8.821 Lecture 01: What you need to know about string theory

Lecturer: McGreevy Scribe: McGreevy

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Since I learned that we would get to have this class, I've been torn between a) starting over with string perturbation theory and b) continuing where we left off. Option a) is favored by some people who didn't take last year's class, and by me when I'm feeling like I should do more research. But because of a sneaky and perhaps surprising fact of nature and the history of science there is a way to do option b) which doesn't leave out the people who missed last year's class (and is only not in the interest of the lazy version of me). The fact is this. It is actually rare that the structure of worldsheet perturbation theory is directly used in research in the subject that is called string theory. And further, one of the most important developments in this subject, which is usually called, synecdochically, the AdS/CFT correspondence, can be discussed without actually using this machinery. The most well-developed results involve only classical gravity and quantum field theory.

So here's my crazy plan: we will study the AdS/CFT correspondence and its applications and generalizations, without relying on string perturbation theory.

Why should we do this? You may have heard that string theory promises to put an end once and for all to that pesky business of physical science. Maybe something like it unifies particle physics and gravity and cooks your breakfast. Frankly, in this capacity, it is at best an idea machine at the moment.

But this AdS/CFT correspondence, whereby the string theory under discussion lives not in the space in which our quantum fields are local, but in an auxiliary curved extra-dimensional space (like a souped-up fourier transform space), is where string theory comes the closest to physics. The reason: it offers otherwise-unavailable insight into strongly-coupled field theories (examples of which: QCD in the infrared, high-temperature superconductors, cold atoms at unitarity), and into quantum gravity (questions about which include the black-hole information paradox and the resolution of singularities), and because through this correspondence, gauge theories provide a better description of string theory than the perturbative one. The role of string theory in our discussion will be like its role in the lives of practitioners of the subject: a source of power, a source of inspiration, a source of mystery and a source of vexation.

The choice of subjects is motivated mainly by what I want to learn better. After describing how to do calculations using the correspondence, we will focus on physics at finite temperature. For what I think will happen after that, see the syllabus on the course webpage. Suggestions in the spirit described above are very welcome.

ADMINISTRIVIA

– please look at course homepage for announcements, syllabus, reading assignments
– please register
– **I promise to try to go more slowly than last year.**

– **coursework:**
1. psets. less work than last year. hand them in at lecture or at my office.
pset 0 posted, due tomorrow (survey).

2. scribe notes. there's no textbook. this is a brilliant idea from quantum computing. method of assigning scribe TBD. as you can see, I am writing the scribe notes for the first lecture.

3. end of term project: a brief presentation (or short paper) summarizing a topic of interest. a list of candidate topics will be posted.
goal: give some context, say what the crucial point is, say what the implications are.
try to save the rest of us from having to read the paper.
(benefits: you will learn this subject much better, you will have a chance to practice giving a talk in a friendly environment)

Next time, we will start from scratch, and motivate the shocking statement of AdS/CFT duality without reference to string theory. It will be useful, however, for you to have some big picture of the epistemological status of string theory. Today's lecture will contain an unusually high density of statements that I will not explain. I explained many of them last fall; experts please be patient.

What you need to know about string theory for this class:

1) It's a quantum theory which
at low energy and low curvature [1] reduces to general relativity
coupled to some other fields
plus calculable higher-derivative corrections.

2) It contains D-branes. These have Yang-Mills theories living on them.

We will now discuss these statements in just a little more detail.

0.1 how to do string theory (textbook fantasy cartoon version)

Pick a background spacetime $\mathcal{M}$, endowed with a metric which in some local coords looks like

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \quad ;$$

there are some other fields to specify, too, but let's ignore them for now.

[1] compared to the string scale M_s, which we'll introduce below

Consider the set of maps

$$\begin{aligned} X^\mu : \text{worldsheet} &\rightarrow \text{target spacetime} \\ \Sigma &\rightarrow \mathcal{M} \\ (\sigma, \tau) &\mapsto X^\mu(\sigma, \tau) \end{aligned}$$

where σ, τ are local coordinates on the worldsheet. Now try to compute the following kind of path integral

$$I \equiv \int [DX(\sigma, \tau)]_\star \exp\left(iS_{ws}[X..]\right).$$

This is meant to be a proxy for a physical quantity like a scattering amplitude for two strings to go to two strings; the data about the external states are hidden in the measure, hence the $\star$. The subscript 'ws' stands for 'worldsheet'; more on the action below.

