

When Goldstone meets Landau

Shenglong Xu¹

¹*Physics Department, University of California, San Diego, CA, 92093*

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Landau Fermi Liquid theory describes a variety of conventional metal, and is robust against many different interaction. Yet, coupling between Goldstone boson and electron results a singularity in the infrared limit of boson self energy, which may lead to non Fermi Liquid behavior depending on the symmetry group generator. Usually this singularity does not appear because the vertex function vanishes in the infrared limit. However, if the symmetry generator does not commute with the momentum operator, it is shown that vertex function is finite in this limit. Its effect on the fermion is also discussed.

INTRODUCTION

The ground state of free fermion system is characterized by sharply defined Fermi surface. The occupation number $n(k)$ jumps from one to zero when k crosses the Fermi surface. Such dramatic drop is due to the δ peaks in the spectral function, which is corresponding to the purely particle-like excitations. In conventional metal, the interaction between fermion is perturbable from free fermion case. This means the electrons behaves more or less the same as one in free case with renormalized particle weight Z . Such systems are well explained by Landau Fermi liquid (FL) theory and have same thermodynamics behavior. For example, the specific heat scales as T and the resistivity scales T^2 .

Although Fermi liquid well describes conventional metal, experimentalists do find some materials have non Fermi liquid behavior. The most famous example is probably the strange metal which appears in high T_C material when temperature is higher than the superconducting critical temperature. Such system behaves like metal but with a resistivity scales linearly as T instead of T^2 . This behavior cannot be captured by *FL* theory. Therefore it is interesting to seek for microscopic mechanism that can destroy the Fermi surface, which may possibly explain the strange metal and leads to other exotic excitation and thermodynamics behavior.

Mathematically, when interaction is turned on, the property of the fermion should be corrected by the self energy. The full single particle propagator is

$$G = \frac{1}{\omega + i0^+ - \xi_k - \Sigma(k, \omega)} \quad (1)$$

The information of interaction is entirely encoded in the self energy $\Sigma(k, \omega)$. The imaginary part of the self energy can significantly effect the spectral function by broadening or even destroy the delta peak. If we assume the pole of the Green function is a simple pole, the quasi particle weight Z , defined as the residue of the pole, is $1/(1 - \partial_\omega \Sigma(\omega))$. In the free electron system, $\Sigma(\omega, k)$ is just zero, and therefore $Z=1$. However, the self energy can make Z zero, which is a sign that quasi particle picture breaks down. Moreover, if this pole occurs at the

original Fermi surface, the system's thermodynamics behavior at low temperature will be deviated from that of FL.

It has been many research suggesting that coupling between electrons and gapless boson can possibly lead to such effect. Gaplessness imposes a strong restriction on the origin of the bosons. The gaplessness should be either obtained by fine tuning, or be protected by physical reason, which means the bosons should be gauge boson or Goldstone boson.

This paper will follow Watanabe's approach [1] to discuss the coupling between Goldstone bosons and fermions, and the condition that such coupling can possibly lead to non FL behavior.

COUPLING BETWEEN GOLDSTONE BOSON AND FERMION

We start with a general Hamiltonian in term of fermion field ψ and boson field ϕ

$$H = H_{0,e}(\psi, \psi^\dagger) + H_{0,b}(\phi) + H_{int} \quad (2)$$

It is possible to obtain this Hamiltonian from purely electron system by Hubbard Stratonovich transformation. Here $H_{0,e}$ is the bare electron propagator, $H_{0,b}$ is the bare boson propagator and H_{int} describes the coupling between these two. Each term should be commute with the momentum and the generator of symmetry group. This requirements impose strong restriction on the form of coupling between fermion and boson after the symmetry is spontaneous broken.

In general, the boson field has n components, and expectation value of one components, denoted as σ is nonzero due to spontaneous symmetry broken and the other components, denoted as π , are small so that we can do perturbation theory latter. Due to symmetry argument, the leading order of the interaction between electrons and π modes can be written in the following form:

$$H_{int}(\psi, \psi^\dagger, \pi) = -[i\pi^\alpha Q_\alpha, H'_{0,e}] \quad (3)$$

where Q_α is the unbroken symmetry generator associated with π^α . $H'_{0,e}$ is the free fermion Hamiltonian including the part that fermion couples to σ mode.

$$H'_{0,e}(\psi, \psi^\dagger) = H_{0,e}(\psi, \psi^\dagger) + H_{int}(\psi, \psi^\dagger, \sigma) \quad (4)$$

The result in equation(3) can be seen as follows. We can rotate (uniform transformation) the boson vector so that it looks that transformed fermion is only coupled to the σ mode, but it is the same interacting Hamiltonian because of the symmetry. The transformed fermion operators depend on the rotation angle, which is determined by the original π^α .

