Neutrino-photon interactions in vacuum and material media

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It is well-established that neutrinos are spin-1/2, nearly massless, electrically neutral fermions. While the fact that they do not carry an electric charge might seem to entail that they do not exhibit electromagnetic behaviour, it can be shown that quantum loop interactions induce electromagnetic properties of neutrinos, in vacuum as well as in material media.

I. INTRODUCTION

The electromagnetic properties of fermions can be expressed in terms of their interaction with photons, with an effective interaction vertex (see FIG. 1) described by $\mathcal{L}_{\text{eff}} = \bar{\psi}\mathcal{O}_\lambda\psi A^\lambda \equiv j_\lambda(x)A^\lambda(x)$. For neutral fermions, the leading contribution to the vertex comes from the one-loop term. The form of this interaction, in general, depends on the nature of the neutrino (Dirac or Majorana). In the present discussion, we shall allude exclusively to Dirac neutrinos.

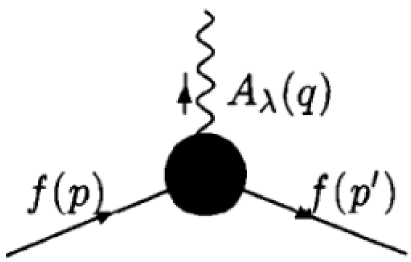

FIG. 1: The effective electromagnetic vertex for a fermion

II. FORM FACTORS OF A DIRAC NEUTRINO

The matrix element of $\mathcal{L}_{\text{eff}}$ between two one-particle states can be described in terms of the interaction amplitude $\Gamma_\lambda(p, p')$. Subject to the constraints impsoed by hermiticity of the Lagrangian, current conservation (Ward identity) and Lorentz invariance, the most general form of Γ_λ is given by

$$\begin{aligned}\Gamma_\lambda(p,p') = &\left(q^2\gamma_\lambda - q_\lambda \not{q}\right)\left[R(q^2) + r(q^2)\gamma_5\right] \\ &+ \sigma_{\lambda\rho}q^\rho\left[D_M(q^2) + iD_E(q^2)\gamma_5\right]\end{aligned} \tag{1}$$

where $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]$, and the form factors R, r, D_M and D_E are all real and manifestly Lorentz invariant (depend only on q^2).

It can be seen that the D_M term reduces to $D_M(0)\boldsymbol{\sigma}\cdot\mathbf{B}$ in the non-relativistic limit and hence $D_M(0)$ can be identified as the magnetic dipole moment. Likewise $D_E(0)$ is the electric dipole moment. In general, $D_M(q^2)$ and $D_E(q^2)$ are called the *magnetic* and *electric form factors*, whereas $R(q^2)$ and $r(q^2)$ are the *charge radius* and *axial charge radius* respectively.

III. KINEMATICS OF RADIATIVE DECAYS

The possibility of the decay of a neutrino $\nu_\alpha \to \nu'_\alpha + \gamma$ is represented by a vertex similar to the one in FIG. 1, with the difference being that the incoming and outgoing fermions are not the same. For Dirac neutrinos, we can write the matrix element as

$$T = \bar{u'}(\mathbf{p'})\Gamma_\lambda u(\mathbf{p})\epsilon^{*\lambda} \tag{2}$$

where ϵ^λ is the photon polarization. For physical photons, the on-shell and the Lorentz gauge conditions require $q^2 = 0$ and $q_\lambda\epsilon^\lambda = 0$, and the general form of Γ_λ reduces to

$$\Gamma_\lambda = \left[F(q^2) + F_5(q^2)\gamma_5\right]\sigma_{\lambda\rho}q^\rho \tag{3}$$

F and F_5 are often called *transition magnetic and electric dipole moments* and their values, in general, are model dependent. In terms of these, the decay rate in the rest frame of the neutrino can be written as

$$\Gamma = \frac{(m_\alpha^2 - m_{\alpha'}^2)^3}{8\pi m_\alpha^3}\left(|F|^2 + |F_5|^2\right) \tag{4}$$

IV. DIPOLE MOMENTS AND RADIATIVE LIFETIME IN AN $SU(2)_L \times U(1)_Y$ MODEL WITH DIRAC NEUTRINOS IN VACUUM

In this section, we shall proceed to calculate the electromagnetic vertex considering only real photons, so that the restrictions leading to Eq. (3) continue to apply.