An analog to keep firmly in mind is the first quantized description of quantum field theory. The Feynman-Kac formula says that a transition amplitude takes the form

$$\int_{x(\tau_1)=x_1}^{x(\tau_2)=x_2} e^{iS_{wl}[x(\tau)]} = \langle x_2, \tau_2 | x_1, \tau_1 \rangle \tag{1}$$

Here

$$\begin{aligned} x^\mu : \text{worldline} &\rightarrow \text{target spacetime} \\ (\tau) &\mapsto x^\mu(\tau). \end{aligned}$$

can be written as a sum over trajectories interpolating between specified inital and final states. Here $x(\tau)$ is a map from the world line of the particle into the target space, which has some coordinates x^μ. For example, for a massless charged scalar particle, propagating in a background spacetime with metric $g_{\mu\nu}$ and background (abelian) gauge field A_μ, the worldline action takes the form

$$S_{wl}[x] = \int d\tau g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu + \int d\tau \dot{x}^\mu A_\mu$$

where $\dot{x} \equiv \partial_\tau x$. Note that the minimal coupling to the gauge field can be written as

$$\int d\tau \dot{x}^\mu A_\mu = \int_{wl} A$$

where the second expression is meant to indicate an integral of the one-form A over the image of the worldline in the target space (The expression on the LHS is often called 'the pullback to the worldline'. Those are words.). A few comments about this worldline integral.

1) The spacetime gauge symmetry $A \rightarrow A + d\Lambda$ requires the worldline not to end, except at a place where we've explicitly stuck some non-gauge-invariant source (x_1, x_2 in the expression above).

2) The path integral in (1) is that of a one-dimensional QFT with scalar fields $x^\mu(\tau)$, and coupling constants determined by $g_{\mu\nu}, A_\mu$. To understand this last statement, Taylor expand around some point in the target space, which let's call 0. Then for example

$$A_\mu(x) = A_\mu(0) + x^\rho \partial_\rho A_\mu(0) + ...$$

and the coupling to the gauge field becomes

$$\int d\tau \left(\dot{x}^\mu A_\mu(0) + \dot{x}^\mu x^\rho \partial_\rho A_\mu(0) + ...\right)$$

so the Taylor coefficients are literally coupling constants, and an infinite number of them are encoded in the background fields g, A.

After all this discussion of the worldline analog, and before we explain more about the $\star$, why might it be good to make this replacement of worldlines with worldsheets? Good Thing number 1: many particles from vibrational modes. From far away, meaning when probed at energies $E \ll M_s$, they look like particles. This suggests that there might be some kind of unification, where all the particles (different spin, different quantum numbers) might be made up of one kind of string.

To see other potential Good Things about this replacement of particles with strings, let's consider how interactions are included. In field theory, we can include interactions by specifying initial and final particle states (say, localized wavepackets), and summing over all ways of connecting them, *e.g.* by position-space feynman diagrams. To do this, we need to include all possible interaction vertices, and integrate over the spacetime points where the interactions occur. UV divergences arise when these points collide.

To do the same thing in string theory, we see that we can now draw smooth diagrams with no special points (see figure). For closed strings, this is just the sewing together of pants. A similar procedure can be done for open strings (though in fact we must also attach handles there). So, Good Thing Number Two is the fact that one string diagram corresponds to many feynman diagrams. Next lets ask

Q: where is the interaction point?

A: nowhere. It's in different places in different Lorentz frames. This is a first (correct) indication that strings will have soft UV behavior (Good Thing Number Three). They are floppy at high energies.