The single fermion operator can be labeled by the mutual eigenstates of $H'_{0,e}$ and momentum. The minimal coupling between fermions and π modes is

$$H_{int}(\psi, \psi^\dagger, \pi) = \sum_{n,n',a} \int \frac{d^d k d^d k'}{(2\pi)^{2d}} v_{k,n,k',n'}^\alpha \psi_{k,n}^\dagger \psi_{k',n'} \pi^\alpha \quad (5)$$

This is just standard vertex term in the language of field theory. The vertex function $v_{k,k',n,n'}^\alpha$ can be obtained from:

$$v_{k,n,k',n'}^\alpha = \langle 0 | \psi_{n,k} H_{int}(\psi, \psi^\dagger, \pi^\alpha) \psi_{n',k'}^\dagger | 0 \rangle \quad (6)$$

We can plug in Eq 3 into above expression and get:

$$v_{k,n,k',n'}^\alpha = -i\pi \langle 0 | \psi_{n,k} Q_\alpha \psi_{n',k'}^\dagger | 0 \rangle (\epsilon_{n',k'} - \epsilon_{n,k}) \quad (7)$$

Here we arrive the main result of this section. The vertex function can possibly has two different behavior in the limit that $n = n'$ and $k - k' \rightarrow 0$. We know that $\epsilon_{n',k'} - \epsilon_{n,k}$ approaches zero as $k - k' \rightarrow 0$ if we assume quadratic band. So, if $\langle 0 | \psi_{n,k} Q_\alpha \psi_{n',k'}^\dagger | 0 \rangle$ is not singular, the vertex function vanishes at this limit. However, if this factor diverges, it is possible that the vertex function remains finite, which contributes to many interesting physics that will be discussed in next section.

Now we need investigate the factor carefully to study its behavior in the limit $k - k' \rightarrow 0$. There, $\langle 0 | \psi_{n,k} Q_\alpha \psi_{n,k}^\dagger | 0 \rangle$ is just the expectation value of the symmetry generator Q_α in the single particle picture, which can be not well defined for some cases, especially when Q_α does not share the same eigenstate with momentum, namely Q_α does not commute with momentum. More rigorously, we can consider the following factor:

$$\langle 0 | \psi_{n,k} [Q_\alpha, e^{i\hat{p}a}] \psi_{n,k'}^\dagger | 0 \rangle = \langle 0 | \psi_{n,k} Q_\alpha \psi_{n,k}^\dagger | 0 \rangle (e^{ik'a} - e^{ika}) \quad (8)$$

When $[Q_\alpha, p] \neq 0$, the left hand side of equation should be some finite value. However, we know that $(e^{ik'a} - e^{ika})$ vanishes as $a(k - k')$, indicating that $\langle 0 | \psi_{n,k} Q_\alpha \psi_{n,k}^\dagger | 0 \rangle$ must diverge as $a(k - k')$. If this is the case, the infrared limit of the bare vertex function should also be finite since this divergence exactly cancels that factor in $(\epsilon_{n',k'} - \epsilon_{n,k})$ that goes to zero in the limit.

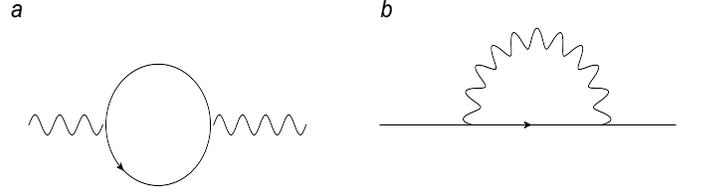


FIG. 1. a: the self energy for bosons; b: the self energy for fermions

Hence we have a general criterion on the condition that $v_{k',n,k,n}^\alpha$ does not vanishes when $k - k' \rightarrow 0$. That is the associated symmetry generator Q_α does not commute with momentum operator. An immediate example of symmetry generator of this kind is the angular momentum. Such effect is important in the system with nematic order [2], which means the spacial rotation symmetry is spontaneously broken.

