

Problem Set 6
Mostly thermodynamics at strong coupling

Due: Never.

Note: These problems fluctuate wildly in difficulty.

1. **3/4.**

Convince yourself that in the $\mathcal{N} = 4$ SYM,

$$\frac{F(\lambda \rightarrow \infty)}{F(\lambda \rightarrow 0)} = \frac{3}{4},$$

where F is the (Helmholtz, *i.e.* canonical ensemble) free energy and λ is the 't Hooft coupling.

2. **A cube's worth of fields?**

Compute the Bekenstein-Hawking entropy of the planar black hole in $AdS_7 \times S^7$. The metric is

$$ds^2 = \frac{L_{AdS}^2}{z^2} \left(-f dt^2 + d\vec{x}^2 + \frac{dz^2}{f} \right) + L_{S^4}^2 d\Omega_4^2,$$

with

$$f = 1 - \frac{z^6}{z_H^6}.$$

The parameters are related to those of the theory of M5-branes by:

$$L = L_{AdS} = 2L_{S^4} = 2l_p(\pi N)^{\frac{1}{3}}$$

where l_p is the eleven-dimensional Planck length, $l_p^9 = 16\pi G_N$. Express the entropy in terms of the BH temperature and the number N of M5-branes.

3. **Warming your feet on the Hawking fire.**

How big a black hole do you need to boil water (at 1 atm, say) with its Hawking radiation?

4. Holographic stress tensor

Using the usual AdS/CFT prescription, compute the expectation value of the stress-energy tensor of the $\mathcal{N} = 4$ theory at finite temperature, at large 't Hooft coupling. Show that the trace vanishes, as it must by the conformal Ward identity.

[If you get stuck, see Balasubramanian-Krauss, hep-th/9902121.]

5. Screening length.

Consider the quark-antiquark potential in $\mathcal{N} = 4$ SYM at large λ, N . Compute the screening length, r_* , the separation of quark and antiquark at which the (regulated) action of the straight-string solution equals that of the hanging string.

[If you get stuck, see hep-th/9803137, hep-th/9803135, 0807.4747.]

6. Confinement and monopole condensation.

There is a beautiful qualitative picture of confinement [due to Polyakov, 't Hooft, Mandelstam] as a *dual Higgs mechanism*, meaning that it should involve the condensation of magnetically charged objects. (This has been explicitly demonstrated in examples by Seiberg-Witten theory.) In 4d YM, this means condensation of magnetic monopoles, which means that magnetic charge should be *screened* in the confining vacuum: the force between magnetically charged object should be independent of distance beyond some minimum distance related to the confinement scale.

Consider the model of confinement obtained from D4-branes on a thermal circle.

a) What is the string theory description of an external magnetic monopole at strong coupling in this model? (Hint: recall pset 1.)

b) Show that magnetic charge is screened by giving a strong-coupling estimate of the force between two such static external monopoles.

[If you get stuck, look in MAGOO §6.]

7. Giant gravitons.

Consider a D3-brane wrapping a (contractible) S^3 whipping around the S^5 of the gravity dual of the $\mathcal{N} = 4$ theory. Using the parametrization given in lecture 19, show that the angular momentum of the brane becomes precisely $\ell = \frac{N}{L}$ when the brane reaches its maximum size.

8. Area theorem in AdS?

The event horizon is a global notion that depends strongly on the asymptotics of the space in which the black hole sits. Prove a version of the Area Theorem for asymptotically AdS spaces. The definition of event horizon for asymptotically AdS spaces, I believe, is (brace yourself): the boundary of (the past of (the intersection of (future timelike infinity) with (the conformal boundary of AdS))).

[I am not sure if this has been done. Please tell me if you find a reference or a proof!]

9. Wald entropy and second law?

In Einstein gravity, the entropy of a black hole is given by the area of its horizon in Planck units (over four). The Einstein-Hilbert term is the leading irrelevant operator in an effective field theory; a microscopic theory of gravity such as a string vacuum predicts definite coefficients for the higher-derivative corrections. In such a more general theory of gravity, where the gravitational action can be written in terms of a lagrangian depending on tensors made from the metric,

$$I_{grav} = \int L(g_{ab}, R_{abcd})$$

a path integral argument shows that the horizon area is replaced by a more general integral over the horizon, often called the Wald entropy. In the case of gravity in four dimensions, the Wald entropy takes the form:

$$S = -2\pi \int_{\text{horizon}} \frac{\partial L}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd}$$

By demanding that the resulting Wald entropy satisfy a second law of thermodynamics, constrain the coefficients of higher-curvature terms in the lagrangian of a sensible effective gravity theory.

[For help, see Jacobson-Kang-Myers, gr-qc/9503020. I believe this is the state-of-the-art on this question.]