Confinement and the Fate of Quantum Spin Liquids

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In this paper I will review the methods of constructing and understanding quantum spin liquids: projective constructions and their emergent gauge theories. Such constructions involve the separation of electronic degrees of freedom into uncharged spin (spinon) and charged spinless (chargon) quasiparticles. The physicality of these degrees of freedom is intimately connected to the role of confinement when coupled with the gauge field describing their dynamics. Focus will be on the case of an emergent U(1) gauge theory in $D = 2 + 1$.

INTRODUCTION

Studying strongly-correlated systems is hard. For some direction of the causality, this also implies they are very interesting. These systems often entail very strange behavior which, on some good days, may even be shown to exist in our world. One such example I would like to discuss is the 'quantum spin liquid'. [15]

It is a 'liquid' because, despite whatever sadistic reductions in temperature or application of pressure, it remains in a disordered state and does not undergo any phase transition. It is 'spin' simply because it is usually a system of interacting spins and this spin-rotational symmetry remains preserved. Finally it is 'quantum' because explaining why such a thing could ever exist involves quantum mechanics.

A good question to ask then is, does a such a thing exist? Maybe Herbertsmithite.[7] For those of us content with lattice simulations the answer is yes. Another question to ask, why should we care? Much more easily answered! Spin liquids carry unique excitations, such as a neutral spin-1/2, and possibly fractional statistics, which correspond to 'topological order'. [16]

The existence of topological order in FQHE is understood (arguably) and is a feature of systems with long-range entanglement and a ground state degeneracy which depends on topology.[1] Topological order provides us a criterion for distinguishing distinct phases from each other, even in the absence of a broken symmetry such as these spin liquid phases. Another reason to consider such systems is they are models for some of the properties of high-T_c superconductivity.[1]

If I have sufficiently interested you then, let us begin by building such a state and exploring its difficulties.

[1] Probably one of the better reasons to name such a thing topological. The other is the use of Chern-Simons terms to describe long-wavelength behavior.

PROJECTIVE CONSTRUCTIONS

Let us begin life on the lattice, specifically one that is square and in $d = 2$. Here we can consider the SU(2) [2] Heisenberg $S = 1$ model:

$$H = J \sum_{<ij>} S_i \cdot S_j \tag{1}$$

Now suppose we would like to attempt some form of mean field theory on (1), replacing a spin operator with some expectation value $\langle S_i \rangle$. This however is rather presumptuous: it will only be effective if in some ordered state where the fluctuations of S_i are small. We're interested in a disordered phase.

Rather than go home in defeat let us be more clever in how we express S_i. Let us define 'spinon' operators: $f_{i\alpha}$ for $\alpha \in \{1, 2\}$.

Now we can decompose the otherwise bosonic spin operator into uncharged spin-1/2 fermions as follows:

$$S_i = \frac{1}{2} f^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \tag{2}$$

Are we free to simply do this? Clearly we have enlarged our Hilbert space to include these seemingly fictious operators, certain sectors of which will be fictitious to any physically meaningful quantities. What we must do then is impose a local constraint on these operators to reduce to one DOF per site:

$$f^\dagger_{i\alpha} f_{i\alpha} = 1 \tag{3}$$

Now we can approach mean field theory on the following operator:

$$\langle f^\dagger_{i\alpha} f_{j\alpha} \rangle = \chi_{ij} = \chi_0 e^{-\boldsymbol{i} A_{ij}} \tag{4}$$

[2] It is important to note that SU(2) here is a global symmetry and not a gauge either now or in the future.

The second equality is chosen with some foresight. We may now rewrite our Hamiltonian (1) as:

$$H_{MF} = -J \sum_{<ij>} f^\dagger_{i\alpha} f_{j\alpha} \chi_0 e^{-iA_{ij}} + h.c. - \sum_i A_0 (f^\dagger_{i\alpha} f_{i\alpha} - 1) \tag{5}$$

Where here the A_0 acts as a Lagrange multiplier to enforce (3). It also not hard to check that this theory is invariant under the gauge transformations:

$$A_{ij} \to A_{ij} + (\theta_i - \theta_j) \qquad f_i \to f_i e^{-i\theta_i} \tag{6}$$

Thus we have an emergent U(1) gauge theory in the mean-field excitations![3] The gauge boson mediates the interaction between spinons and should 'glue' them together to create the original bosonic spin.[9]

What can we say about the results of this method? We need a sanity check. If these spinon excitations are always paired off in close proximity to each other, if they're confined, then we would expect that the results we get from this more fancy mean field calculation would be identically that of the standard calculation.