The crucial point: near any point in the worldsheet (contributing to some amplitude), it is just free propagation. This implies a certain uniqueness (Good Thing Number Four): once the free theory is specified, the interactions are determined. The structure of string perturbation theory is very rigid; it is very diffcult to mess with it (*e.g.* by including nonlocal interactions) without destroying it. [2]

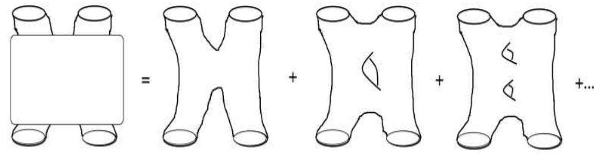
BUT: Only for some choices of $(\mathcal{M}, \star)$ can we even define I. To see why, let's consider the simplest example where $\mathcal{M} = \mathbb{R}^{D-1,1}$ with $g_{\mu\nu} = \eta_{\mu\nu} \equiv \mathrm{diag}(-1, 1, 1, 1...1)$. Then the worldsheet action is

$$S_{ws}[X] = \frac{1}{4\pi\alpha'} \int d^2\sigma \partial_\alpha X^\mu \partial^\alpha X^\nu \eta_{\mu\nu}. \tag{2}$$

Here $\alpha = 1, 2$ is a worldsheet index and $\mu = 0...D-1$ indexes the spacetime coords.

The quantity $\frac{1}{4\pi\alpha'}$ has dimensions of length^{-2}. It is the *tension* of the string, because it suppresses configurations with large gradients of X^μ; when it is large, the string is stiff. This is the mass scale

[2] Why stop at one-dimensional objects? Why not move up to membranes? It's still true that the interaction points get smoothed out. But the divergences on the worldvolume are worse. $d = 2$ is a sort of compromise. In fact, some sense can be made of the case of 3-dimensional base space, using the BFSS matrix theory. They turn out to be already 2d quantized! The spectrum is a continuum, corresponding to the degree of freedom of bubbling off parts of the worldvolume. It's not entirely clear to me what to make of this.



Crudely rendered perturbative interactions of closed strings. Time goes upwards. Slicing this picture by horizontal planes shows that these are diagrams which contribute to the path integral for 2 to 2 scattering. Tilting the horizontal plane corresponds to boosting your frame of reference and moves the point where you think the splitting and joining happened.

alluded to twice so far:

$$M_s^2 \equiv \frac{1}{4\pi\alpha'};$$

people will disagree about the factors of 2 and π here.

(2) is the action for free bosons. We can compute anything we want about them. The equations of motion $0 = \frac{\delta S_{ws}}{\delta X}$ give the massless 2d wave equation:

$$0 = \left(-\partial_\tau^2 + \partial_\sigma^2\right) X^\mu = \partial_+ \partial_- X^\mu$$

where $\sigma^\pm \equiv \tau \pm \sigma$. This is solved by superpositions of right-movers and left-movers:

$$X^\mu(\sigma, \tau) = X_L^\mu(\sigma^+) + X_R^\mu(\sigma^-).$$

To study closed strings, we can treat the spatial direction of the worldsheet as a circle $\sigma \equiv \sigma + 2\pi$, in which case we can expand in modes:

$$X^\mu(\sigma, \tau) = \sum_{n \in \mathbb{Z}} \alpha_n^\mu e^{in\sigma^+} + \sum_{n \in \mathbb{Z}} \tilde{\alpha}_n^\mu e^{in\sigma^-}.$$

For open strings, the boundary conditions will relate the right-moving and left-moving modes:

$$\tilde{\alpha} \propto \alpha.$$

We can quantize this canonically, and find

$$[\alpha_n^\mu, \alpha_m^\nu] = n\delta_{n+m}\eta^{\mu\nu}, \quad [\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu] = n\delta_{n+m}\eta^{\mu\nu}, [\alpha, \tilde{\alpha}] = 0.$$

This says that we can think of $\alpha_{n<0}$ as creation operators and $\alpha_{n>0}$ as annihilation operators. Let $|0\rangle$ be annihilated by all the $\alpha_{n>0}$. The zeromodes $\alpha_0, \tilde{\alpha}_0$ are related to the center-of-mass position and momentum p^μ of the string. The worldsheet hamiltonian is

$$H_{ws} = \alpha' p^2 + \sum_{n>0} \alpha_{-n}^\mu \alpha_{n\ \mu} - a,$$

where a is a 'normal-ordering' constant, which turns out to be 1.

