Strong Correlations with String Theory

John McGreevy
Bold claim: string theory is useful.

My goal for today is to convince you that string theory can be useful for physics.

The physical systems about which we can hope to say something have in common strong coupling or strong correlations. This feature is a big problem for our usual techniques.

This opportunity comes about in a very sneaky way, and to explain it, we have to back up a bit.
Unity of purpose in hep-th and cond-mat

A string theorist’s instruction manual for doing theoretical physics:

**Step 1:** Identify the quantum field theory (QFT) that describes your system.

*e.g.* there’s one for QED, QCD, ferromagnets, high-Tc, ...
Unity of purpose in hep-th and cond-mat

A string theorist’s instruction manual for doing theoretical physics:

**Step 1:** Identify the quantum field theory (QFT) that describes your system.
* e.g. there’s one for QED, QCD, ferromagnets, high-Tc, ...

**Step 2:** Figure out what happens:
What is the groundstate?
What are the low-energy excitations above the groundstate?
(In favorable cases: ‘elementary particles’ or ‘quasiparticles’)

Sometimes we can answer these questions using ordinary tricks.
Basically, perturbation theory around a ‘solvable’ theory.

When the interactions are not a small perturbation, this fails.
Plan for this talk

Here’s the sneaky way of using string theory:

We can answer these questions about some field theories using an **auxiliary** string theory, some ground state of string theory that looks nothing like ours:
Plan for this talk

Here’s the sneaky way of using string theory:

We can answer these questions about some field theories using an auxiliary string theory, some ground state of string theory that looks nothing like ours:

This relation is called ‘the AdS/CFT Correspondence’ or ‘Holographic Duality’. I’m going to explain its origins.
Plan for this talk

Here’s the sneaky way of using string theory:

We can answer these questions about some field theories using an auxiliary string theory, some ground state of string theory that looks nothing like ours:

This relation is called ‘the AdS/CFT Correspondence’ or ‘Holographic Duality’. I’m going to explain its origins.

Then I’ll talk about three classes of real physical systems (which involve strong interactions between the constituents) where usual techniques have been having a hard time and where we’ve been trying to use these ideas to learn something about physics.
Universality and coarse-graining in field theory

and other systems with extensive degrees of freedom.
Old-school universality

Experimental universality [1960s]:
same critical exponents from (microscopically) very different systems.

Near a (continuous) phase transition (at $T = T_c$), scaling laws:
observables depend like power laws on the deviation from the critical point.
Old-school universality

Experimental universality [1960s]:
same critical exponents from (microscopically) very different systems.

Near a (continuous) phase transition (at $T = T_c$), scaling laws:
observables depend like power laws on the deviation from the critical point.

E.g. ferromagnet near the Curie transition

(let $t \equiv \frac{T_c - T}{T_c}$)

specific heat: $c_v \sim t^{-\alpha}$
magnetic susceptibility: $\sim t^{-\gamma}$

[MIT TSG]
Old-school universality

Experimental universality [1960s]:
same critical exponents from (microscopically) very different systems.

Near a (continuous) phase transition (at $T = T_c$), scaling laws:
observables depend like power laws on the deviation from the critical point.

E.g. ferromagnet near the Curie transition
(let $t \equiv \frac{T_c - T}{T_c}$)

Specific heat: $c_v \sim t^{-\alpha}$
Magnetic susceptibility: $\sim t^{-\gamma}$

Water near its liquid-gas critical point:

Specific heat: $c_v \sim t^{-\alpha}$
Compressibility: $\sim t^{-\gamma}$

With the same $\alpha, \gamma$!
Renormalization group idea

This phenomenon is explained by the Kadanoff-Wilson idea:

\[
\begin{align*}
\text{eg: } S_i &= \pm 1 \\
H &= \sum_{\text{neighbors, } \langle ij \rangle} J_{ij} S_i S_j + \sum_{\text{next neighbors, } \langle \langle ij \rangle \rangle} K_{ij} S_i S_j + \ldots
\end{align*}
\]
Renormalization group idea

This phenomenon is explained by the Kadanoff-Wilson idea:

\[ S_i = \pm 1 \quad H = \sum_{\text{neighbors}, \langle ij \rangle} J_{ij} S_i S_j + \sum_{\text{next neighbors}, \langle \langle ij \rangle \rangle} K_{ij} S_i S_j + \ldots \]

Idea: measure the system with coarser and coarser rulers.
Let ‘block spin’ = average value of spins in block.
Renormalization group idea

This phenomenon is explained by the Kadanoff-Wilson idea:

\[ S_i = \pm 1 \quad H = \sum_{\text{neighbors, } \langle ij \rangle} J_{ij} S_i S_j + \sum_{\text{next neighbors, } \langle\langle ij \rangle\rangle} K_{ij} S_i S_j + \ldots \]

Idea: measure the system with coarser and coarser rulers.

Let ‘block spin’ = average value of spins in block.

Define a Hamiltonian \( H(r) \) for block spins so long-wavelength observables are the same.

\[ \rightarrow \text{a ‘renormalization group’ (RG) flow on the space of hamiltonians: } H(r) \]
RG fixed points give universal physics

Universality: fixed points are rare. Many microscopic theories will flow to the same fixed-point. \( \implies \) same critical exponents.

The fixed point theory is scale-invariant:
if you change your resolution you get the same picture back.
RG fixed points give universal physics

Universality: fixed points are rare. Many microscopic theories will flow to the same fixed-point. $\implies$ same critical exponents.