In this case the spinon pairs are the only low energy excitation and the dynamics of the gauge field are irrelevant.[11] Good, but useless in describing new physics.[4]

What we would like then is to understand the deconfined phase and when such a phase is stable, which turns out to be a very tricky question.

CONFINEMENT

Let us retreat from this discussion of spin liquids and focus momentarily on the problem of confinement in electrodynamics.

Here it is necessary we make the distinction between compact U(1) gauge and what's called non-compact U(1) gauge theory.[5]

What does gauge compactness do for us? Well you may recall that a compact U(1) theory realizes topological defects in the form of magnetic monopoles and this is responsible for the quantization of charge.

Connected between a monopole/anti-monopole pair is a string defect where the gauge field is no longer exact.

If I compute the holonomy about this string then I get some measurable phase: $e^{i\oint A\cdot \mathrm{d}x} = e^{2\pi eq}$ where here e and q are the electric and magnetic charges respectively.

In a sane universe I need this phase to vanish exactly so the product of charges needs to be an integer. The string defect is no longer physical and can be moved around by gauge transformation.[17]

Good. In materials, and indeed in our universe at large, the electric charge does appear to be quantized.[6] Indeed it appears the photon doesn't have a mass either so it would appear we live in an electrically deconfined phase. Is this true in general?

Suppose, for some strange reason, we would like to study the case of electrodynamics in $d = 2$. For a moment let us consider just a pure compact gauge theory without matter fields.

It is a famous result of Polyakov that in this case the photon is always gapped and, at all strengths of the coupling, the theory is in an electrically confining phase.[14]

Indeed this result is a direct consequence of our once friend the monopole. The appearance of monopoles in compact U(1) gauge theory is an example of an instanton effect: a critical point of the Euclidean action which connects topologically distinct vacuum states of the classical theory.

In the path integral formulation of quantum mechanics, with a double well potential, these take the form of kink solutions which allow for tunneling between the two wells, a result invisible to naive perturbation theory. [2]

For definiteness, the Euclidean U(1) gauge theory is expressed as:

$$\mathcal{L}_{U(1)} = \tfrac{1}{g^2} F_{\mu\nu} F^{\mu\nu} \qquad B_\mu = \epsilon_{\mu\nu\lambda} F^{\nu\lambda} \tag{7}$$

But let's not stop there; consider a dual description of this theory using the XY model:

$$\mathcal{L}_{XY} = g^2 (\partial_\mu \sigma)^2 \qquad B_\mu = g^2 \partial_\mu \sigma \tag{8}$$

Now we can relate the Maxwell equation $\partial_\mu B^\mu = 0$ to the equation of motion for the dual photon: $\partial^2 \sigma = 0$. [7]

But wait, the whole point is to allow for the fluctuations of instantons!

These correspond to $\partial_\mu B^\mu \neq 0$ and the insertion of some quantized flux: $\oint \vec{B} \cdot \mathrm{d}\vec{A} = 2\pi q$.

In the XY model the insertion/removal of this flux corresponds to the following operator(s): $e^{\pm i\sigma}$

Adding both of these operators to (8) in equal weight gives us:

$$\mathcal{L}_{XY} = g^2 (\partial_\mu \sigma)^2 + K \cos(\sigma) \tag{9}$$

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[3] The gauge kinetic term has $g = \infty$ and does not appear explicitly
[4] A confining ground state is not unreasonable however
[5] For those worried that the circle is no longer the circle what is actually going on is that the Lie group of the non-compact theory is $\mathbb{R}$ under addition. They share the same Lie algebra so it isn't so bad of a name.

