MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
String Theory (8.821) – Prof. J. McGreevy – Fall 2007

Solution Set 7

D-branes and orientifolds

Reading: Polchinski, Chapter 13 and v. I pp. 189-192, 226-229, §8.8, v. II pp. 29-31, §10.8, §13.2; Uranga, chapter 10.

Due: Thursday, November 29, 2007 at 11:00 AM in lecture or in the box.

1. **BPSness and multiple branes.**

   (a) Which of the 32 supercharges of type II are conserved in the presence of a Dp-brane and a D(p+2k)-brane which lie along the 01...p and 01..(p+2k) coordinate axes, respectively? How much supersymmetry does the D0-D4 system preserve?

   A Dp-brane along the $m = 0...p$ coordinate directions preserves the supercharges of the form

$$Q^p \equiv Q_\alpha + (\beta_p^\perp \tilde{Q})_\alpha$$

   with

$$\beta_p^\perp \equiv \prod_{m=0}^{p} \beta^m.$$

   We can write the corresponding $\beta_{p+2k}^\perp$ for a D$(p+2k)$ brane as

$$\beta_{p+2k}^\perp = \beta_p^\perp \beta_p^{\perp\dagger} \beta_{p+2k}^\perp$$

   using the fact that β is unitary. The supercharges which are shared among Q^p and Q^{p+2k} are then those invariant under

$$\beta_p^{\perp\dagger} \beta_{p+2k}^\perp.$$

   We can diagonalize this by making complex pairs out of the 2k coordinates along the D$(p+2k)$-brane which are perpendicular to the Dp-brane,

$$\beta_p^{\perp\dagger} \beta_{p+2k}^\perp = e^{\pi i(s_1+...+s_k)},$$

with $s_\alpha = \pm\frac{1}{2}$. For $k = 1, 3, 5$, this is $e^{\pm i\pi/2} = \pm i$, and hence no Qs are shared. For $k = 2$, *e.g.* the D0-D4 system, this is ± 1 and half of the Qs (namely 8) are preserved.

(b) Consider on the worldvolume of $N > 1$ D4-branes a configuration of the gauge field F which carries a topological charge

$$\frac{1}{16\pi^2}\int_{\text{space}} \text{tr}_{\,N\times N} F \wedge F = 1$$

(here 'space' is meant to denote an integral over the four spatial dimensions of the D4-brane at fixed time); this quantity is called the 'instanton number' because in 4d gauge theory there exist finite-action solutions of the euclidean EOM which carry such charge[1].

Show that any such configuration carries the RR charge of one D0-brane. (Such configurations which preserve supersymmetry describe the condensation of the massless scalars arising from the 0-4 strings – on this *Higgs branch* of the moduli space, the D0-brane can be said to dissolve into the D4-brane.)

$$S_{D4} \supset i\mu_4 \int_5 \text{tr}\left(e^{2\pi\alpha' F_2 + B_2} \wedge \sum_{q=1,3,\ldots} C_q\right) \supset \frac{i}{2}(2\pi\alpha')^2\mu_4 \int \text{tr}\ F_2 \wedge F_2 \wedge C_1.$$

The charge of the fourbrane and of the zerobrane are related by

$$2(4\pi\alpha')^2\mu_4 = \mu_0$$

So a configuration of the worldvolume gauge fields on the D4-brane where

$$\text{tr}\ F \wedge F = 8\pi^2$$

will carry the RR charge of the D0-brane.

(c) [More optional than usual] Show that if we take a pair of coincident Dp-branes and rotate one relative to the other, the amount of preserved supersymmetry is determined by whether the rotation matrix can be written as an element of a subgroup of $SO(n)$ which preserves a spinor.

[1] There are of course many configurations which carry the charge but do not solve the equations of motion. The gauge field configurations which actually describe instantons are (anti-)self-dual: $F = \pm \star F$. The instanton number is also called the 'second Chern class' or 'first Pontryagin class'.

Consider the case when the branes share 3+1 dimensions, so that the relative rotation M is in general an element of $SO(6)$. Show that if $M \in SU(3)$ but $M \notin SU(2)$, the spectrum contains chiral matter under the *relative* $U(1)$ which couples to the strings stretched between the two branes.

If you get stuck, see BDL `hep-th/9606139`.

The rotation matrix relating the branes acts just like a holonomy matrix on the spinors. If it's in a subgroup of $SU(3)$, there will be an invariant spinor by the same argument we used to see that CY compactification preserves a spinor.

2. **Dp-D(p+2) system.**

(a) In type II string theory in flat space, consider a Dp-brane and a D(p+2)-brane which are parallel in $p+1$ flat dimensions. Show that the spectrum of strings stretching between them includes a tachyon if they are close enough together.

The NS zero-point energy for a string stretched between a Dp-brane and a D$(p + \#_{ND})$ brane sharing $p+1$ dimensions is

$$E_0^{NS} = -\frac{1}{2} + \frac{\#_{ND}}{8}$$

[Polchinski (13.4.8)]. For this situation, $\#_{ND} = 2$ and we get

$$E_0^{NS} = -\frac{1}{4}.$$

There is therefore a tachyon in this sector if the branes are close enough. The mass of the ground state is

$$m^2 = \left(\frac{(\delta x)^2}{2\pi\alpha'}\right)^2 - \frac{1}{4\alpha'}.$$

(b) Make a conjecture for the endpoint of the condensation of this tachyon.