The fixed point theory is scale-invariant:
if you change your resolution you get the same picture back.

Sometimes the fixed point theory is also ‘Conformally invariant’.
This is the ‘C’ in AdS/CFT.
Quantum field theory (QFT)

Questions about long-wavelength modes, wavelength $\gg$ lattice spacing:

lattice details absorbed in couplings between long-wavelength modes

$\rightarrow$ continuum description: this is a QFT.

In general: QFT = a perturbation of an RG fixed point.

The same theoretical construct is used to describe high-energy particle physics.
Quantum field theory (QFT)

Questions about long-wavelength modes, wavelength $\gg$ lattice spacing:

lattice details absorbed in couplings between long-wavelength modes
$\rightarrow$ continuum description: this is a QFT.

In general: QFT = a perturbation of an RG fixed point.

The same theoretical construct is used to describe high-energy particle physics.

**BUT**: This procedure (the sums) is hard to do in practice!
The answer is not always freely-propagating sound waves.
Not everything is harmonic oscillators with small nonlinearities!
Strongly coupled QFTs are at the heart of central problems of modern physics.
Quantum field theory (QFT)

Questions about long-wavelength modes, wavelength $\gg$ lattice spacing:

lattice details absorbed in couplings between long-wavelength modes
$\rightarrow$ continuum description: this is a QFT.
In general: QFT = a perturbation of an RG fixed point.

The same theoretical construct is used to describe high-energy particle physics.

**BUT**: This procedure (the sums) is hard to do in practice!
The answer is not always freely-propagating sound waves.
Not everything is harmonic oscillators with small nonlinearities!
Strongly coupled QFTs are at the heart of central problems of modern physics.

Wouldn’t it be nice if the picture satisfied some nice equation?
Some remarks on the curious scientific status of string theory
What is string theory?

String theory is an alien artifact, discovered in the wreckage of hadronic resonances.

- 1960s: it was used as a model of the Strong Interactions.
What is string theory?

String theory is an alien artifact, discovered in the wreckage of hadronic resonances.

• 1960s: it was used as a model of the Strong Interactions.

• 1970s: people realized that it actually contains gravity.
What is string theory?

String theory is an alien artifact, discovered in the wreckage of hadronic resonances.

- 1960s: it was used as a model of the Strong Interactions.

- 1970s: people realized that it actually contains gravity.

- 1980s: people realized that it has vacua that look like the Standard Model of particle physics, coupled to gravity.

→ much rejoicing.
What is string theory?

We still don’t know!
What is string theory?

We still don’t know!

Our current description of string theory is something like this:

We understand limits by various approximate descriptions. We have a machine doing perturbation theory around free strings (many harmonic oscillators).
What is string theory?

We still don’t know!

Our current description of string theory is something like this:

We understand limits by various approximate descriptions. We have a machine doing perturbation theory around free strings (many harmonic oscillators).
What you need to know about string theory for this talk:

1. String theory is a quantum theory of gravity.
What you need to know about string theory for this talk:

1. String theory is a quantum theory of gravity.
2. In the vacua we understand, at low energies, string theory reduces to Einstein gravity (GR).
What you need to know about string theory for this talk:

1. String theory is a quantum theory of gravity.
2. In the vacua we understand, at low energies, string theory reduces to Einstein gravity (GR).
3. It is a unique theory which has many ground states, which has many ground states,
What you need to know about string theory for this talk:

1. String theory is a quantum theory of gravity.
2. In the vacua we understand, at low energies, string theory reduces to Einstein gravity (GR).
3. It is a unique theory which has many ground states, some of which look like our universe...
   (with 3+1 dimensions, particle physics, ...)
   These vacua ARE NOT the subject of today’s talk.)
What you need to know about string theory for this talk:

1. String theory is a quantum theory of gravity.
2. In the vacua we understand, at low energies, string theory reduces to Einstein gravity (GR).
3. It is a unique theory, which has many ground states, some of which look like our universe...
   (with 3+1 dimensions, particle physics, ...)
   These vacua ARE NOT the subject of today’s talk.)
   most of which don’t.
The holographic principle

Gravity is different.
Black hole thermodynamics

Gravity $\implies$ black holes. (regions of no escape)

There are close parallels between black hole (BH) mechanics and the Laws of Thermodynamics. [70s]

Consistent laws of thermo require BH has entropy: $(k_B = 1)$

$$S_{BH} = \frac{\text{area of horizon}}{4\ell_p^2}. \quad \ell_p \equiv \sqrt{\frac{G_N \hbar^2}{c^3}}.$$ 

‘Generalized 2d Law’: $S_{total} \equiv S_{\text{ordinary stuff}} + S_{BH}$

$\Delta S_{total} \geq 0$ in processes which happen. [Bekenstein]
Holographic principle

Recall: In an ordinary $d$-dim’l system without gravity (a chunk of stuff, the vacuum...) DoFs at each point $\Rightarrow$ max entropy in some region of space $\sim$ volume $L^d$
Holographic principle

Recall: In an ordinary $d$-dim’l system without gravity (a chunk of stuff, the vacuum...)  
DoFs at each point $\implies$  
max entropy in some region of space $\sim$ volume $L^d$

Holographic Principle: In a gravitating system,  
max entropy in a region of space $V =$  
entropy of the biggest black hole that fits.

$$S_{max} = S_{BH} = \frac{1}{4\pi G_N} \times \text{horizon area}$$

$\propto \text{area of } \partial V \text{ in planck units.}$  
[t’Hooft, Susskind 1990s]

Why: suppose the contrary, a configuration with  
$S > S_{BH} = \frac{A}{4G_N}$  but $E < E_{BH}$ (biggest BH fittable in $V$)  
Then: throw in junk (increases $S$ and $E$) until you make a BH.  
$S$ decreased, violating 2d law.
Punchline: Gravity in \( d + 1 \) dimensions

has the same number of degrees of freedom as

a QFT in fewer \( (d) \) dimensions.