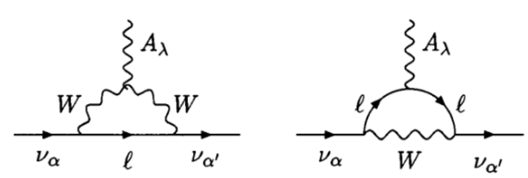

FIG. 2: One-loop diagrams for the decay $\nu_\alpha \to \nu'_\alpha + \gamma$

The one-loop diagrams contributing to the electromagnetic vertex of neutrinos are shown in FIG. 2. Under the assumptions $m_\alpha \ll M_W, m_\ell$, one obtains

$$\begin{aligned}\Gamma_\lambda = &-\frac{eG_F}{4\sqrt{2}\pi^2}\left(m_\alpha R + m_{\alpha'}L\right) \\ &\times \sigma_{\lambda\rho}q^\rho\sum_\ell U_{\ell\alpha}U^*_{\ell\alpha'}f(r_\ell)\end{aligned} \tag{5}$$

where U is the mixing matrix in the leptonic sector and we have defined $r_\ell \equiv (m_\ell/M_W)^2$ and $f(r) \equiv \frac{3}{4(1-r)^2}\left[-2+5r-r^2+\frac{2r^2\ln r}{1-r}\right] \simeq -\frac{3}{2}+\frac{3}{4}r+...$

When $\nu_\alpha = \nu_{\alpha'}$, then $(m_\alpha R + m_{\alpha'} L)$ is simply m_α, and thus the electric dipole moment vanishes (no γ_5 term). The magnetic dipole moment of ν_α is then given by

$$\mu_\alpha = -\frac{eG_F}{4\sqrt{2}\pi^2} m_\alpha \sum_\ell U_{\ell\alpha} U^*_{\ell\alpha} f(r_\ell) \simeq \frac{3eG_F}{8\sqrt{2}\pi^2} m_\alpha \quad (6)$$

where in the second equality, we have used the leading term of $f(r_\ell)$ and the unitarity of the mixing matrix U. Thus, for Dirac neutrinos with ordinary weak interactions, the magnetic moment is given by

$$\mu_\alpha \simeq 3.1 \times 10^{-19} \mu_B \left(\frac{m_\alpha}{1 \text{ eV}}\right) \quad (7)$$

For the case where $\nu_\alpha \neq \nu_{\alpha'}$, we can write $F = K(m_\alpha + m_{\alpha'})$ and $F_5 = K(m_\alpha - m_{\alpha'})$, where

$$K = -\frac{eG_F}{8\sqrt{2}\pi^2} \sum_\ell U_{\ell\alpha} U^*_{\ell\alpha'} f(r_\ell) \quad (8)$$

Here it is worth noting that the contribution from the constant term in the expansion of $f(r_\ell)$ sums up to zero in the above expression because of the unitarity of U. Thus, the leading contribution to the decay rate comes from the $\mathcal{O}(r_\ell)$ term in the expansion. This effect is akin to the GIM mechanism which suppresses flavor changing neutral currents in the quark sector.

The decay rate in vacuum can now be obtained using Eq. (4). In a simplifed model with only two families of fermions (say e and τ), the leading contribution to the decay rate can be calculated to be

$$\Gamma \approx \frac{\alpha}{2}\left(\frac{3G_F}{32\pi^2}\right)^2 m_\alpha^5 \sin^2\theta \cos^2\theta \left(\frac{m_\tau}{M_W}\right)^4 \quad (9)$$

where we have also assumed $m_{\alpha'} \ll m_\alpha$. For $m_\alpha = 1$ eV, this gives a lifetime of order 10^{36} years. Thus, neutrinos in vacuum are practically stable to radiative decay over the age of the universe (10^{10} years).

V. ELECTROMAGNETIC PROPERTIES OF NEUTRINOS IN A MEDIUM

In previous sections, we examined the electromagnetic properties of neutrinos in vacuum. Next, we attempt to re-calculate the vertex in a background of electrons.

