# Condensed matter applications of holographic duality

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How can we use AdS/CFT to study cond-mat?

Obviously $\mathcal{N} = 4$ SYM is not a good model of a solid.

How can we use AdS/CFT to study cond-mat?

Obviously $\mathcal{N} = 4$ SYM is not a good model of a solid.

• apply to strongly-coupled liquids (*e.g.* QGP, unitary cold atoms).

• apply to cond-mat situations where the lattice is forgotten (*e.g.* quantum critical points).

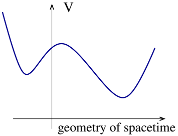
• find states of holographic systems described by the same low-energy EFT.

(In some cases, the relevant EFT is unknown, *e.g.* critical metals.)

• try to find gravity duals of more realistic systems.

# A word about string theory

String theory is a (poorly-understood) quantum theory of gravity
which has a 'landscape' of **many** groundstates
some of which look like our universe
($3 + 1$ dimensions, particle physics...)
most of which don't.




A difficulty for particle physics, a virtue for many-body physics:
by AdS/CFT, each groundstate (with $\Lambda < 0$) describes a universality class of critical behavior and its deformations
This abundance mirrors 'landscape' of many-body phenomena.

Note: tuning on both sides.
An opportunity to connect string theory and experiment.

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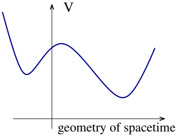


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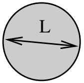
Note: tuning on both sides.
An opportunity to connect string theory and experiment.
New perspective on the structure of QFT: access to

| uncalculable things | in | uncalculable situations |
|---|---|---|
| $G(\omega, k, T)$ | | at strong coupling |
| potentials for moving probes | | far from equilibrium |
| entanglement entropy | | in real time |
| | | with a finite density of fermions |

# *e.g.*: Entanglement entropy

If $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\overline{A}}$ (*e.g.* in local theory, $A$ is a region of space)
If ignorant of $\overline{A} \to \rho_A = \mathrm{tr}_{\mathcal{H}_{\overline{A}}} \rho$ *e.g.* $\rho = |\Omega\rangle\langle\Omega|$.
$S_A \equiv -\mathrm{tr}_A \rho_A \ln \rho_A$. (notoriously hard to compute)




- 'order parameter' for topologically ordered states

in 2+1d, $S(L) = \gamma \frac{L}{a} + S_{\text{top}}$ [Levin-Wen, Preskill-Kitaev 05]

- scaling with region-size characterizes simulability: [Verstraete,Cirac,Eisert]

boundary law $\leftrightarrow$ matrix product state ansatz (DMRG) will work.

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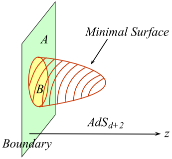
[Ryu-Takayanagi] $$S_A = \mathrm{extremum}_{\partial M = \partial A} \frac{\mathrm{area}(M)}{4G_N}$$




outcome from holography:

which bits are universal in CFT? in $d$ space dims,

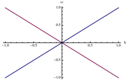
$S_A =$

$$p_1 \left(\frac{L}{a}\right)^{d-1} + p_3 \left(\frac{L}{a}\right)^{d-3} \cdots + \begin{cases} p_{d-1} \frac{L}{a} + \tilde{c} \ , & d: \text{ even} \\ p_{d-2} \left(\frac{L}{a}\right)^2 + \tilde{c} \log (L/a) \ , & d: \text{ odd} \end{cases}$$

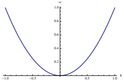
In fact, the area law coeff is also a universal measure of # of dofs, can be extracted from mutual information $S_A + S_B - S_{A\cup B}$ for colliding regions. [Swingle]

# Plan for this talk

1. introduction (over)

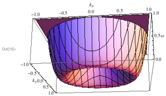


2. relativistic CFT liquid (towards QGP)

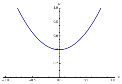


3. Galilean CFT liquid (towards cold atoms at unitarity)

4. relativistic CFT at finite density of some conserved charge
   (towards non-BCS superconductors and non-Fermi liquids)



5. strongly-coupled gapped phases



   (fractional topological insulators)

6. some concluding remarks

# Relativistic CFT plasma

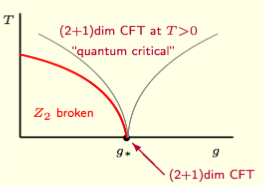
# Application 1: quantum critical transport

Quantum critical points provide one situation
where short-distance physics is unimportant.

And they present a challenge for ordinary QFT techniques:

The usual theory of transport (Boltzman eqn)
depends on a description in terms of *particles*.

For $T > m(g) = \frac{1}{\xi(g)} \propto (g - g_c)^\nu$, CFT at finite $T$:
excitations are not particles.




Also, many examples where one can argue for relevance of QCP
may involve strongly coupling:

cuprates, heavy fermions, quantum Hall plateau transitions, graphene

# Quantum critical transport from holography

*e.g.* Fluctuations of Maxwell field in *AdS* BH
$\rightarrow$ Density-density response function (or longitudinal conductivity)

in *some* thermal CFT

$$0 = \frac{\delta S}{\delta A_\nu(\omega,q,r)} \propto \partial_\mu \left(\sqrt{g} g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}\right) \quad A = e^{-i\omega t + iqx}\left(dt A_t(r) + dx A_x(r)\right)$$

$$\Longrightarrow 0 = A_t''' + \frac{f'}{f} A_t'' + \frac{1}{f^2}\left(\omega^2 - f q^2\right) A_t'$$

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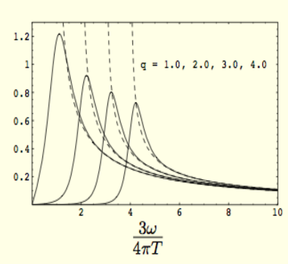
[Herzog-Kovtun-Sachdev-Son] $\mathrm{Im}\, G^R_{J^t J^t}(\omega, q; T)/q^2 \quad \longrightarrow$




Up to a factor of $T^{-1}$ depends only on $\frac{\omega}{T}, \frac{q}{T}$.