Questions:

- Who is the QFT on the boundary?
- From its point of view, what is the extra dimension?
- Where do I put the boundary?
Holographic duality (AdS/CFT)

gravity in some spacetime $AdS_{d+2}$

\[
(ds^2 = \frac{R^2}{r^2} (-dt^2 + d\vec{x}^2) + \frac{R^2 dr^2}{r^2})
\]
Holographic duality (AdS/CFT)

gridy in some spacetime $AdS_{d+2}$

\[
(\quad ds^2 = \frac{R^2}{r^2} (-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} dr^2 \quad )
\]

EQUALS
Holographic duality (AdS/CFT)

gravity in some spacetime $AdS_{d+2}$

$$ds^2 = \frac{R^2}{r^2} (-dt^2 + d\vec{x}^2) + \frac{R^2 dr^2}{r^2}$$

EQUALS

[McDacena]

a CFT in $d + 1$ spacetime dimensions

• No proof yet. A zillion checks.
• LHS: 'bulk' RHS: 'boundary'. You'll see why on next slide.
Holographic duality (AdS/CFT)

First check: symmetries of AdS = the relativistic conformal group. including scale invariance: $\vec{x} \to \lambda \vec{x}, t \to \lambda t, r \to \lambda r$ preserves $ds^2$. 

$$ds^2 = \frac{R^2}{r^2} (-dt^2 + d\vec{x}^2) + \frac{R^2 dr^2}{r^2}$$
Holographic duality (AdS/CFT)

First check: symmetries of AdS = the relativistic conformal group. including scale invariance: \( \vec{x} \rightarrow \lambda \vec{x}, t \rightarrow \lambda t, r \rightarrow \lambda r \) preserves \( ds^2 \).

The extra (‘radial’) dimension is the resolution scale!
The bulk picture is a hologram:
Things with different wavelengths get put in different places.

Boundary at \( r = 0 \): UV data is ‘initial’ conditions for RG flow.

\[
ds^2 = \frac{R^2}{r^2} (-dt^2 + d\vec{x}^2) + \frac{R^2 dr^2}{r^2}
\]
Best-understood example (of infinitely many)

Role of string theory: identify precise dual pairs. "\( \mathcal{N} = 4 \text{ SYM} \)" is a CFT. (a supersymmetric, relativistic gauge theory).
Best-understood example (of infinitely many)

Role of string theory: identify precise dual pairs.
“$\mathcal{N} = 4$ SYM” is a CFT.
(a supersymmetric, relativistic gauge theory).
A gauge theory comes with two parameters:
– a coupling constant $\lambda$,
– an integer, the number of colors $N$. 
Best-understood example (of infinitely many)

Role of string theory: identify precise dual pairs.

“$\mathcal{N} = 4$ SYM” is a CFT.
(a supersymmetric, relativistic gauge theory).

A gauge theory comes with two parameters:
– a coupling constant $\lambda$,
– an integer, the number of colors $N$.

$$\mathcal{N} = 4 \text{ SYM}_{N,\lambda} = \text{IIB strings in } AdS_5 \times S^5 \text{ of size } \lambda, \hbar = 1/N$$

• large $N$ makes gravity classical
(suppresses splitting and joining of strings)

• strong coupling (large $\lambda$) makes the geometry big.
Best-understood example (of infinitely many)

Role of string theory: identify precise dual pairs.

“$\mathcal{N} = 4$ SYM” is a CFT.
(a supersymmetric, relativistic gauge theory).
A gauge theory comes with two parameters:
– a coupling constant $\lambda$,
– an integer, the number of colors $N$.

\[
\mathcal{N} = 4 \text{ SYM}_{N,\lambda} = \text{IIB strings in } AdS_5 \times S^5 \text{ of size } \lambda, \hbar = 1/N
\]

- large $N$ makes gravity classical
  (suppresses splitting and joining of strings)
- strong coupling (large $\lambda$) makes the geometry big.

**strong/weak duality:** hard to check, very powerful
Holographic duality at finite temperature

Black holes radiate like blackbodies [Hawking].

gravity in $AdS_{d+2}$ with a black hole
Holographic duality at finite temperature

Black holes radiate like blackbodies [Hawking].

g

gravity in $AdS_{d+2}$ with a black hole

EQUALS [Witten]

CFT in $d + 1$ spacetime dimensions at finite temperature
Holographic duality at finite temperature

Black holes radiate like blackbodies [Hawking].

gravity in $AdS_{d+2}$ with a black hole

EQUALS [Witten]

CFT in $d+1$ spacetime dimensions at finite temperature

GR ‘no-hair theorem’:
black holes labelled by few parameters
(mass, charge)

←→

Thermal equilibrium states labelled by few parameters
(temperature, chemical potential)
What can be computed

[Ref: Gubser-Klebanov-Polyakov, Witten]

fields in the bulk $\leftrightarrow$ local operators in the QFT

Compute correlation functions by solving classical wave equations.