In vacuum the most general form of the interaction amplitude Γ_λ given by Eq. (1) involves only the photon 4-momentum q. Now however, another 4-vector makes an appearance, namely the 4-velocity u of the medium. As a result, there can be additional terms in the interaction amplitude, which under the previously imposed constraints can be quantified as

$$\begin{aligned}\Gamma'_\lambda = &\left(q \cdot u \gamma_\lambda - u_\lambda \not{q}\right)\left[C_E + iD'_E\gamma_5\right] \\ &+ \varepsilon_{\lambda\rho\alpha\beta}\gamma^\rho q^\alpha u^\beta \left[C_M + iD'_M\gamma_5\right]\end{aligned} \quad (10)$$

It can be verified that, in the non-relativistic limit, C_E and C_M vanish, whereas the D'_E and D'_M terms contribute to the electic and magnetic dipole moments respectively. The latter two are thus the matter-induced form factors of the neutrino.

At 1-loop level, the interaction vertices can be obtained by attaching a photon line to every possible internal line in FIG. 3. FIG. (3a) has two terms corresponding to a photon attaching to either of the two internal lines, whereas FIG. (3b) gives one term for each charged fermion.

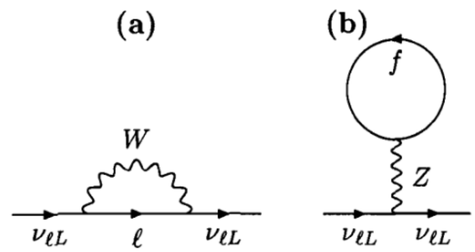

FIG. 3: One-loop diagrams for the decay $\nu_\alpha \to \nu'_\alpha + \gamma$

In a medium whose effects come predominantly from thermal electrons, the average number of particles at a certain energy is given by a Fermi-Dirac distribution, and hence we have

$$\left\langle f_s^\dagger(\mathbf{k}') f_s(\mathbf{k})\right\rangle = \delta^3(\mathbf{k}-\mathbf{k}') f_F(k,\mu,\beta), \quad (11)$$

$$\left\langle \hat{f}_s^\dagger(\mathbf{k}') \hat{f}_s(\mathbf{k})\right\rangle = \delta^3(\mathbf{k}-\mathbf{k}') f_F(k,-\mu,\beta) \quad (12)$$

This allows us to calculate the thermal propagator

$$\begin{aligned}iS_F(k,\mu,\beta) = (\not{k}+m)&\left[\frac{i}{k^2-m^2+i0}\right. \\ &\left. - 2\pi\delta(k^2-m^2)\eta_F(k,\mu,\beta)\right]\end{aligned} \quad (13)$$

where we define $\eta_F(k,\mu,\beta) = \Theta(k\cdot u) f_F(k,\mu,\beta) + \Theta(-k\cdot u) f_F(-k,-\mu,\beta)$

Let us write $\Gamma'_\lambda \equiv \mathcal{T}_{\lambda\rho}\gamma^\rho L$, where $\mathcal{T}_{\lambda\rho} = \mathcal{T}^{(Z)}_{\lambda\rho} + \mathcal{T}^{(W)}_{\lambda\rho}$, indicating the contributions from each of the two diagrams. $\mathcal{T}_{\lambda\rho}$ can be evaluated from these diagrams using the propagator from Eq. (13). After some cumbersome (although not difficult) gymnastics, we are motivated to write $\mathcal{T}_{\lambda\rho} = \mathcal{T}_T R_{\lambda\rho} + \mathcal{T}_L Q_{\lambda\rho} + \mathcal{T}_P P_{\lambda\rho}$, where

$$\begin{aligned}R_{\lambda\rho} &= g_{\lambda\rho} - \frac{q_\lambda q_\rho}{q^2} - Q_{\lambda\rho} \\ Q_{\lambda\rho} &= \frac{\tilde{u}_\lambda \tilde{u}_\rho}{\tilde{u}^2} \\ P_{\lambda\rho} &= \frac{i}{Q}\varepsilon_{\lambda\rho\alpha\beta} q^\alpha u^\beta\end{aligned} \quad (14)$$

where $\tilde{u}_\lambda = u_\lambda - \omega q_\lambda/q^2$, $\omega = q \cdot u$ and $Q = \sqrt{\omega^2 - q^2}$ (apologies for the notation within notation within notation, i.e. notationception!). The form factors can now be expressed as

$$\begin{aligned}\mathcal{T}_T &= -2eG_F(g_V+1)\left(A - \frac{B}{\tilde{u}^2}\right),\\ \mathcal{T}_L &= -4eG_F(g_V+1)\frac{B}{\tilde{u}^2},\\ \mathcal{T}_P &= 4eG_F(g_A-1)QC \end{aligned} \tag{15}$$

