x), A_\mu \to A_\mu. \tag{1}$$

   The phrase "$\mathcal{O}$ has charge Q" means $\mathcal{O}(x) \mapsto e^{\mathbf{i}Q\alpha(x)}\mathcal{O}(x)$. (Notice that (1) is *not* the gauge transformation, since A does not transform.) Derive the Schwinger-Dyson equation which follows from demanding that the path integral is invariant under the change of variables (1) to first order in α. (We are supposed to call the result a Ward-Takahashi identity.)

(b) Consider an amplitude in scalar QED with an external photon of polarization ϵ: $\mathcal{M} = \epsilon^\mu \mathcal{M}_\mu$. Using the LSZ reduction formula, show the result of the previous part implies the Ward identity $p^\mu \mathcal{M}_\mu = 0$. Conclude that the longitudinal photons decouple in scalar QED as well.