New perspective on the structure of QFT: access to otherwise uncalculable things

$G(\omega, k, T)$ potentials for moving probes

entanglement entropy

uncalculable situations at strong coupling far from equilibrium in real time with a finite density of fermions
Applications of holographic duality to quantum liquids

Next I’ll discuss three example systems to which we can apply these ideas.
Example 1: The Strong Interactions

The theory of the Strong Interactions (QCD) is also a gauge theory. Unlike $\mathcal{N} = 4$ SYM, it’s not a CFT; the coupling runs.

For length scales longer than $1\text{GeV}^{-1}$:

CONFINEMENT.

We still lack a quantitative theoretical understanding of this phenomenon.
**Example 1: The Strong Interactions**

The theory of the Strong Interactions (QCD) is also a gauge theory. Unlike $\mathcal{N} = 4$ SYM, it’s not a CFT; the coupling runs.

For length scales longer than $1\text{GeV}^{-1}$:

**CONFINEMENT.**

We still lack a quantitative theoretical understanding of this phenomenon.

Holography provides a useful image:

Spectrum of hadrons: resonances in this cavity.
Example 1: The Strong Interactions

The theory of the Strong Interactions (QCD) is also a gauge theory. Unlike $\mathcal{N} = 4$ SYM, it’s not a CFT; the coupling runs.

For length scales longer than $1 GeV^{-1}$:

CONFINEMENT.

We still lack a quantitative theoretical understanding of this phenomenon.

Holography provides a useful image:

Spectrum of hadrons: resonances in this cavity.

A new state of condensed matter [RHIC, LHC]:

...
Quark-gluon plasma is strongly coupled

QGP is strongly coupled: a liquid, not a gas.
(RHIC, LHC not in asymptotically free regime.)
Quark-gluon plasma is strongly coupled

QGP is strongly coupled: a liquid, not a gas. (RHIC, LHC not in asymptotically free regime.)

1. It is opaque:

2. It exhibits rapid thermalization, rapid hydro-ization to a fluid with very low viscosity. It exhibits collective motion (‘elliptic flow’):

[O’Hara et al]
Holographic gauge theory plasma

Positive outcomes of approximating QCD in this regime by a QFT with a gravity dual:

▶ String theorists have learned lots of physics.
▶ The holographic plasma provided a proof of principle that low viscosity $\frac{\eta}{s} \sim \frac{1}{4\pi}$ was possible
  (vs: perturbation theory prediction of $\frac{\eta}{s} = \frac{1}{g^4 \ln g}$ with $g \ll 1$).
▶ Beautiful studies of hydrodynamics by BH horizon fluctuations

Where’s the dissipation? Energy falls into BH. [Horowitz-Hubeny, 99]
Holographic gauge theory plasma

Positive outcomes of approximating QCD in this regime by a QFT with a gravity dual:

- String theorists have learned lots of physics.
- The holographic plasma provided a proof of principle that low viscosity $\eta/s \sim \frac{1}{4\pi}$ was possible (vs: perturbation theory prediction of $\frac{\eta}{s} = \frac{1}{g^4 \ln g}$ with $g \ll 1$).
- Beautiful studies of hydrodynamics and its onset by BH horizon fluctuations and gravitational collapse.

Where’s the dissipation? Energy falls into BH. [Horowitz-Hubeny, 99]

[Chesler-Yaffe] (PDEs!)
but:
RHIC and LHC unwieldy.
The QGP lasts for a time of order a few light-crossing times of a nucleus.

Wouldn’t it be nice if we could do a quantum gravity experiment on a table top...
Example 2:

Galilean CFT liquid from holography
(towards cold atoms at unitarity)
Cold atoms at unitarity

Most of the work on AdS/CFT involves relativistic CFTs. Strongly-coupled Galilean-invariant CFTs exist, even experimentally. [Zwierlein et al, Hulet et al, Thomas et al]

Consider nonrelativistic fermionic particles (‘atoms’) interacting via a short-range attractive two-body potential $V(r)$, e.g.:

Case (b): $\sigma$ saturates bound on scattering cross section from unitarity
Range of interactions $\rightarrow 0$, scattering length $\rightarrow \infty \Rightarrow$ no scale.
Lithium atoms
have a boundstate with a different magnetic moment.
Zeeman effect $\Rightarrow$ scattering length can be controlled using an external magnetic field:
Strongly-coupled NRCFT

The fixed-point theory ("fermions at unitarity") is a strongly-coupled nonrelativistic CFT (‘Schrödinger symmetry’) [Nishida-Son].

Universality: it also describes neutron-neutron scattering [Mehen-Stewart-Wise] Two-body physics is completely solved.
Many body physics is mysterious.
Experiments: very low viscosity, $\frac{\eta}{s} \sim \frac{5}{4\pi}$ [Thomas, Schafer] $\rightarrow$ strongly coupled.
The fixed-point theory ("fermions at unitarity") is a strongly-coupled nonrelativistic CFT (‘Schrödinger symmetry’) [Nishida-Son].

Universality: it also describes neutron-neutron scattering [Mehen-Stewart-Wise].
Two-body physics is completely solved.

Many body physics is mysterious.

Experiments: very low viscosity, $\frac{\eta}{s} \sim \frac{5}{4\pi}$ [Thomas, Schafer]

$\rightarrow$ strongly coupled.

AdS/CFT?

Clearly we can’t approximate it as a relativistic CFT.
Different hydro: conserved particle number.
A holographic description?