As $T \to 0$, CFT behavior $= \frac{\text{const}}{\sqrt{q^2 - \omega^2}}, \omega > q$ (else zero).

$T \neq 0$: nonzero outside lightcone $\omega < q$.

peak becomes diffusion peak for $\omega \ll q$.

Redo in presence of $B_{\text{ext}}$ and $\mu \neq 0$

- 'cyclotron resonance': pole at $\omega_c = \frac{\rho B}{\epsilon + P}$

[Hartnoll-Kovtun-Mueller-Sachdev-Son, Hartnoll-Herzog]

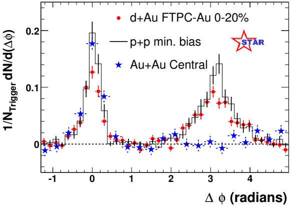
new hydro result, predicted by AdS/CFT.

- in $d = 3 + 1$: new hydro term from parity anomaly.

# Application 2: far from equilibrium dynamics

Quark-gluon plasma (QGP) is condensed matter.

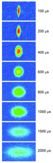
QGP is strongly coupled: a liquid, not a gas. [RHIC]




1. It is opaque:
2. It exhibits rapid thermalization,

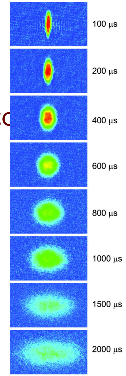
rapid hydro-ization to a fluid with very low visco

It exhibits collective motion ('elliptic flow'):



→

[O'Hara et al]



# Model QGP as relativistic CFT plasma

QGP is a deconfined phase, and hence it may not matter that one is studying (the dual of) a gauge theory that never confines at any scale.

One can hope that there is some universal physics of deconfined gauge theory plasma, or perhaps even strongly-coupled QFT stuff.

Practical note: In a relativistic QFT, the vacuum is also interesting, and can also maybe be studied using the stringy dual.

Vacuum is hard, finite-energy-density configurations exhibit more universal behavior.

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Vacuum is hard, finite-energy-density configurations exhibit more universal behavior.

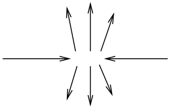
Many questions have been asked of this model.

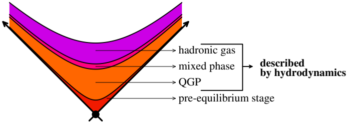
Many involve hard probes of the medium: not yet a common measurement made in strongly-correlated electron systems.

# Q: Hydro-ization of strongly-coupled glue

Important question for interpreting RHIC data: how long does it take before hydro sets in?

initially in gold-gold collision: anisotropic momentum-space distribution






[Heller-Janik-Peschanski]

after time $\tau_{th}$: locally thermal distribution and hydrodynamics.

At RHIC: $\tau_{th}$ much smaller than perturbation theory answer.
($\tau_{th}$ affects measurement of viscosity:
good elliptic flow requires both low $\eta$ and early applicability of hydro)

Thermal equilibrium of CFT stuff $\leftrightsquigarrow$ AdS black hole
$T \leftrightsquigarrow$ location of horizon $r = r_H$

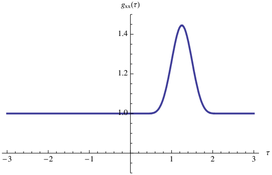
*Local* thermal equilbrium (hydro) $\leftrightsquigarrow$ slowly-varying deformations of AdS BH: $r = r_H(\vec{x}, t)$. [Janik-Peschanski, Bhattacharyya et al]

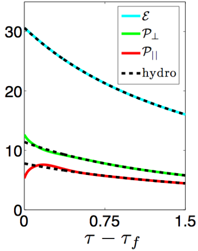
# Approach to hydrodynamic equilibrium

bulk picture: dynamics of gravitational collapse.
dissipation: energy falls into BH [Horowitz-Hubeny, 99]

- quasinormal modes of a small BH [Freiss et al, 06] $\tau_{th} \sim \frac{1}{8\,T_{peak}}$.
- far-from equilibrium processes: [Chesler-Yaffe, 08, 09] (PDEs!)

$ds^2 = -A\,d\tau^2 + \Sigma^2\left[e^B d\mathbf{x}_\perp^2 + e^{-2B} dy^2\right] + 2dr\,d\tau$

input: 


output: 

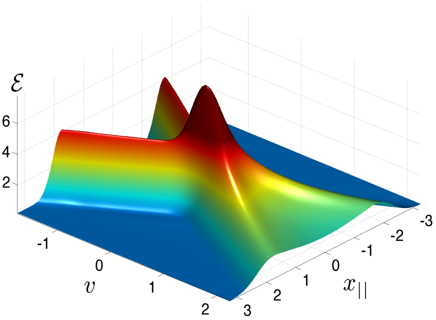

black hole forms from vacuum initial conditions.
brutally brief summary: all relaxation timescales $\tau_{th} \sim T^{-1}$.

- Lesson: In these models, breakdown of hydro in this model is not set by higher-derivative terms, but from non-hydrodynamic modes.

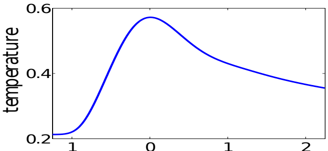
# Update

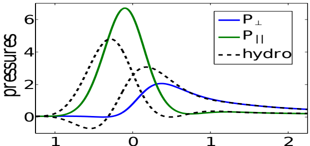
This model of plasma formation misses some physics of the heavy-ion collision.

First collisions achieved: [Chesler-Yaffe, last week]




at $x_\perp = 0$:



# Quantum quenches

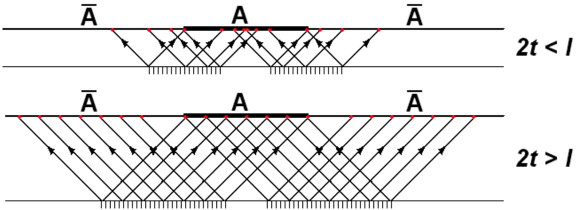
Quenches are hard to study in real solids (times too short),
but hard not to study in artificial solids (atoms in an optical lattice).

