1. **Brain-warmer.** Find the coefficient $N_s$ in the coherent state representation of the spin operator for general spin $s$

$$S^a = N_s \int dn \mid \hat{n} \rangle \langle \hat{n} \mid .$$

2. **Topological charge.** How does the theta term appear in the $\mathbb{CP}^1$ representation of the NLSM on $S^2$? Show that

$$\epsilon_{abc} n^a dn^b \wedge dn^c = \alpha dA$$

for some constant $\alpha$, and find $\alpha$.

3. **Large $n$.** Consider the NLSM on $S^{n+1}$ in terms of the $\hat{n}$ variables, in $D$ space-time dimensions. Impose the constraint $\hat{n} \cdot \hat{n} = 1$ by Lagrange multiplier, $\int [d\sigma] e^{if \sigma(n^2 - 1)}$, so that the integral over $n$ is Gaussian. Do the gaussian integral and find an effective action for $\sigma$. Find the saddle point equation for $\sigma$. Find a translation-invariant saddle point. Compare and contrast the saddle point condition for $D = 2$ and $D > 2$. For $D > 2$ you should find a critical value of the coupling.

Compare the behavior near the critical point with the large-$n$ limit of the Wilson-Fisher fixed point in the $\epsilon$ expansion.

Evaluate the two point function $\langle n^a(x)n^a(0) \rangle$ at the saddle point.

4. **Reminder.** If you didn’t do the problem on the Haldane phase on the previous problem set, try it now.

5. **Fermionic coherent state exercise.**

Consider a collection of fermionic modes $c_i$ with quadratic hamiltonian $H = \sum_{ij} h_{ij} c_i^\dagger c_j$, with $h = h^\dagger$.

(a) Compute $\text{tr} e^{-\beta H}$ by changing basis to the eigenstates of $h_{ij}$ (the single-particle hamiltonian) and performing the trace in that basis: $\text{tr}... = \prod \epsilon \sum_{n_x = 0,1}^c c_i^\dagger c_x = 0,1 ...$

(b) Compute $\text{tr} e^{-\beta H}$ by coherent state path integral. Compare!

(c) [super bonus problem] Consider the case where $h_{ij}$ is a random matrix. What can you say about the thermodynamics?