THE EFFECT OF NON VANISHING VERTEX FUNCTION IN THE INFRARED LIMIT

Now we discuss the consequence of the non vanishing vertex function. Basically this property make the imaginary part of boson self energy diverge with $\nu/|q|$ in the infrared limit. When plugging the self energy into the Dyson equation for the boson propagator, we can obtain an over damping pole, which implies the particle-like boson description break down. If we further calculate the fermion self energy, we can find the imaginary part goes $\omega^{d/3}$, where d is the dimension of the space, when ω approaches zero. This indicates that the quasi particle peak disappears when $d \leq 3$ and the system cannot be captured by FL theory.

To explicitly show this, we define the free bosonic propagator and fermionic propagator.

$$D^0(\nu, q) = \frac{1}{\nu - \rho_s q^2} \quad (9)$$

$$G^0(\omega, q) = \frac{1}{\omega - \xi_k}$$

We need calculate the correction to $D^0(\nu, q)$ at one loop level. The self energy is corresponding to Feynman diagram drawn in Fig 1(a)

$$\Pi_{ab}(\nu, q) = \int \frac{d^d k}{(2\pi)^{d+1}} d\omega v_{k,k+q}^\alpha v_{k+q,k} G^0(\omega, k) G^0(\omega + \nu, k + q) \quad (10)$$

The integration of ω can be done by contour integration. Then one can find:

$$Im \Pi_{ab}(\nu, q \rightarrow 0) \sim -\frac{\nu k_f^{d-3}}{q m} v_{k_f, k_f}^\alpha v_{k_f, k_f}^\alpha = -A \frac{\nu}{q} \quad (11)$$

As we can see it has $1/q$ divergence as $q \rightarrow 0$ if v_{n,k_f, n, k_f}^α is not zero, which happens only when $[Q^\alpha, p] \neq 0$. We

can plug it into the Dyson equation to obtain the new boson propagator:

$$D(\nu, q) = \frac{1}{\nu - \rho_s q^2 + iA \frac{\nu}{q}} \quad (12)$$

Now we need use the updated boson propagator to calculate the fermion self energy(Fig1(b)):

$$\Sigma(k, \omega) = \int \frac{d^d q d\nu}{(2\pi)^{d+1}} v_{k, k+q}^\alpha v_{k+q, k}^\alpha D(q, \nu) G_0(\omega + \nu, k + q) \quad (13)$$

Within certain approximation, it can be shown that the imaginary part scales as $\omega^{d/3}$ (refer to appendix for detail calculation). When d is small than 3, the derivative, which appears at the denominator of the quasi particle weight, diverges at $\omega \rightarrow 0$. Therefore Z is zero for this case, and the quasi particle picture does not hold anymore. A more systematic study on this effect can be found here[3].

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- [1] H. Watanabe and A. Vishwanath, , 5 (2014), arXiv:1404.3728
 - [2] M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075127 (2010)
 - [3] D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Physical Review B **82**, 045121 (2010)

Calculating the fermion self energy

In the infrared limit, the fermion self energy is

$$\begin{aligned}
\Sigma(k, \omega) &\sim \int \frac{d^d q d\nu}{(2\pi)^{d+1}} v_{k, k+q}^\alpha v_{k+q, k}^\alpha D(q, \nu) G_0(\omega + \nu, k + q) \\
&\sim \int \frac{d^d q d\nu}{(\nu - \rho_s q^2 + iA\frac{\nu}{q})(\omega + \nu - \xi_{k+q} + i0^+)} \\
&\sim \int \frac{1}{(\nu - \frac{\rho_s}{A^2} q^3 (q - iA))(\omega + \nu - \xi_{k+q} + i0^+)} \quad (14)
\end{aligned}$$

Here we are only interested in the most singular part therefore only the leading order of q is kept. The integration over ν can be carried out most easily in Masubara frequency at zero temperature by contour integration.

Then the self energy becomes:

$$\Sigma(k, \omega) \sim \int \frac{\theta(\omega - \xi_k)}{\omega + \frac{i\rho_s q^3}{A} - \xi_k - mkq \cos\theta} q^{d-1} dq \cos\theta d\theta \quad (15)$$

We can integrate out θ first. At the original Fermi surface, the self energy becomes

$$\Sigma(k, \omega) = \int \theta(\omega) \ln(\omega + i\frac{\rho q^3}{A}) q^{d-1} dq \quad (16)$$

We can expand the \log function in the infrared limit, and the imaginary part of the self energy is

$$\text{Im}(\Sigma(k, \omega)) \sim \theta(\omega) \int \frac{q^3}{\omega} q^{d-1} dq \sim \omega^{d/3} \quad (17)$$