[many people]: quenches in gaussian models.

[Cardy-Calabrese]: quantum quenches in 1+1d CFT.

$$S(l,t) \simeq \frac{c}{3}\ln\frac{2\tau_0}{\pi} + \begin{cases} \frac{\pi c t}{6\tau_0} & t < l/2\,, \\ \frac{\pi c l}{12\tau_0} & t > l/2\,. \end{cases}$$

outcome: entanglement propagates like particles w/ $v = c$




Supported by holographic examples (not just 1+1d)

[Abajo-Arrastia, Aparicio, Lopez 1006; Albash, Johnson 1008] but:

heavy-ion colliders are unwieldy.

The QGP lasts for a time of order a few light-crossing times of a nucleus.

Wouldn't it be nice if we could do a quantum gravity experiment on a table top...

# Gravity duals of Galilean CFTs

(towards cold atoms at unitarity)

Note restriction to Gal.-invariance $\quad \partial_t - \vec{\nabla}^2$

distinct from: Lifshitz-like fixed points $\quad \partial_t^2 - (\vec{\nabla}^2)^2$

are not relativistic, but have antiparticles.

gravity duals of those: S. Kachru, X. Liu, M. Mulligan, 0808.1725

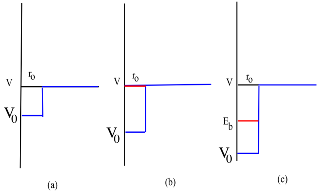
# Cold atoms at unitarity

Most of the work on holographic duality involves relativistic CFTs.

Strongly-coupled Galilean-invariant CFTs exist, even experimentally.

[Zwierlein et al, Hulet et al, Thomas et al]

Consider nonrelativistic fermionic particles ('atoms') interacting via a short-range attractive two-body potential $V(r)$, *e.g.*:



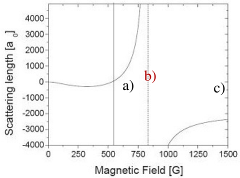

Case (b): $\sigma$ saturates bound on scattering cross section from unitarity

Range of interactions $\to 0$, scattering length $\to \infty$ $\implies$ no scale.

Lithium atoms

have a boundstate with a different magnetic moment.

Zeeman effect $\implies$ scattering length can be controlled using an external magnetic field:

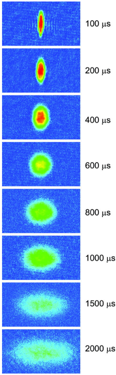


# Strongly-coupled NRCFT

The fixed-point theory ("fermions at unitarity") is a strongly-coupled nonrelativistic CFT ('Schrödinger symmetry') [Mehen-Stewart-Wise, Nishida-Son].

Universality: it also describes neutron-neutron scattering.
Two-body physics is completely solved.

Many body physics is mysterious.
Experiments: very low viscosity, $\frac{\eta}{s} \sim \frac{5}{4\pi}$ [Thomas, Schafer]
$\longrightarrow$ strongly coupled.



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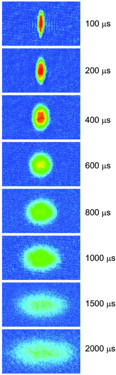


AdS/CFT?
Clearly we can't approximate it as a *relativistic* CFT.
Different hydro: conserved particle number.

# A holographic description?

Method of the missing box

AdS : relativistic CFT

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Method of the missing box

AdS : relativistic CFT

$\boxed{\textit{"Schrodinger spacetime"}}$ : galilean-invariant CFT

A metric whose isometry group is the Schrödinger group:

[Son; K Balasubramanian, JM 0804]

$$L^{-2}ds^2 = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - 2\beta^2 \frac{dt^2}{r^2}$$

This metric solves reasonable equations of motion.

Holographic prescription generalizes naturally.

# A holographic description?

Method of the missing box

| | | |
|---|---|---|
| AdS | : | relativistic CFT |
| $\boxed{\text{“\textit{Schrodinger spacetime}”}}$ | : | galilean-invariant CFT |

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This metric solves reasonable equations of motion.
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But: the vacuum of a galilean-invariant field theory is extremely boring:
no antiparticles! no stuff!
How to add stuff?

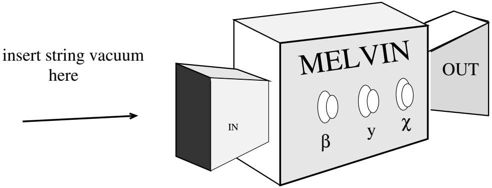
# A holographic description of more than zero atoms?

A black hole (BH) in Schrödinger spacetime.

[A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was extremely useful:

A solution-generating machine named Melvin [Ganor et al]




IN: $AdS_5 \times S^5$ OUT: Schrödinger $\times S^5$

IN: $AdS_5$ BH $\times S^5$ OUT: Schrödinger BH $\times$ squashed $S^5$

[since then, many other stringy realizations: Hartnoll-Yoshida, Gauntlett, Colgain, Varela, Bobev, Mazzucato...]

# Results so far

This black hole gives the thermo and hydro of some NRCFT ('dipole theory' [Ganor et al] ).

$$\text{Einstein gravity} \implies \frac{\eta}{s} = \frac{1}{4\pi}$$

[Iqbal-Liu].

Satisfies laws of thermodynamics, correct scaling laws, correct kubo relations.

[Rangamani-Ross-Son, McEntee-JM-Nickel]

But it's a different class of NRCFT from unitary fermions:

$$F \sim -\frac{T^4}{\mu^2}, \quad \mu < 0$$

This is because of an

Unnecessary assumption: all of Schröd must be realized geometrically.

We now know how to remove this assumption, can find more realistic models.