Method of the missing box

AdS : relativistic CFT

: Galilean-invariant CFT

This metric solves reasonable equations of motion.
Holographic prescription generalizes naturally.
But: the vacuum of a Galilean-invariant field theory is extremely boring:
no antiparticles! no stuff!
How to add stuff?
A holographic description?

**Method of the missing box**

\[
\text{AdS} \quad : \quad \text{relativistic CFT}
\]

\[
\text{“Schrödinger spacetime”} \quad : \quad \text{Galilean-invariant CFT}
\]

A spacetime whose isometry group is the Schrödinger group:

[Son; K Balasubramanian, JM]

\[
L^{-2} ds^2 = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - \frac{dt^2}{r^2}
\]

This metric solves reasonable equations of motion. Holographic prescription generalizes naturally.

**But:** the vacuum of a Galilean-invariant field theory is extremely boring: no antiparticles! no stuff! How to add stuff?
A holographic description of more than zero atoms

A black hole in Schrödinger spacetime.

[A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was extremely useful:

A solution-generating machine named Melvin: [Ganor et al]

\[
\begin{align*}
\text{IN: } & AdS_5 \times S^5 \\
\text{OUT: } & \text{Schrödinger } \times S^5
\end{align*}
\]
A holographic description of more than zero atoms

A black hole in Schrödinger spacetime.

[A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was extremely useful:

A solution-generating machine named Melvin: [Ganor et al]

\[
\text{IN: } AdS_5 \times S^5 \\
\text{IN: } AdS_5 \text{ BH } \times S^5 \\
\text{OUT: } \text{Schrödinger BH } \times \text{squashed } S^5 \\
\text{OUT: } \text{Schrödinger } \times S^5
\]
A holographic description of more than zero atoms

A black hole in Schrödinger spacetime.

[A. Adams, K. Balasubramanian, JM; Maldacena et al; Rangamani et al]

Here, string theory was extremely useful:

A solution-generating machine named Melvin: [Ganor et al]

\[
\begin{align*}
\text{IN: } & AdS_5 \times S^5 & \quad \text{OUT: } & \text{Schrödinger BH } \times \text{squashed } S^5 \\
\text{IN: } & AdS_5 \text{ BH } \times S^5 & \quad \text{OUT: } & \text{Schrödinger BH } \times \text{squashed } S^5
\end{align*}
\]

This black hole gives the thermo and hydro of some NRCFT.

Not unitary fermions: \( F \sim -\frac{T^4}{\mu^2}, \quad \mu < 0. \)

Unnecessary assumption: all of Schröd must be realized geometrically.

We now know how to remove this assumption, can seek more realistic models.
Example 3:

Strange metals from holography

Towards universal physics of interacting Fermi surfaces
Hierarchy of understoodness

systems with a gap (insulators)

Effective field theory (EFT) is a topological field theory
Hierarchy of understoodness

systems with a gap (insulators)

Effective field theory (EFT) is a topological field theory

systems at critical points or topological insulators with gapless boundary DoFs

EFT is a CFT
Hierarchy of understoodness

systems with a gap (insulators)

Effective field theory (EFT) is a topological field theory

systems at critical points or topological insulators with gapless boundary DoFs

EFT is a CFT

systems with a Fermi surface (metals)

??
Slightly subjective musical classification of states of matter

insulator
Slightly subjective musical classification of states of matter

- insulator
- rel. critical point or TI

(Kenny G image)
Slightly subjective musical classification of states of matter

- insulator
- rel. critical point or TI
Fermi Liquids

Basic question: What is the effective field theory for a system with a Fermi surface (FS)?
Basic question: What is the effective field theory for a system with a Fermi surface (FS)?

Lore: must be Landau Fermi liquid [Landau, 50s].

Recall [8.044, 8.06]:
if we had free fermions, we would fill single-particle energy levels $\epsilon(k)$ until we ran out of fermions: $\rightarrow$

Low-energy excitations:
remove or add electrons near the Fermi surface $\epsilon_F, k_F$. 

![Diagram of Fermi surface and energy levels](image-url)
Fermi Liquids

Basic question: What is the effective field theory for a system with a Fermi surface (FS)?

Lore: must be Landau Fermi liquid [Landau, 50s].

Recall [8.044, 8.06]:

if we had free fermions, we would fill single-particle energy levels $\epsilon(k)$ until we ran out of fermions: $\rightarrow$

Low-energy excitations:
remove or add electrons near the Fermi surface $\epsilon_F, k_F$.

Idea [Landau]: The low-energy excitations of the interacting theory are still weakly-interacting fermionic, charged ‘quasiparticles’.

Elementary excitations are free fermions with some dressing:
The standard description of metals

The metallic states that we understand well are described by Landau’s Fermi liquid theory. Landau quasiparticles $\rightarrow$ poles in single-fermion Green function $G_R$ at $k_\perp \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_*(k_\perp) \sim 0$: $G_R \sim \frac{Z}{\omega - v_F k_\perp + i\Gamma}$ Measurable by ARPES (angle-resolved photoemission): $\omega_{\text{out}} = \omega_{\text{in}} - eV$ Intensity $\propto$ spectral density: $A(\omega, k_\perp) \equiv \text{Im} G_R(\omega, k_\perp)$ $k_\perp \rightarrow 0 \rightarrow Z\delta(\omega - v_F k_\perp)$ Landau quasiparticles are long-lived: width is $\Gamma \sim \omega^2\star$, residue $Z$ (overlap with external $e^-$) is finite on Fermi surface. Reliable calculation of thermodynamics and transport relies on this.
The standard description of metals