Here comes the problem: consider the state $|BAD\rangle \equiv \alpha_{-n}^{\mu=0}|0\rangle$ for $n > 0$. Because of the commutator $[\alpha_n^0, \alpha_{-n}^0] = -n$, we face the choice

$$\||BAD\rangle\|^2 < 0 \quad \text{or} \quad E_{BAD} < 0.$$

Neither is OK, especially since E_{BAD} becomes arbitrarily negative as n gets larger.

This kind of problem is familiar from QED or Yang-Mills theory, where the time component of the gauge field presents with very similar symptoms. We need to gauge some symmetry of the theory to decouple the negative-norm states.

One way to do this is to gauge the 2d conformal group. We're going to talk a lot more about conformal symmetry soon, so I'll postpone even the definition. There are fancy motivations for this involving 2d gravity on the worldsheet that we are not going to talk about now. Of course, like any gauge symmetry, this symmetry is a fake, and one could imagine formulations of the theory wherein it is never introduced. One necessary condition arising from this is that the worldsheet QFT needs to be a conformal field theory (CFT). This is *not* the CFT of the AdS/CFT correspondence.

The generators of the conformal group include, in particular, the hamiltonian, so we must impose

$$0 = H_{ws}|phys\rangle$$

as a constraint, like Gauss' law in QED. Acting on a state with N oscillator excitations, this equation takes the form

$$0 = \alpha' p^2 + N - 1 \propto p^2 + m^2,$$

which looks like an on-shell condition for a particle in minkowski space. This tells us the target-space mass of a string state in terms of its worldsheet data. Now we can build physical states[3]. The result looks like

STRING STATE	FIELD	MASS^2
$\|0\rangle$	scalar $\mathcal{T}$	$m^2 = -M_s^2$
$\alpha_{-1}^{\{\mu}\tilde{\alpha}_{-1}^{\nu\}}\|0\rangle$	sym tensor $g_{\mu\nu}$	$m^2 = 0$
$\alpha_{-1}^{[\mu}\tilde{\alpha}_{-1}^{\nu]}\|0\rangle$	AS tensor $B_{\mu\nu}$	$m^2 = 0$
$\alpha_{-1}^{\mu}\tilde{\alpha}_{-1\ \mu}\|0\rangle$	scalar Φ	$m^2 = 0$
$\alpha_{-1}^{\mu_1}...\alpha_{-1}^{\mu_{n-1}}\tilde{\alpha}_{-n}^{\mu_n}\|0\rangle$	$T_{\mu_1...\mu_n}$	$m^2 = nM_s^2$
...	...	...

(where [..] is meant to indicate antisymmetrization of indices, {..} is meant to indicate symmetrization of indices). The scalar Φ is called the dilaton. For open strings, we would have, among other states,

STRING STATE	FIELD	MASS^2
$\alpha_{-1}^{\mu}\|0\rangle$	vector A_μ	$m^2 = 0$

[3]Theres one important constraint I didnt mention, called *level-matching*, which says that the number of right-moving excitations N and the number of left-moving excitations $\tilde{N}$ should be equal. This follows from demanding that there be no special point on the worldsheet.

The worldsheet gauge symmetry leads directly to the spacetime gauge symmetries under which *e.g.*

$$A \to A + d\Lambda^{(0)}$$

and the two-form

$$B \equiv B_{\mu\nu} dx^\mu \wedge dx^\nu \to B + d\Lambda^{(1)}.$$

There's a general theorem that a massless symmetric tensor like $g_{\mu\nu}$ which interacts must couple to stress energy, and hence is a graviton. Using the machinery built from the above picture, this mode can be seen to participate in nontrivial scattering amplitudes, which indeed reduce to those computed from GR at low energies. So string theory, however unwieldy this description, is a quantum theory of gravity.

0.2 the spacetime effective action

Almost every string theory result goes through the *spacetime* effective action:

$$S_{st}[g, B, \Phi ...] \sim \int d^D x \sqrt{g} e^{-2\Phi} \left[\mathcal{R}^{(D)} + (\partial\Phi)^2 + H^2 + ... \right]$$

where $H \equiv dB$ *i.e.*$H_{\mu\nu\rho} \equiv \partial_{[\mu} B_{\nu\rho]}$, $\mathcal{R}^{(D)}$ is the Ricci scalar made from $g_{\mu\nu}$, and more about the dots below. This action is correct in two a priori different ways.
1) It reproduces scattering amplitudes of the fluctuations $\delta g_{\mu\nu}, \delta B_{\mu\nu}, \delta\Phi ...$ computed by the worldsheet path integral, *at leading order* in $\alpha' E^2$, where E is say the biggest energy of a state involved in the scattering event. That is to say, that the effective action is the leading term in a derivative expansion, where the higher-dimension operators are suppressed by the appropriate number of powers of α'; the coefficients of the corrections are definite and computable.