# A byproduct of the new realization of Schrod

A solution of 11d SUGRA which is asymptotically Schrod $\times$ stuff with non-zero particle number density which ends smoothly at $r = r_0$. [Balasubramanian, JM 1007.2184]

(like the geometries describing confining gauge theories.) [KS, MN]

no horizon $\implies$ no entropy.

Real BCs in IR $\implies$ discrete spectrum of charge excitations.

This is a "Mott" "insulator":

$\rho \neq 0$ but there is a gap to charge excitations
("Mott": there are strong interactions,

and it's not a band insulator or an Anderson insulator)

But: translation invariance $\implies \sigma(\Omega) \propto \delta(\Omega)$.

As specified, it's a perfect conductor.

**Conjecture:** if we pinned down the center-of-mass mode, it would be an insulator.

Asymptotically AdS solutions like this should exist.

An easier way to choose a rest frame (and break conformal invariance) is to study CFT at finite density.

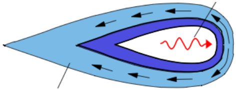
# Non-Fermi Liquids from Holography

Systems with Fermi surfaces need to be understood better.

# The standard description of fermions at finite density

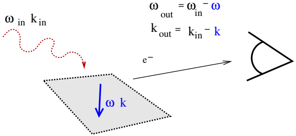
The metallic states that we understand are described by Landau's Fermi liquid theory.

Idea: Elementary excitations are free fermions with some dressing:



Landau quasiparticles $\xrightarrow{\textit{in medium}}$ surface of poles at $k_\perp \equiv |\vec{k}| - k_F = 0$ in the single-fermion Green function $G_R$.

Measurable by ARPES (angle-resolved photoemission spectroscopy):




$$\text{spectral density}: \quad A(\omega, k) \equiv \frac{1}{\pi} \mathrm{Im}\, G_R(\omega, k) \overset{k_\perp \to 0}{\to} Z\delta(\omega - v_F k_\perp)$$

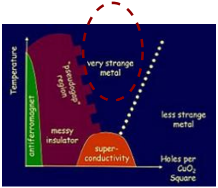
Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_\star^2$.

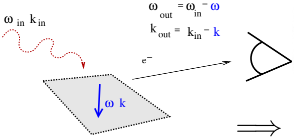
Residue $Z$ (overlap with external $e^-$) is finite on Fermi surface.

Reliable calculation of thermodynamics and transport relies on this.

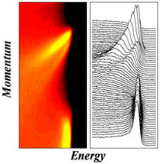
# Non-Fermi liquids exist but are mysterious

*e.g.*: 'normal' phase of optimally-doped cuprates: ('strange metal')




$\Longrightarrow$



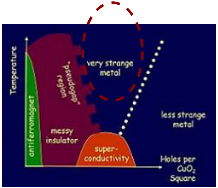

among other anomalies: ARPES shows gapless modes at finite $k$ (FS!) with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \overset{k_\perp \to 0}{\to} 0$.

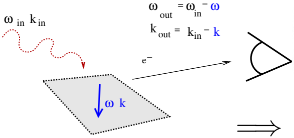
Working defintion of NFL:

Still a sharp Fermi surface
but no long-lived quasiparticles.

# Non-Fermi liquids exist but are mysterious

*e.g.*: 'normal' phase of optimally-doped cuprates: ('strange metal')

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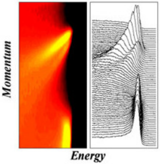


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Working defintion of NFL:

Still a sharp Fermi surface

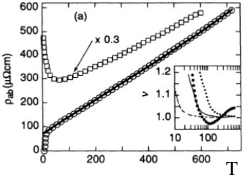
but no long-lived quasiparticles.

Most prominent

mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, e-phonon: $\rho \sim T^5$, ...

no known robust effective theory: $\rho \sim T$.



# Can string theory be useful here?

It would be valuable to have a non-perturbative description of such a state in more than one dimension.

**Gravity dual?**

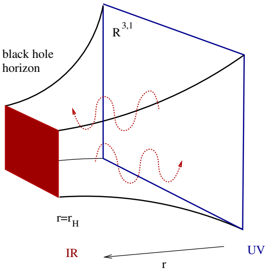
We're not going to look for a gravity dual of the whole material.

Rather: lessons for principles of “non-Fermi liquid”.

# Strategy to find a holographic Fermi surface

Consider any relativistic CFT with a gravity dual
a conserved $U(1)$ symmetry proxy for fermion number $\to A_\mu$
and a charged fermion proxy for bare electrons $\to \psi$.
$\exists$ many examples. Any $d > 1+1$, focus on $d = 2+1$.

CFT at finite density:
charged black hole (BH) in $AdS$.
To find FS: look for sharp features in fermion Green functions at finite momentum and small frequency.

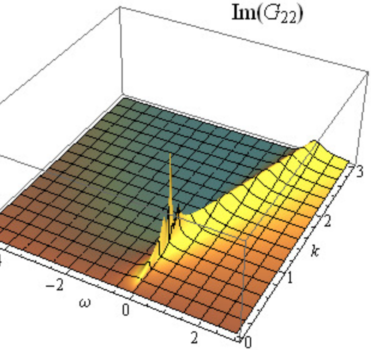



To compute $G_R$: solve Dirac equation in charged BH geometry.
'Bulk universality': for two-point functions, the interaction terms don't matter.
Results only depend on $q, m$.

# Fermi surface:

The system is rotation invariant, $G_R$ depends on $k = |\vec{k}|$.