Hint:

$$S_{Dp} \supset \int_{Dp} \sum_q C_{RR}^{(q)} \wedge \mathrm{tr}\, e^{aF} = \int_{Dp} \left(C_{RR}^{(p+1)} + a \mathrm{tr}\, F \wedge C_{RR}^{(p-1)} + ... \right)$$

where a is some numerical stuff with factors of α'.

On the worldvolume of the D(p+2)-brane, there are terms

$$S_{D(p+2)} \subset \mu_{p+2} \int \left(C_{RR}^{(p+3)} + C_{RR}^{(p+1)} 2\pi\alpha' \text{tr } F + ... \right)$$

so that magnetic flux tr $F = 2\pi$ **on the D(p+2) worldvolume carries Dp charge. This configuration can be shown to have lower energy than the pair of Dp and D(p+2) branes.**

3. **O-plane charge.**

   How many Dp-branes cancel the RR charge of an Op$^+$ plane (the SO kind)?

   Hint: consider what happens when you T-dualize type I, and notice that the number of connected components of the fixed locus of $x^i \to -x^i, \quad i = 1..q$, with x^i periodic, is 2^q.

   [OK, this is an uninspired question, but I didn't get to mention it in lecture.]

   Compactifying type I on an n**-torus, the action** Ω **T-dualizes to an action**

   $$\Omega(-1)^{nF_L}\mathcal{I}_n$$

   where

   $$\mathcal{I}_n : x^i \to -x^i, \qquad i = 1..n.$$

   This map has 2^n **fixed points on the** n**-torus, each of which is an O(9-n)-plane. The 16 D9-branes of type I turn into 16 D(9-n)-branes. Note that there are 16 D9-branes in the sense that the** *rank* **of** $SO(32)$ **is 16, and hence at a generic point in the moduli space of wilson lines,** $SO(32)$ **is broken down to** $U(1)^{16}$**. Therefore the RR charge of 16 D(9-n) branes cancels the charge of** 2^n **O(9-n) branes. We have**

   $$2^n \mu_{Op} = -16\mu_p$$

   $$\mu_{Op} = -2^{4-(9-p)}\mu_p = -2^{p-5}\mu_{Dp}.$$

   Note, for example, that this means that an O3-plane carries the charge of a quarter of a D3-brane.

4. **A string duality.**

   Consider the worldvolume theory of a D-string in the type I theory in ten flat dimensions. (Recall that unlike the type I F-string, the D-string carries a charge because the RR 2-form potential survives the orientifold projection.) Stretch the D1-brane along the 01 directions. What is the spectrum of 11-strings after performing the orientifold projection? Recall that the type I vacuum contains 32 D9-branes; what is the spectrum of 1-9 strings?

Show that the orientifold projection leaves a $\mathbb{Z}_2$ subgroup of the worldvolume gauge group which acts on the 1-9 strings.

Where have you seen this theory before?[2]

Put the D1-brane along $x^{0,1}$ at $x^2 = ... = x^9 = 0$.

Consider first the sector of 1-1 strings, *i.e.* those with both ends on the D1-brane. Before the orientifold, they give the spectrum of the D1-brane in type IIB, which is the dimensional reduction of the 4d $\mathcal{N} = 4$ $U(1)$ gauge theory to two dimensions.

Ω acts differently on the 01 oscilators (NN) and the $2..9$ oscillators (DD); this is because worldsheet orientation reversal acts with a -1 on the tangential derivative to the boundary (and hence on the NN vertex operators), but with a $+1$ on the normal derivative (and on the DD vertex operators). This means that the tranvserse scalars $X^{2..9}$ which move the D1-brane around survive, but the gauge field does not. The holonomy of the gauge field $e^{i\oint A}$ is mapped to $e^{-i\oint A}$ by the action of Ω. A $\mathbb{Z}_2$ survives, which acts on charge 1 fields with a -1; this is a $\mathbb{Z}_2$ gauge symmetry.

Of the fermions, only the $\mathbf{8}_s$ survives the Ω projection. The action of Ω on the α^μ is related by worldsheet susy to the action on ψ^μ – this amounts to an extra -1 in the action on $\psi^{2..9}$ relative to those in the 9-9 sector. This means that on the spinor ground states $|s_0, s_1 ... s_4\rangle$, Ω acts as

$$\Omega = -e^{\pi i (s_1+s_2+s_3+s_4)}.$$

The $\mathbf{8}_s$ is the representation where $\sum_{a=1}^{4} s_a$ is even.

Now consider strings stretched between the D1 and the D9. Before the orientifold, there are independent 1-9 strings and 9-1 strings in the $(-, N)$ and $(+, \bar{N})$ of the $U(1) \times U(N)$ gauge symmetry. These are mapped to each other by Ω (and at the same time $U(N = 32)$ gets broken to $SO(32)$).

For $\#_{ND} = 8$, there are no massless particles in the NS sector from the formula used already in problem two. The R-ground state gives

[2] This wonderful observation is from `hep-th/9510169`. See also JP §14.3.

a chiral fermion, of the opposite D-string worldsheet chirality from the $\mathbf{8}_s$ in the 11-sector, in the $(+, \mathbf{32})$ of $\mathbb{Z}_2 \times SO(32)$, ($i.e.$ the $+$ is meant to denote the fact that it transforms with a minus sign under the residual $\mathbb{Z}_2$ gauge group on the D1).

This spectrum is exactly that of an SO(32) heterotic string stretched along $x^{0,1}$, including the GSO projection.