The metallic states that we understand well are described by Landau’s Fermi liquid theory. Landau quasiparticles → poles in single-fermion Green function $G_R$ at $k_{\perp} \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_*(k_{\perp}) \sim 0$: $G_R \sim \frac{Z}{\omega - v_F k_{\perp} + i\Gamma}$

Measurable by ARPES (angle-resolved photoemission):

$\omega_{\text{in}}$ $k_{\text{in}}$ $\omega_{\text{out}} = \omega_{\text{in}} - \omega$

$k_{\text{out}} = k_{\text{in}} - k$

Intensity $\propto$ spectral density: $A(\omega, k) \equiv \text{Im} G_R(\omega, k) \overset{k_{\perp} \to 0}{\rightarrow} Z \delta(\omega - v_F k_{\perp})$

Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_*^2$, residue $Z$ (overlap with external $e^-$) is finite on Fermi surface.

Reliable calculation of thermodynamics and transport relies on this.
Ubiquity of Landau Fermi liquid

Physical origin of lore:
1. Landau FL successfully describes $^3$He, metals studied before $\sim 1980$s, ...

2. RG: Landau FL is stable under almost all perturbations.

[Shankar, Polchinski, Benfatto-Gallivotti 92]
Non-Fermi liquids exist but are mysterious

e.g.: ‘normal’ phase of optimally-doped cuprates: (‘strange metal’)

among other anomalies: ARPES shows gapless modes at finite $k$ (FS!)
with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \xrightarrow{k_\perp \to 0} 0$.

Working definition of NFL:
Still a sharp Fermi surface
but no long-lived quasiparticles.
Non-Fermi liquids exist but are mysterious

\[ \omega_{\text{out}} = \omega_{\text{in}} - \omega \]
\[ k_{\text{out}} = k_{\text{in}} - k \]

among other anomalies: ARPES shows gapless modes at finite \( k \) (FS!) with width \( \Gamma(\omega_*) \sim \omega_* \), vanishing residue \( Z \to 0 \).

Working definition of NFL:
Still a sharp Fermi surface but no long-lived quasiparticles.

Most prominent mystery of the strange metal phase:
e-e scattering: \( \rho \sim T^2 \), e-phonon: \( \rho \sim T^5 \), ...
no known robust effective theory: \( \rho \sim T \).
Non-Fermi Liquid from non-Holography

- Luttinger liquid in $1+1$ dimensions. ✓
- loophole in RG argument:
  couple a Landau FL perturbatively to a bosonic mode
  (e.g.: magnetic photon, slave-boson gauge field, statistical gauge field,
  ferromagnetism, SDW, Pomeranchuk order parameter...)

\[ G^R(\omega) \sim \frac{1}{v_F k_\perp + \epsilon \omega^{2\nu}} \text{ at FS: NFL.} \]
Non-Fermi Liquid from non-Holography

- Luttinger liquid in 1+1 dimensions. ✓
- loophole in RG argument: couple a Landau FL perturbatively to a bosonic mode
  (e.g.: magnetic photon, slave-boson gauge field, statistical gauge field, ferromagnetism, SDW, Pomeranchuk order parameter...)

\[ G^R(\omega) \sim \frac{1}{v_F k_\perp + c \omega^{2\nu}} \text{ at FS: NFL.} \]

Not strange enough:
These NFLs are not strange metals in terms of transport.
FL killed by gapless bosons:
small-angle scattering dominates
\[ \implies \text{`transport lifetime' } \neq \text{`single-particle lifetime'} \]
Can string theory be useful here?

It would be valuable to have a non-perturbative description of such a state in more than one dimension.

Gravity dual?

We’re not going to look for a gravity dual of the whole material.

Rather: lessons for universal physics of “non-Fermi liquid”.

[an un-doped Cu-O plane, from the New Yorker]
Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual $g_{\mu\nu}$
a conserved $U(1)$ symmetry proxy for fermion number $A_\mu$
and a charged fermion proxy for bare electrons $\psi$.

$\exists$ many examples. Any $d > 1 + 1$, focus on $d = 2 + 1$.

Holographic CFT at finite density*: charged black hole (BH) in $AdS$. 

$\text{charged black hole horizon}$

$R^{3,1}$

UV

$r=r_H$
Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual

\[ g_{\mu\nu} \]

a conserved \( U(1) \) symmetry  proxy for fermion number \( A_\mu \)

and a charged fermion proxy for bare electrons \( \psi \).

\( \exists \) many examples. Any \( d > 1 + 1 \), focus on \( d = 2 + 1 \).

Holographic CFT at finite density*:
charged black hole (BH) in \( AdS \).

*: If we ignore the back-reaction of other fields. More soon.
Minimal ingredients for a holographic Fermi surface

Consider any relativistic CFT with a gravity dual

\[ g_{\mu\nu} \]

a conserved \( U(1) \) symmetry proxy for fermion number \( A_\mu \)

and a charged fermion proxy for bare electrons \( \psi \).

∃ many examples. Any \( d > 1 + 1 \), focus on \( d = 2 + 1 \).

Holographic CFT at finite density*: charged black hole (BH) in \( \text{AdS} \).

To find FS: look for sharp features in fermion Green functions \( G_R \) at finite momentum and small frequency.  [S-S Lee]

To compute \( G_R \): solve Dirac equation in charged BH geometry.

‘Bulk universality’: for two-point functions, the interaction terms don’t matter. Results only depend on \( q, m \).

*: If we ignore the back-reaction of other fields. More soon.
Fermi surface!