At $T = 0$, we find numerically:




*For* $q = 1, m = 0:$ $\quad k_F \approx 0.918528499$

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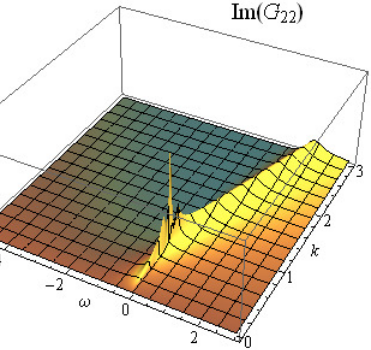



*For* $q = 1, m = 0$ : $k_F \approx 0.918528499$

## But it's not a Fermi liquid:

The peak moves
with dispersion relation $\omega \sim k_\perp^z$ with

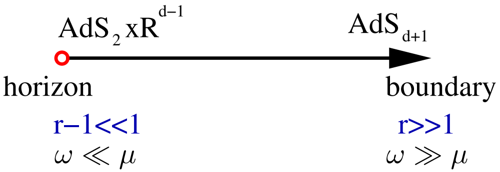
$$z = 2.09 \text{ for } q = 1, m = 0$$
$$z = 5.32 \text{ for } q = 0.6, m = 0$$

and the residue vanishes.

# Emergent quantum criticality

## Whence these exponents?

Near-horizon geometry of black hole is $AdS_2 \times \mathbb{R}^{d-1}$.
The conformal invariance of this metric is **emergent**.
(We broke the microscopic conformal invariance with finite density.)




AdS/CFT says that the low-energy physics is governed by the dual **IR CFT**.

The bulk geometry is a picture of the RG flow from the $CFT_d$ to this NRCFT.

# Analytic understanding of Fermi surface behavior

$$G_R(\omega, k) = K \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

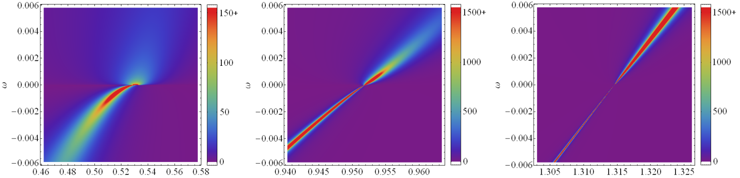
The location of the Fermi surface ($a_+^{(0)}(k = k_F) = 0$) is determined by short-distance physics (analogous to band structure –

find normalizable sol'n of $\omega = 0$ Dirac equation in full BH)

but the low-frequency scaling behavior near the FS is universal

(determined by near-horizon region – IR CFT $\mathcal{G} \sim \omega^{2\nu}$).

Depending on the dimension of the operator $(\nu + \frac{1}{2})$ in the IR CFT, we find Fermi liquid behavior (but not Landau) or non-Fermi liquid behavior:



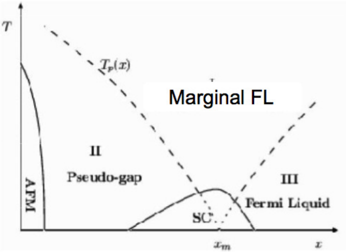

$\nu < \frac{1}{2}$ $\quad$ $\nu = \frac{1}{2}$ $\quad$ $\nu > \frac{1}{2}$

## $\nu = \frac{1}{2}$: Marginal Fermi liquid

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \ln \omega + c_1 \omega}, \quad \tilde{c}_1 \in \mathbb{R}, \quad c_1 \in \mathbb{C}$$

$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp \to 0}{\to} \text{const}, \qquad Z \sim \frac{1}{|\ln \omega_\star|} \overset{k_\perp \to 0}{\to} 0.$$

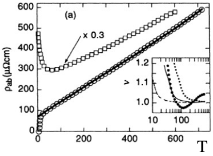
A well-named phenomenological model of high-$T_c$ cuprates near optimal doping




[Varma et al, 1989].

# Charge transport by holographic Fermi surfaces

Most prominent mystery
of strange metal phase: $\sigma_{\rm DC} \sim T^{-1}$ $\rightarrow$

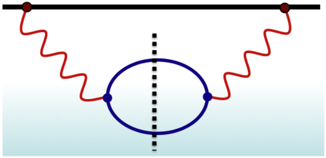



We can compute the contribution
to the conductivity from the Fermi surface.

[Faulkner, Iqbal, Liu, JM, Vegh, 1003.1728 and to appear]

$$\sigma^{\rm FS}_{\rm DC} \propto \lim_{\Omega\to 0} \frac{1}{\Omega} {\rm Im}\, \langle jj \rangle \sim T^{-2\nu}$$

from spinor particles falling into the horizon.
dissipation of current is controlled by
the decay of the fermions into the $AdS_2$ DoFs.



$\Longrightarrow$ single-particle lifetime controls transport.

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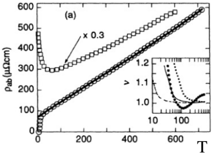


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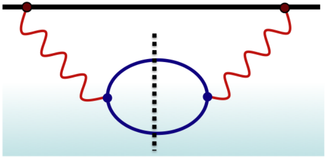

$\Longrightarrow$ single-particle lifetime controls transport.

marginal Fermi liquid: $\nu = \frac{1}{2} \Longrightarrow$ $\boxed{\rho_{FS} = \left(\sigma^{DC}\right)^{-1} \sim T}$ .

# Charged AdS black holes and frustration

Entropy density of black hole:

$$s(T = 0) = \frac{1}{V_{d-1}} \frac{A}{4G_N} \propto \rho$$

This is a large low-energy density of states!

not supersymmetric ... lifted at finite $N$

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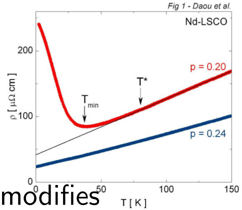
not supersymmetric ... lifted at finite $N$

pessimism: $S(T=0) \neq 0$ violates third law of thermodynamics, unphysical, weird string-theorist nonsense.

optimism:
we're describing the state
where the SC instability is removed by hand
(**here:** don't include charged scalars, **expt:** large $\vec{B}$).




[Hartnoll-Polchinski-Silverstein-Tong, 0912]: bulk density of fermions modifies extreme near-horizon region (out to $\delta r \sim e^{-N^2}$), removes residual entropy. (Removes non-analyticity in $\Sigma(\omega)$ for $\omega < e^{-N^2}\mu$)

$\exists$ other possible endpoints, *e.g.* from neutral scalars [Goldstein et al, Kiritsis et al, Gursoy].

