University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215C QFT Spring 2025
Assignment 7

Due 11:59pm Wednesday, May 15, 2025

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1. **Gross-Neveu model.** [optional] This problem uses *very* similar steps to the analysis of the $\mathsf{O}(n)$ model, but leads to very different physical conclusions. I include it here to emphasize the many applications of this method. It also illustrates the manipulations we did in describing the BCS phenomenon. And it's an application of fermionic path integrals,

   Consider the partition function for an N-vector of fermionic spinor fields in D dimensions:
   $$Z = \int [d\psi d\bar{\psi}] e^{\mathbf{i}S[\psi]}, \quad S[\vec{\psi}] = \int \mathrm{d}^D x \left(\bar{\psi}^a \mathbf{i} \partial\!\!\!/ \psi^a - \frac{g}{N} \left(\bar{\psi}^a \psi^a \right)^2 \right).$$

   (a) At the free fixed point, what is the dimension of the coupling g as a function of the number of spacetime dimensions D? Show that it is classically marginal in $D = 2$, so that this action is (classically) scale invariant.

   (b) We will show that this model in $D = 2$ exhibits dimensional transmutation in the form of a dynamically generated mass gap. Here are the steps: first use the Hubbard-Stratonovich trick to replace ψ^4 by $\sigma\psi^2 + \sigma^2$ in the action, where σ is a scalar field. Then integrate out the ψ fields. Find the saddle point equation for σ; argue that the saddle point dominates the integral for large N. Find a translation invariant saddle point. Plug the saddle point configuration of σ back into the action for ψ and describe the resulting dynamics.

2. **Galilean transformation of non-relativistic fields.**

   Show that the action
   $$S = \int dt d^d x \left(\Phi^\star \mathbf{i} \partial_t \Phi - \frac{1}{2m} \vec{\nabla}\Phi^\star \cdot \vec{\nabla}\Phi - V(|\Phi|) \right) \tag{1}$$
   is invariant under Galilean boosts, in the form
   $$\Phi(\vec{x}, t) \to \Phi'(\vec{x}', t') \quad \text{with} \quad \Phi(\vec{x}, t) = e^{-\frac{\mathbf{i}}{2} m v^2 t + \mathbf{i} m \vec{v}\cdot\vec{x}} \Phi'(\vec{x}', t') \tag{2}$$
   with $t' = t, x'_i = x_i - v_i t$.

   Note that this is also how the nonrelativistic single-particle wavefunction must transform in order to preserve the Schrödinger equation.

   Don't forget that $\frac{\partial}{\partial x^\mu} = \frac{\partial x'^\nu}{\partial x^\mu} \frac{\partial}{\partial x'^\nu}$.

   How does the boost act on the Goldstone mode in the symmetry-broken phase?