The system is rotation invariant, $G_R$ depends on $k = |\vec{k}|$.

At $T = 0$, we find numerically [H. Liu-JM-D. Vegh]:

For $q = 1, m = 0$: $k_F \approx 0.92$

But it's not a Fermi liquid:

The peak has a nonlineardispersion relation $\omega \sim k^z_\perp$ with

$z = 2.09$ for $q = 1, \Delta = 3/2$

$z = 5.32$ for $q = 0.6, \Delta = 3/2$.

and the residue vanishes.
Emergent quantum criticality

Whence these exponents? [T. Faulkner-H. Liu-JM-D. Vegh]

Near-horizon geometry of black hole is $\text{AdS}_2 \times \mathbb{R}^{d-1}$:

$$
ds^2 \sim -\frac{dt^2}{u^2} + \frac{du^2}{u^2} + d\vec{x}^2 \quad u \equiv r - r_H
$$

The conformal invariance of this spacetime is emergent. (We broke the microscopic conformal invariance with finite density.)

$$
t \to \lambda t, \quad x \to \lambda^{1/z} x \quad \text{with} \quad z \to \infty.
$$

The bulk geometry is a picture of the RG flow from the CFT$_d$ to this NRCFT.

$\text{AdS}_2 \times \mathbb{R}^{d-1}$  $\quad$  $\text{AdS}_{d+1}$

horizon  $\quad$  boundary

$r-r_H \ll 1$  $\quad$  $r \ll 1$

$\omega \ll \mu$  $\quad$  $\omega \gg \mu$

AdS/CFT $\Rightarrow$ low-energy physics is governed by the dual IR CFT.
Analytic understanding of Fermi surface behavior

\[ G_R(\omega, k) = K \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + c(k)\omega^{2\nu} \left( b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2) \right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + c(k)\omega^{2\nu} \left( a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2) \right)} \]

The location of the Fermi surface \((a_+^{(0)}(k = k_F) = 0)\) is determined by short-distance physics (analogous to band structure – find normalizable sol’n of \(\omega = 0\) Dirac equation in full BH) but the low-frequency scaling behavior near the FS is universal (determined by near-horizon region – IR CFT \(\mathcal{G}\)).
The location of the Fermi surface \((a_+^0(k = k_F) = 0)\) is determined by short-distance physics (analogous to band structure – find normalizable sol’n of \(\omega = 0\) Dirac equation in full BH) but the low-frequency scaling behavior near the FS is universal (determined by near-horizon region – IR CFT \(\mathcal{G}\)).

In hindsight: “semi-holographic” interpretation [FLMV, Polchinski-Faulkner] quasiparticle decays by interacting with \(z = \infty\) IR CFT degrees of freedom.
Depending on the dimension of the operator \((\nu + \frac{1}{2})\) in the IR CFT, we find Fermi liquid behavior or non-Fermi liquid behavior:

\[\nu < \frac{1}{2}\]

\[\nu = \frac{1}{2}\]

\[\nu > \frac{4}{2}\]
Depending on the dimension of the operator \((\nu + \frac{1}{2})\) in the IR CFT, we find Fermi liquid behavior or non-Fermi liquid behavior:

\[
\nu < \frac{1}{2} \quad \nu = \frac{1}{2} \quad \nu > \frac{1}{2}
\]

\[
\nu = \frac{1}{2} : \quad G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \ln \omega + c_1 \omega}
\]

A well-named phenomenological model of high-\(T_c\) cuprates near optimal doping

[Varma et al, 1989].
Charge transport by holographic Fermi surfaces

Most prominent mystery of strange metal phase: $\rho_{\text{DC}} \sim T$

We can compute the contribution to the conductivity from the Fermi surface

[Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and to appear (???)]:

$$\rho_{\text{FS}} \sim T^{2\nu}$$
Charge transport by holographic Fermi surfaces

Most prominent mystery of strange metal phase: $\rho_{\text{DC}} \sim T$

We can compute the contribution to the conductivity from the Fermi surface

[Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and to appear (???)]:

$$\rho_{\text{FS}} \sim T^{2\nu}$$

Dissipation of current is controlled by the decay of the fermions into the $AdS_2$ DoFs.

$\implies$ single-particle lifetime controls transport.
Charge transport by holographic Fermi surfaces

Most prominent mystery of strange metal phase: $\rho_{\text{DC}} \sim T$

We can compute the contribution to the conductivity from the Fermi surface:

[Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and to appear (???)]:

$$\rho_{\text{FS}} \sim T^{2\nu}$$

Dissipation of current is controlled by the decay of the fermions into the $AdS_2$ DoFs. $\implies$ single-particle lifetime controls transport.

marginal Fermi liquid: $\nu = \frac{1}{2} \implies \rho_{\text{FS}} \sim T$. 

[Important disclaimer: this is NOT the leading contribution to $\sigma_{\text{DC}}$!]
Charge transport by holographic Fermi surfaces

Most prominent mystery of strange metal phase: \( \rho_{\text{DC}} \sim T \)

We can compute the contribution to the conductivity from the Fermi surface:

\[
\rho_{\text{FS}} \sim T^{2\nu}
\]

Dissipation of current is controlled by the decay of the fermions into the \( AdS_2 \) DoFs.