# Stability of the groundstate

Charged bosons: In many explicit dual pairs, $\exists$ charged scalars.
• At small $T$, they can condense spontaneously breaking the $U(1)$ symmetry, changing the background [Gubser, Hartnoll-Herzog-Horowitz].

spinor: $G_R(\omega)$ has poles only in LHP of $\omega$ [Faulkner-Liu-JM-Vegh, 0907]

scalar: $\exists$ poles in UHP $\quad \langle \mathcal{O}(t) \rangle \sim e^{i\omega_\star t} \propto e^{+\mathrm{Im}\,\omega_\star t}$

$\Longrightarrow$ growing modes of charged operator: holographic superconductor

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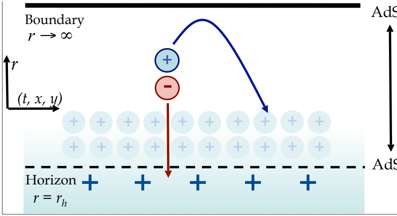
**why:** black hole *spontaneously* emits charged particles [Starobinsky, Unruh, Hawking].

AdS is like a box: they can't escape.

Fermi: negative energy states get filled.

Bose: the created particles then cause *stimulated emission* (superradiance).

A holographic superconductor is a "black hole laser".

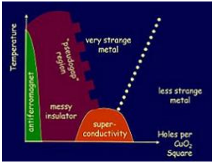


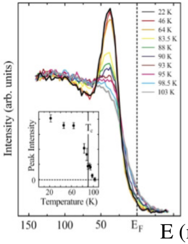
# Photoemission 'exp'ts' on holographic superconductors

$S_{BH}(T=0) \implies$ instability.

With charged scalars in bulk, groundstate is superconducting. [Gubser; Hartnoll et al 2008]

In SC state: a sharp peak forms in $A(k,\omega)$.



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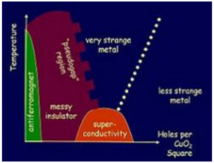
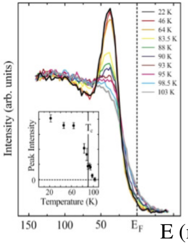



With a suitable coupling between $\psi$ and $\varphi$, the superconducting condensate opens a gap in the fermion spectrum.

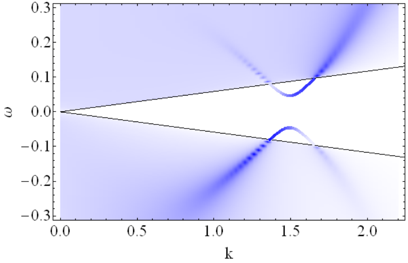
[Faulkner, Horowitz, JM, Roberts, Vegh, 0911.3402]

For $q_\varphi = 2q_\psi$

$$L_{\text{bulk}} \ni \eta_5 \varphi \bar{\psi} \mathcal{C} \Gamma^5 \bar{\psi}^T + \text{h.c}$$

The (gapped) quasiparticles are exactly stable in a certain kinematical regime

(outside the lightcone of the IR CFT) – the condensate lifts the IR CFT modes into which they decay.



# Frameworks for non-Fermi liquid

- a Fermi surface coupled to a critical boson field

[Halperin-Lee-Read, Polchinski, Altshuler-Ioffe-Millis, Nayak-Wilczek, ... More recently: S-S Lee, Metlitski-Sachdev, Mross-JM-Liu-Senthil]

$$L = \bar{\psi}\left(\omega - v_F k_\perp\right)\psi + \bar{\psi}\psi a + L(a)$$

small-angle scattering dominates $\Longrightarrow$ transport is not that of strange metal.

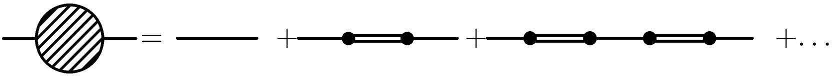
---

- a Fermi surface mixing with a bath of critical fermionic fluctuations with large dynamical exponent

[FLMV 0907.2694, Faulkner-Polchinski 1001.5049, FLMV+Iqbal 1003.1728]

$$L = \bar{\psi}\left(\omega - v_F k_\perp\right)\psi + \bar{\psi}\chi + \psi\bar{\chi} + \bar{\chi}\mathcal{G}^{-1}\chi$$

$\chi$: IR CFT operator



$$\langle\bar{\psi}\psi\rangle = \frac{1}{\omega - v_F k_\perp - \mathcal{G}} \qquad \mathcal{G} = \langle\bar{\chi}\chi\rangle = c(k)\omega^{2\nu}$$

$\nu \le \frac{1}{2}$: $\bar{\psi}\chi$ coupling is a relevant perturbation.

# Comments

1. The green's function near the FS is of the form ('local quantum criticality', analytic in $k$.) found previously in perturbative calculations, but the nonanalyticity can be order one.
   This is an *input* of many studies (dynamical mean field theory)
2. [Sachdev, 1006.3794]: Slave-particle solution of large-$d$, large-spin limit of *random* antiferromagnet produces a very similar IR CFT [Sachdev-Ye].
3. The leading $N^{-1}$ contribution to the free energy exhibits quantum oscillations in a magnetic field. [Denef-Hartnoll-Sachdev]
   Shape different from FL. [Hartnoll-Hofman]
4. Main challenge: step away from large $N$. So far:
   - Fermi surface is a small part of a big system.
   - Fermi surface does not back-react on IR CFT.

