1. **Brain-warmer.** Show that the relation $e^{2J'} = \cosh 2J$ can be rewritten as $v' = v^2$ in terms of $v \equiv \tanh J$.

2. **Decimation of 1d Ising model in a field.**

Consider again a closed (periodic) chain of $N$ classical spins $s_i = \pm 1$ with Hamiltonian

$$H = -J \sum_i s_is_{i+1} - h \sum_i s_i + \text{const}, \quad s_{N+1} = s_1$$

The partition function is $Z(\beta J, \beta h) = \sum \{s\} e^{-\beta H}$; let’s measure $J, h$ in units of temperature, i.e. set $\beta = 1$.

Suppose that $N$ is even.

(a) Decimate the even sites:

$$\sum_{s_{\text{even}}} e^{-H(s)} \equiv \Delta e^{H_{\text{eff}}(s_{\text{odd}})}.$$ 

More explicitly, identify the terms in $H(s)$ that depend on any one even site, $H_2(s)$ and define its contribution to $H_{\text{eff}}$ by

$$\sum_{s_2} e^{-H_2(s)} \equiv \Delta e^{-H_{\text{eff}}(s_1,s_3)}$$

Rewriting $H_{\text{eff}}(s_{\text{odd}}) = -J'\sum ss - h'\sum s - \text{const}$ in the usual form, find $J', h'$ and the constant in terms of the microscopic parameters $J, h$.

(b) Let $w \equiv \tanh \beta J, v \equiv \tanh \beta h$. Plot some RG trajectories in the $v, w$ plane.

(c) Find all the fixed points and compute the exponents near each of the fixed points.

3. **High temperature expansion for Ising model.**

In lecture, we rewrote the partition function of the nearest-neighbor Ising model (on any graph) as a sum over closed loops. Without a magnetic field, the loops were weighted by their length, just like in our discussion of SAWs. If we turn on a magnetic field, how does it change the form of the sum?