2) To see the second way of thinking about S_{st}, consider a string propagating in a background of $g_{\mu\nu}, B_{\mu\nu} ...$ Its action is

$$S_{ws}[X] = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} \left[g_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu \gamma^{\alpha\beta} + B_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu \epsilon^{\alpha\beta} + \Phi(X) \mathcal{R}^{(2)}_\gamma + ... \right] \quad (3)$$

Here, $\gamma_{\alpha\beta}$ is an auxiliary metric on the worldsheet that we can introduce for convenience using the gauge symmetry.

This is a 2d QFT with fields X^μ. The bosons X^μ are free is the spacetime is flat, and the other fields are trivial. Otherwise, they interact, and the coupling constants are determined by the background fields, like on the worldline above. If the target space is nearly flat, we can treat these interactions perturbatively. The effective $\hbar$, *i.e.* strength of interaction, is determined by $\sim \alpha' \mathcal{R}^{(D)}$. In general, these coupling constants will run with scale. A necessary condition for this QFT to be a CFT is that the beta functions for all the coupling constants vanish. It turns out that the leading-order beta functions for $g_{\mu\nu}, B_{\mu\nu}, \Phi$ are exactly the equations of motion arising from varying the spacetime effective action S_{st}!

We can learn some more simple lessons from the *worldsheet* action (3).
1) The worldsheet metric is innocuous[4]. If you tried to take it seriously and give it some dynamics, you would try to add the Einstein-Hilbert term,

$$\Delta S_{ws} = \int \sqrt{\gamma}\mathcal{R}^{(2)}.$$

But this term, which is already present, with coefficient determined by the spacetime zeromode Φ_0 of the dilaton field Φ, is topological – it is the Euler characteristic of the worldsheet. On a worldsheet Σ with h handles (genus h) and b boundaries, it evaluates to

$$\int \sqrt{\gamma}\mathcal{R}^{(2)} = \chi(\Sigma) = 2 - 2h - b.$$

Letting $g_s \equiv \ln\langle\Phi_0\rangle$, this means that the contribution to the sum over worldsheets of worldsheets with h splittings and joinings is proportional to

$$g_s^{2h-2}.$$

Thus g_s is the quantity which determines how much strings want to interact, the string coupling. It is the $\hbar$ in spacetime. This means that the effective action above is also the leading term in an expansion in powers of g_s. What I wrote above is the *tree-level* effective action. A more correct expression is

$$S_{st}[g, B, \Phi ...] \sim \int d^D x \sqrt{g} e^{-2\Phi} \left[\mathcal{R}^{(D)} + (\partial\Phi)^2 + H^2 + ...\right] \left(1 + \mathcal{O}(\alpha')\right) \left(1 + \mathcal{O}(g_s^2)\right)$$

2) Recall that on the worldline with coupling $\int d\tau A_\mu \dot{x}^\mu = \int A$, gauge invariance under $A \to A + d\Lambda^{(0)}$ implied that the worldline was not allowed to end randomly, and that the worldline was a monopole source for the gauge field A. The coupling of the string to the AS tensor can similarly be written as a minimal coupling:

$$\int d\sigma d\tau \partial_\sigma X^\mu \partial_\tau X^\nu B_{\mu\nu} = \int_{X(\Sigma)} B.$$

The gauge invariance

$$B \to B + d\Lambda^{(1)}$$

($\Lambda^{(1)}$ is an arbitrary, well-behaved one-form in the target space) implies that the worldsheet cannot end just anywhere. More on this soon. It also means that fundamental strings are charged under the AS tensor, which is usually called the NSNS B-field, or Kalb-Ramond field.

[To be continued]

[4]modulo the issue of the conformal anomaly