University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Field Theory C (215C) Spring 2013
Assignment 0

Posted April 1, 2013 **Due 11am April 4, 2013**

Please remember to put your name at the top of your homework.

Announcements

- The 215C web site is:

    http://physics.ucsd.edu/∼mcgreevy/s13.

Problem Set 0

1. There is still some room for flexibility in the planning of this class. In order to help maximize the general happiness, I need some information from you. Strive for brevity, please.

    Below is a list of facts and statements about quantum field theory (QFT) which one might learn in the first two quarters of a sequence of QFT classes. Please rate your level of intimacy with each of them according to the following rough system:

    1   'I've never heard of it.'
    3   'I ran into it once on the street on a rainy weekend evening in 1992.'
    5   'I tried it a few times, but it made my neck hurt.'
    8   'I ate it for breakfast this morning.'
    10   'I wrote a 900-page book about it.'
    It will help to keep in mind that the goal here is not to impress, but to convey what it is you need to learn.

    (a) The loop expansion in QFT perturbation theory is an expansion in powers of $\hbar$.

    (b) In quantum Maxwell theory (*i.e.* QED without the electrons), the electric field is canonically conjugate to the vector potential.

    (c) The quantum Maxwell field in a box is a bunch of coupled harmonic oscillators. The oscillators labelled by wavenumber decouple from each other.

    (d) The energy of the vacuum in free quantum field theory gets contributions from the groundstate energies of each of these oscillators Changing the boundary conditions on the field, we change the allowed modes; this results in a finite, measurable force on the walls of the box.

(e) A free quantum field is like a mattress. Phonons are a pretty literal realization of this statement.

(f) We can think of the Coulomb force between electric charges as arising from the exchange of virtual photons with spacelike momenta. This momentum $\vec{k}$ is the integration variable in the Coulomb potential created by a static charge at $r^2 \equiv \vec{x} \cdot \vec{x} = 0$:
$$\int d^3 k\, e^{i\vec{k}\cdot\vec{x}} \propto \frac{1}{r}$$

(g) The sum of the disconnected Feynman diagrams contributing to some process equals the exponential of the connected Feynman diagrams associated with the same process.

(h) Noether's theorem says a continuous symmetry implies a conserved charge. It has a converse: A conserved charge Q generates the associated symmetry, in the sense that the variation of any field ϕ is of the form

$$\delta\phi = \epsilon[Q, \phi].$$

Classically, this is accomplished by Poisson brackets; quantumly this is accomplished by commutators.

(i) The Ward identity associated with gauge invariance guarantees that the photon remains massless in interacting QED.

(j) In order for the photon to acquire a mass, it would have to gain an extra degree of freedom somehow, associated with the 'longitudinal' polarization state with $\vec{k} \cdot \vec{A}(k) \neq 0$.

(k) The generator of Lorentz transformations on a Dirac spinor field is

$$\mathbf{J}^{\mu\nu} = \mathbf{L}^{\mu\nu} + \mathbf{\Sigma}^{\mu\nu}$$

where $\mathbf{L}^{\mu\nu} = \mathbf{i}\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right)$ is the orbital part and $\mathbf{\Sigma}^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$ acts on the spinor indices.

(l) In 2+1 dimensions we can use the Pauli sigma matrices for gamma matrices.

(m) In a field theory with massless particle excitations, such as QED, IR divergences arising from loops of the massless particles precisely cancel in physical observables with divergences arising from the emission of low frequency particles in the final state. IR divergences only occur in unmeasurable exclusive amplitudes where we artificially specify the number of massless particles in the final state.

(n) Quantum field theories which are not renormalizable are still useful.

2. Complete this sentence as many times as is applicable: I would be disappointed if June 2013 arrived and I still didn't know about ...