[6] It is actually the case that any U(1) gauge theory embedded in some larger gauge group, such as SU(2), must be compact.
[7] For those curious about this map we have abused Hodge duality by $\star \mathrm{d}A = \mathrm{d}\sigma$ and also common convention by denoting B as the Maxwell form but there's no matter here so no E. Also bouncing around are some silly factors of 2.

Here the coefficient K is proportional to the energy of the monopole configuration. This interaction term is now what gaps out the dual photon. A simple momentum space RG calculation will also show this term is relevant in $D = 3$ to all strengths of the coupling g.[8]

For a picture of what's happening here we can make the analogy to a charged plasma. Unlike the case of $D = 2$ there is no Kosterlitz-Thouless transition possible[8] and thus it will remain in the Coulomb phase. Through Debye screening of the fields it provides an effective mass for the dual photon.[4]

So how bad is it? If we think about a pair of electrical test charges of opposite sign, this corresponds to an vortex/anti-vortex pair in the XY model.

In the presence of the $\cos(\sigma)$ term, the energy of the configuration to grow linearly in separation scaled by the mass of the dual photon.[9] This is opposed to the usual logarithmic potential one would expect. [16]

SURVIVAL OF THE SPINNIEST

Things are looking fairly pesimistic for at least those U(1) spin liquids in $D = (2 + 1)$.[10] By the above argument it would seem that our spinon excitations are doomed to a destructively codependent relationship.

However, there is the effect of matter fields which I have so far neglected. This gives the photon the opportunity to spend a portion of its life as a pair of spinons via vacuum polarization corrections. This then adjusts the potential between a spinon pair to something less confining through the running of the gauge coupling.[10]

When is this enough to provide a stable spin liquid? It is a fairly subtle thing to extract. Indeed, several have argued in the past that the monopole effect will always overtake other contributions.[6]

The most recent calculations however have shown that in a $1/N$-expansion in the number of spinon flavors[11] the model goes to a stable deconfined fixed point for large N.[8]

Is there a finite value of N should we expect this to be stable? Here there is more variety. One calculation places the number around $N = 1.1$, and there is a recent hard upper bound of $N = 6$, but the question of whether the important SU(2) spin model is stable remains open.[13][5]

Of course we shouldn't be so pessimistic about this business. The effect of instantons is restricted to $d = 2$, though it is an important case for condensed matter. General principles for what allows a U(1) spin liquid to be stable are still unknown.

There's still quite the variety in 'matter fields' including whether the spinons are gapless, whether bosonic matter is included, and even the possibility of a spinon fermi surface; U(1) spin liquids come in many flavors![12] There also other ways out which involve breaking the U(1) gauge to some smaller discrete gauge theory and creating topological order.

In $2d$ we have 2 options: the Anderson-Higgs mechanism and the inclusion a Chern-Simons term.[3] The first allows for a deconfining phase of the spinons in a manner directly analogous to a superconductors and generates the $\mathbb{Z}_2$ spin liquids.[13] The possibility of adding a CS term to a spin-1/2 model creates the so called chiral spin liquid which carries fractional statistics.[15][16]

SUMMARY

Obviously there is a lot more that remains to be said about spin liquid phases, including several open questions about their properties and stability. What we have hopefully shown here though is that the projective construction is a useful and interesting tool in exploring the properties of novel phases of matter and that spin liquids in particular have some significant overlap with issues and ideas from high energy physics, particularly confinement.

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[2] Sidney Coleman. The uses of instantons. In *The Whys of Subnuclear Physics*, pages 805–941. Springer, 1979.

[8] Much to the disappointment of Dan Arovas
[9] This scaling is referred to as the string tension of the flux tube connecting the vortex/anti-vortex.
[10] It is worth mentioning again that our spin liquids correspond to compact theories though, if we can neglect instantons, it is common to treat the theory in a non-compact limit
[11] Consider (1) but allowing the spin index to run to N, $S = N/2$, and a $1/N$ scaling of the coupling.
[12] In the fermi surface it seems to be that the deconfined phase is stable; imagine coupling to a continuum of flavors. Also pardon the bad pun.[12]
[13] I've been a bit misleading in saying (5) has an emergent U(1) gauge theory; the most general gauge transformations on the spinons are SU(2). Pretend I Higgs'd it.

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