# Gapped states at strong coupling

(towards fractional topological insulators)

[Following A. Karch]

# EFT of 3+1d insulators (is boring!)

low-energy dofs: $A_\mu$ symmetries: rotations, translations, $\mathcal{T}$

$$S_{\text{eff}} = \frac{1}{8\pi}\int dtd^3x \left(\epsilon\vec{E}^2 + \frac{1}{\mu}\vec{B}^2\right.$$

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$$\mathcal{T} : \vec{E} \to \vec{E}, \vec{B} \to -\vec{B} \qquad \theta \to -\theta$$

flux quantiz'n $\Longrightarrow \frac{\alpha}{32\pi^2} \int_M \vec{E}\cdot\vec{B} = \frac{\alpha}{32\pi^2} \int_M F \wedge F = n \in \mathbb{Z}$ $M$ closed.

$$Z(\theta) = \sum_{n\in\mathbb{Z}} \mathcal{A}_n e^{i\theta n} \Longrightarrow \theta \simeq \theta + 2\pi$$

$$\Longrightarrow -\theta \simeq \theta \Leftrightarrow \theta = 0 \text{ or } \pi \bmod 2\pi. \qquad \text{(trivial insulator or TI)}$$

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So? $\theta F\wedge F = \theta d(A\wedge F)$.

Boundary of TI (= domain wall of $\theta$) has Hall response (half IQH):

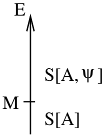
$$j^\mu = \frac{e^2}{2\pi h}\epsilon^{\mu\nu\rho\sigma}\partial_\nu\theta\partial_\rho A_\sigma \implies \sigma_H = \frac{e^2}{h}\frac{\Delta\theta}{2\pi} = \frac{e^2}{h}\left(\frac{1}{2}+n\right)$$

This EFT predicts many novel properties of TI response to external fields.

(Faraday and Kerr effects, Witten effect on image charge)

From protected gapless edge modes.

# Slightly more microscopically




$$L = \bar{\psi}\left(i\left(\partial_\mu + iqA_\mu\right)\gamma^\mu - M\right)\psi$$

$\mathcal{T} : M \mapsto M^\star$. $\qquad$ $\mathcal{T}$-invt $\implies M = \pm|M|$, real.

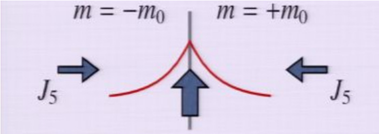
chiral rotation: $\psi \to e^{-i\varphi\gamma_5/2}\psi$ is a classical sym of $L|_M = 0$.

$$\bar{\psi}\psi \to e^{-i\varphi}\bar{\psi}\psi \implies M \to e^{i\varphi}M$$

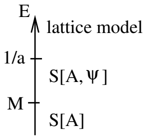
ABJ anomaly: $\Delta L = \mathcal{C}\alpha \frac{\varphi}{32\pi^2} \mathrm{tr}\ \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$
$\mathcal{C} = \sum_{\text{fields}} q^2$. $\implies \theta \mapsto \theta - \mathcal{C}\varphi$.

integrate anomaly: $S_{\text{eff}}[A] = \ln \int [d\psi] e^{iS[\psi, A]} = \mathcal{C}\frac{\alpha}{16\pi}\int F \wedge F$

boundary zeromode: $(i\not{D} - M)\psi = 0$
has an $E = 0$ normalizable
soln at domain wall of $\theta$. (anomaly inflow)



# TI from free electrons on lattice




$\frac{\theta}{2\pi}$ = top invt of band structure [Kane,Mele,Fu,Zhang,Qi...]

single-particle wavef'ns live in

a vector bundle over the Brillouin zone $\simeq T^3$

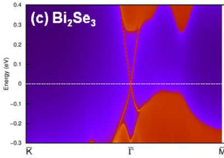
$$\frac{\theta}{2\pi} = \frac{1}{16\pi^2}\int d^3k\left(a \wedge f - \frac{2}{3} a \wedge a \wedge a\right)$$

$a_i^{\alpha\beta} \equiv -i\langle \alpha, \vec{k}|\partial_{k^i}|\beta, \vec{k}\rangle$ Berry connection

$\alpha, \beta$ band indices, $\vec{k} \in BZ$, $f = da$

An important role is played by spin-orbit couplings.

Edge states observed by ARPES $\longrightarrow$




K-theory classification of vector bundles classifies *free*-electron TI in various dims

[Kitaev; Ludwig, Ryu et al]

Not coincidentally, same as classification of D-brane charges.

[Ryu-Takayanagi]

| | Bulk topological insulator | | Boundary theory With anomaly | |
|---|---|---|---|---|
| 4+1 | Callan-Harvey, Kaplan | Z | axial anomaly | 3+1 |
| 3+1 | TRI topological insulator | $Z_2$ | parity anomaly | 2+1 |
| 2+1 | Quantum Hall | Z | chiral anomaly | 1+1 |

# Interacting Topological Insulators?

| IQHE | : | FQHE | :: | TI | : | ? |
|---|---|---|---|---|---|---|
| $\sigma_H = n\frac{e^2}{h}$ | | $\sigma_H = \frac{n}{m}\frac{e^2}{h}$ | | $\sigma_H^{\text{bdy}} = \frac{1}{2}\frac{e^2}{h}$ | | $\sigma_H^{\text{bdy}} = \frac{1}{m}\frac{1}{2}\frac{e^2}{h}$ |
| 1-particle physics | | interactions shatter electron | | | | |

"slave particle construction" for FQHE [Wen]:

- suppose $e = c_a c_b c_c \epsilon^{abc}$ $\quad$ $c^\dagger$ create charge $1/3$ 'partons'

$c_a \mapsto U_a^b c_b$, $U \in SU(3)$ preserves $e$. gauge redundancy.

- suppose each $c$ lives in a $\nu = 1$ IQHE state. gapped.

EFT below gap (includes $SU(3)$ gauge field):

$$e^{iS_{\text{eff}}[a,A]} = \int [dc] e^{iS_{IQH}[c,a,A]} = e^{iC(CS[A+a])}$$

$\Longrightarrow$ $a$ is massive and we can ignore it.

$\Longrightarrow$ $\nu = 1/3$ QH response.

