University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Field Theory C (215C) Spring 2013
Assignment 3

Posted April 23, 2013 **Due 11am, Thursday, May 2, 2013**

Relevant reading: Zee, Chapter IV.3. Zee, Chapter III.3 also overlaps with recent lecture material.

Problem Set 3

1. **Propagator corrections in a solvable field theory.**

   Consider a theory of a scalar field in D dimensions with action

   $$S = S_0 + S_1$$

   where

   $$S_0 = \int d^D x \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi - m_0^2 \phi^2 \right)$$

   and

   $$S_1 = - \int d^D x \frac{1}{2} \delta m^2 \phi^2 \ .$$

   We have artificially decomposed the mass term into two parts. We will do perturbation theory in small δm^2, treating S_1 as an 'interaction' term. We wish to show that the organization of perturbation theory that we've seen lecture will correctly reassemble the mass term.

   (a) Write down all the Feynman rules for this perturbation theory.

   (b) Determine the 1PI two-point function in this model.

   (c) Show that the (geometric) summation of the propagator corrections correctly produces the propagator that you would have used had we not split up $m_0^2 + \delta m^2$.

2. **Coleman-Weinberg potential.**

   (a) [Zee problem IV.3.4] What set of Feynman diagrams are summed by the Coleman-Weinberg calculation?

   [Hint: expand the logarithm as a series in V''/k^2 and associate a Feynman diagram with each term.]

(b) [Zee problem IV.3.3] Consider a massless fermion field coupled to a scalar field ϕ by a coupling $g\phi\bar{\psi}\psi$ in $D = 1+1$. Show that the one loop effective potential that results from integrating out the fluctuations of the fermion has the form

$$V_F = \frac{1}{2\pi}\,(g\phi)^2 \log\left(\frac{\phi^2}{M^2}\right)$$

after adding an appropriate counterterm.