University of California at San Diego – Department of Physics – Prof. John McGreevy

Quantum Field Theory C (215C) Spring 2015
Assignment 3

Posted April 21, 2015 **Due 11am, Thursday, April 30, 2015**

Relevant reading: Zee, Chapter IV.3. Zee, Chapter III.3 also overlaps with recent lecture material.

Problem Set 3

1. **Propagator corrections in a solvable field theory.**

   Consider a theory of a scalar field in D dimensions with action

   $$S = S_0 + S_1$$

   where

   $$S_0 = \int d^D x \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi - m_0^2 \phi^2 \right)$$

   and

   $$S_1 = -\int d^D x \frac{1}{2} \delta m^2 \phi^2 \ .$$

   We have artificially decomposed the mass term into two parts. We will do perturbation theory in small δm^2, treating S_1 as an 'interaction' term. We wish to show that the organization of perturbation theory that we've seen lecture will correctly reassemble the mass term.

   (a) Write down all the Feynman rules for this perturbation theory.

   (b) Determine the 1PI two-point function in this model, defined by

   $$\mathbf{i}\Sigma \equiv \sum (\text{all 1PI diagrams with two nubbins})\,.$$

   (c) Show that the (geometric) summation of the propagator corrections correctly produces the propagator that you would have used had we not split up $m_0^2 + \delta m^2$.

2. **Coleman-Weinberg potential.**

   (a) [Zee problem IV.3.4] What set of Feynman diagrams are summed by the Coleman-Weinberg calculation?

   [Hint: expand the logarithm as a series in V''/k^2 and associate a Feynman diagram with each term.]

(b) [Zee problem IV.3.3] Consider a massless fermion field coupled to a scalar field ϕ by a coupling $g\phi\bar{\psi}\psi$ in $D = 1+1$. Show that the one loop effective potential that results from integrating out the fluctuations of the fermion has the form

$$V_F = \frac{1}{2\pi}(g\phi)^2 \log\left(\frac{\phi^2}{M^2}\right)$$

after adding an appropriate counterterm.

3. **2PI generating functional.**

In this problem, we will consider $0+0$-dimensional ϕ^4 theory for definiteness, but the discussion also works with labels. Parametrize the integral as

$$Z[K] \equiv \int_{-\infty}^{\infty} dq \; e^{-\frac{1}{2}m^2q^2 - \frac{g}{4!}q^4 - \frac{1}{2}Kq^2} \equiv e^{-W[K]} \;;$$

we will regard K as a source for the two-point function.

The 2PI generating functional is defined by the Legendre transform

$$\Gamma[G] \equiv W[K] - K\partial_K W[K]$$

with $K = K(G)$ determined by $G = \partial_K W[K]$. Here G is the 2-point Green's function $G \equiv \langle q^2 \rangle_K$.

(a) Show from the definition that Γ satisfies the Dyson equation

$$\partial_G \Gamma[G] = -\frac{1}{2}K.$$

(b) Show that the perturbative expansion of Γ can be written as

$$\Gamma[G] \overset{\text{A}}{=} \frac{1}{2}\log G^{-1} + \frac{1}{2}m^2 G + \gamma_{2PI} + \text{const.}$$

(The A over the equal sign means that this is a perturbative statement.) Here γ_{2PI} is defined as everything beyond one-loop order in the perturbation expansion.

(c) Show that $-\gamma_{2PI}$ is the sum of connected two-particle-irreducible (2PI) vacuum graphs, where 2PI means that the diagram cannot be disconnected by cutting any *two* propagators.

(d) Define the 1PI self-energy in this Euclidean setting as

$$-\Sigma \overset{\text{A}}{=} \sum \left(\text{1PI diagrams with two nubbins}\right).$$

Show that

$$\Sigma \overset{\text{A}}{=} \frac{\partial \gamma_{2PI}}{\partial K}$$

by examining the diagrams on the BHS.

(e) Show that the expression for G that results from the Dyson equation

$$G^{-1} = K + 2\partial_G \gamma_{2PI}$$

agrees with our summation of 1PI diagrams.

(f) I may add another part to this problem which shows why 2PI effective actions are useful.