where (more notation!) $g_V = -\frac{1}{2} + 2\sin^2\theta_W$, $g_A = -\frac{1}{2}$, and

$$\begin{aligned} A &= \left\langle \frac{2me^2 - 2k\cdot q}{q^2 + 2k\cdot q}\right\rangle_+ + (q \to -q),\\ B &= \left\langle \frac{2\Omega_k^2 + 2\Omega_k\omega - k\cdot q}{q^2+2k\cdot q}\right\rangle_+ + (q\to -q),\\ C &= \left\langle \frac{k\cdot\tilde{u}/\tilde{u}^2}{q^2+2k\cdot q}\right\rangle_- + (q\to -q)\end{aligned} \tag{16}$$

The sums $\langle\rangle_\pm$ stand for

$$\langle x\rangle_\pm = \int \frac{d^3k}{(2\pi)^3 2\Omega_k} x(f_F(k,\mu,\beta) \pm f_F(k,\mu,-\beta)) \tag{17}$$

A. Induced electric charge of neutrinos

The effective charge of a fermion can be obtained from the effecttive electromagnetic vertex in the static, low-momentum transfer limit

$$e_{\text{eff}} = \frac{1}{2E}\bar{u}(p)\Gamma_0(\omega = 0, Q\to 0)u(p) \tag{18}$$

It is easy to see that the vertex function in vacuum vanishes in this limit, implying a zero effective charge, but the same need not happen inside a medium. For $\omega = 0$, the only nonvanishing contribution comes from $Q_{0\rho}$, thus only the form factor $\mathcal{T}_L$ is relevant. For a non-relativistic background of electrons, the effective charge can be calculated to be

$$e_{\text{eff}} = \frac{1}{4}eG_F n_e \beta(g_V+1) \tag{19}$$

for ν_e, whereas for $\nu_{\mu,\tau}$ one has to replace $g_V + 1$ by g_V (no charge-current interaction).

B. Radiative neutrino decay in a medium

As long as flavor-changing neutral currents are assumed to be absent, only the charge-current vertex $\Gamma_\lambda^{(W)}$ contributes here. In this case, the vertex is evaluated at $q^2 = 0$, and hence the form factors $\mathcal{T}_{L,P}$ vanish, leavign us with only $\mathcal{T}_T$. Limiting ourselves to a 2-flavor model and assuming $m_\alpha \ll m_{\alpha'}$, the decay rate in the rest frane of the medium can be calculated to be

$$\Gamma' = \frac{\alpha}{2}G_F^2 \sin^2\theta\cos^2\theta F(\mathcal{V}) \times \frac{m_\alpha n_e^2}{m_e^2} \tag{20}$$

where $\mathcal{V}$ is the 3-velocity of the incoming neutrino in the rest frame of the medium, with $F(\mathcal{V}) = \sqrt{1-\mathcal{V}^2}\left[\frac{2}{\mathcal{V}}\ln\frac{1+\mathcal{V}}{1-\mathcal{V}} - 3 + \frac{m_{\alpha'}^2}{m_\alpha^2}\right]$. Thus it can be seen that matter effects can enhance the decay rate by a factor

$$\frac{\Gamma'}{\Gamma} = 9\times 10^{23} F(\mathcal{V})\left(\frac{n_e}{10^{24}\ \text{cm}^{-3}}\right)^2\left(\frac{m_\alpha}{1\ \text{eV}}\right)^{-4} \tag{21}$$

The main reason for this enhancement is that the GIM mechnism, which suppresses the decay rate in vacuum, becomes irrelevant in a medium because of its flavor asymmetry (only electrons present). Unlike Eq. (8), there is no sum over flavor indices, and hence we do not see any cancellations at lowest order.

VI. CONCLUSION

We have thus seen how quantum loop interactions can induce electromagnetic properties in neutrinos. And the other takeaway message from this writeup is that the breaking of Lorentz and flavor symmetries by a background medium can cause matter-induced one-loop corrections to differ significantly from those in vacuum.

[1] Rabindra N. Mohapatra and Palash B. Pal, "Massive Neutrinos in Physics and Astrophysics (3rd ed.)," World Scientific (2004).