This gives a trial wavefunction for $e$

# Towards fractional TI

Try $e = c_a c_b c_c \epsilon^{abc}$ put each $c$ into a 3+1d TI band structure

[Maciejko-Karch-Qi-Zhang 1004.3628]

then $\int [dc] e^{iS_{TI}[c,a,A]} = e^{iC \int E \cdot B + S[a]}$

in 3+1-d no gauge-invt mass terms! strongly interacting $a$ with $\theta = \pi$. IR dynamics?

Need: deconfined phase with $\mathcal{T}$ not spontaneously broken.

[Witten 97]: AdS/CFT $\implies$ at large $N$ $\mathcal{T}$ is spontaneously broken in YM at $\theta = \pi$

Note: can distinguish (in principle) from ITI + FQH on surface.

[Swingle-Barkeshli-JM-Senthil 1005.1076] (non-holographic):

- examples of robust FTI states: Higgs $SU(3)$
- FTI states of bosons: $b = c_\alpha c_\beta$ with $G = SU(2)$

$H \ni c^\dagger_{a\alpha} \vec{d}_{ab} \cdot \vec{\sigma}_{\alpha\beta} c_\beta$ ($\vec{d}_{ab} = \langle higgs \rangle$) *is* the spin-orbit coupling.

- fractional $\theta/2\pi$ and a gap *requires* topological order.

[Karch et al 1007.3253, 1009.2991]: Holographic (brane) constructions of FTI existence proof for this phase.

# Concluding remarks

# Other progress I would have liked to have discussed

- models of spin physics [Iqbal et al, Gursoy]:

  holographic superconductor phenomenon happens even for $q = 0$

- holographic mechanisms for quantum phase transitions:

  [Iqbal et al, Jensen et al]: In geometries with an $AdS_2$, let the IR mass of some field vary with a parameter.

  If it hits the BF bound, a BKT transition.

  [Faulker-Horowitz-Roberts]: In geometries with an $AdS_2$, if one finds a holographic superconductor instability of a field with $m^2_{IR} > m^2_{IR\ BF}$, multitrace deformations of the UV theory can cause a phase transition.

- interacting Lifshitz fixed points [Kachru-Liu-Mulligan 08]: The free Lifshitz scalar theory $L = (\partial_t \phi)^2 - \left(\nabla^2 \phi\right)^2$ at finite $T$ has ultralocal correlations [Ghaemi et al 06].

  Interacting versions are more normal [Balasubramanian, JM 09] .

- brane models of transport [Karch et al, Tong et al]

# Remarks on the roles of SUSY and string theory

Supersymmetry

- constrains the form of interactions.

  fewer candidates for dual, more checks
- allows *lines* of fixed points (*e.g.* $\mathcal{N} = 4$ SYM).

  coupling = dimensionless parameter
- is broken by finite $T, \mu$, in these pplications anyway.
  It's not clear what influence it has on the resulting states.

String theory

- tells us consistent ways to UV complete our gravity model.

  So far, no known constraints that aren't visible from EFT.

  And we can't find the physics we want in *any* gravity model ...
- can provide a microscopic description of the dual QFT.

  Honesty: Any $L_{micro}$ that we would get from string theory is so far from $L_{Hubbard}$ anyway that it isn't clear how it helps.
- suggests interesting resummations of higher-derivative terms, protected by stringy symmetries.

  *e.g.* the DBI action $L_{DBI} \sim \sqrt{1 - F^2}$ is 'natural' in string theory because its form is protected by the T-dual Lorentz invariance.

# What's special about Einstein gravity?

a) It describes our universe well and we've studied it a lot.

The bending-of-light-by-the-sun experiments say nothing about the gravity theory describing the possible dual of some condensed matter.

b) It's what comes from perturbative string theory in flat space at leading order in the $\alpha'$ and $g_s$ expansions.

c) The einstein term is the leading irrelevant operator (besides the cosmological constant) by which we can couple metric fluctuations. RG flows in the bulk correspond to some kind of uber-motion in the space of field theories.

Holographic universality: many *classes* of RG fixed points (univ. classes!) may be described by the same gravity dual

(near a fixed point in the space of bulk gravity theories.).

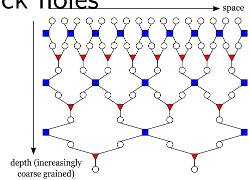
(perhaps they are all very special (large $N$, hierarchy in operator dimensions))

[Heemskerk et al, FItzpatrick et al]

(eg: spectral density calculation only depended on *quadratic terms* in bulk action.)

# Lessons for gravity from many-body physics

- violation of no-hair expectations for AdS black holes




- entanglement RG [G. Vidal]:
a real space RG which keeps track of entanglement
builds an extra dimension
$ds^2 = dS^2$ [Swingle 0905.1317, Raamsdonk 0907.2939]

- information is not lost in BH evaporation

- basic facts about QM forbid traversable wormholes in $AdS$
(information can't propagate between decoupled theories) [Swingle, to appear]
even at finite $N$, small $\lambda \Longrightarrow \exists$ "quantum horizons"

- weak evidence for weak gravity conjecture [Arkani et al] from studies of holographic superconductors [Denef-Hartnoll, 0901]

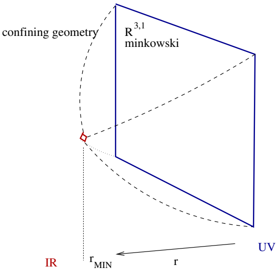
# Are there new strongly-coupled phases of matter?

Ab initio prediction of liquid phase? [Weisskopf]

# An old strongly-coupled phase of matter from holography

If we didn't happen to be made from the excitations of a confining gauge theory (QCD), [H Liu]
we would have predicted color confinement using AdS/CFT.

A cartoon by which we would have discovered confinement:




(hologram:

if IR region is missing, no low-energy excitations, mass gap.)

# Are there new strongly-coupled phases of matter?

Ab initio prediction of liquid phase? [Weisskopf]
of confinement, of superconductivity, of fractional quantum Hall states...

Our ability to imagine possibilities for phases of matter so far has been limited by weak coupling descriptions
and by our ability to build things.

**What other phases of matter may still be hidden?**

The end. Thanks for listening.