\( \Rightarrow \) single-particle lifetime controls transport.

\[
\text{marginal Fermi liquid: } \nu = \frac{1}{2} \quad \Rightarrow \quad \rho_{\text{FS}} \sim T
\]

[Important disclaimer: this is NOT the leading contribution to \( \sigma_{\text{DC}} \)!]
Drawbacks of this construction

1. The Fermi surface degrees of freedom are a small part \( o(N^0) \) of a large system \( o(N^2) \). (More on this in a moment.)
Drawbacks of this construction

1. The Fermi surface degrees of freedom are a small part \( o(N^0) \) of a large system \( o(N^2) \). (More on this in a moment.)

2. *Too much* universality! If this charged black hole is inevitable, how do we see the myriad possible dual states of matter (e.g. superconductivity...)?

Problems 2 and 3 solve each other: degeneracy \( \Rightarrow \) instability. The charged black hole describes an intermediate-temperature phase.
Drawbacks of this construction

1. The Fermi surface degrees of freedom are a small part \( o(N^0) \) of a large system \( o(N^2) \). (More on this in a moment.)

2. Too much universality! If this charged black hole is inevitable, how do we see the myriad possible dual states of matter (e.g. superconductivity...)?

3. The charged black hole we are studying violates the 3rd Law of Thermodynamics (Nernst’s version):
   \[
   S(T = 0) \neq 0 \quad \text{– it has a groundstate degeneracy.}
   \]

   This is a manifestation of the black hole information paradox:
   classical black holes seem to eat quantum information.

Problems 2 and 3 solve each other: degeneracy \( \implies \) instability.

The charged black hole describes an intermediate-temperature phase.
Stability of the groundstate

Often, \( \exists \) charged bosons.
At small \( T \), the dual scalar can condense spontaneously breaking the \( U(1) \) symmetry;
BH acquires hair [Gubser, Hartnoll-Herzog-Horowitz].
Stability of the groundstate

Often, \( \exists \) charged bosons.

At small \( T \), the dual scalar can condense spontaneously breaking the \( U(1) \) symmetry; BH acquires hair [Gubser, Hartnoll-Herzog-Horowitz].

Why: black hole spontaneously emits charged particles [Starobinsky, Unruh, Hawking]. AdS is like a box: they can’t escape.

Fermi: negative energy states get filled.

Bose: the created particles then cause stimulated emission (superradiance).

A holographic superconductor is a black hole laser.

Photoemission ‘exp’ts’ on holographic superconductors:

[Faulkner-Horowitz-JM-Roberts-Vegh]

In SC state: a sharp peak forms in \( A(k, \omega) \).

The condensate lifts the IR CFT modes into which they decay.
Superconductivity is a distraction

Look ‘behind’ superconducting dome by turning on magnetic field:

Strange metal persists to $T \sim 0$!
So we want to look for a theory of this intermediate-scale physics
(like Fermi liquid theory).
Drawbacks of this construction, revisited

1. The Fermi surface degrees of freedom are a small part ($o(N^0)$) of a large system ($o(N^2)$).

2. The extremal black hole we are studying violates the 3rd Law of Thermodynamics (Nernst’s version): $S(T = 0) \neq 0$ – it has a groundstate degeneracy.
The problem we really want to solve

\[ \mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + \kappa \bar{\psi} i (\mathcal{D} - m) \psi \]
The problem we really want to solve

$$\mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + \kappa \bar{\psi} i (\slashed{D} - m) \psi$$

(with AdS boundary conditions, with a chemical potential.)
Electron stars

Choose $q, m$ to reach a regime where the bulk fermions can be treated as a (gravitating) fluid (Oppenheimer-Volkov aka Thomas-Fermi approximation).

$\rightarrow$ “electron star”
Electron stars

Choose $q, m$ to reach a regime where the bulk fermions can be treated as a (gravitating) fluid (Oppenheimer-Volkov aka Thomas-Fermi approximation).

$\rightarrow$ “electron star”

But:

• Because of parameters (large mass) required for fluid approx, the dual Green’s function exhibits many Fermi surfaces.

$\rightarrow$ stable quasiparticles at each FS.

To do better, we need to take into account the wavefunctions of the bulk fermion states: a quantum electron star.
A (warmup) quantum electron star

\[ \mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F^2 + \kappa \bar{\psi} i (\Phi - m) \psi \]

A solution of QED in AdS [A. Allais, JM, S. J. Suh].

In retrospect, the dual system describes

a Fermi Surface coupled to relativistic CFT.
A (warmup) quantum electron star

\[ \mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F^2 + \kappa \bar{\psi} i (\mathcal{D} - m) \psi \]

A solution of QED in AdS [A. Allais, JM, S. J. Suh].

In retrospect, the dual system describes

a **Fermi Surface** coupled to relativistic CFT.

- FS quasiparticles survive this:
  - FS at \( \{ \omega = 0, |\vec{k}| = k_F \neq 0 \} \)
  - is outside IR lightcone \( \{ |\omega| \geq |\vec{k}| \} \).

Interaction is kinematically forbidden.

[Landau: minimum damping velocity in superfluid; Gubser-Yarom; Faulkner-Horowitz-JM-Roberts-Vegh]
A (warmup) quantum electron star

\[ \mathcal{L}_{d+1} = \mathcal{R} + \Lambda - \frac{1}{g^2} F^2 + \kappa \bar{\psi} i (\mathcal{D} - m) \psi \]

A solution of QED in AdS [A. Allais, JM, S. J. Suh].

In retrospect, the dual system describes a Fermi Surface coupled to relativistic CFT.

- FS quasiparticles survive this:
  FS at \( \{ \omega = 0, |\vec{k}| = k_F \neq 0 \} \)
  is outside IR lightcone \( \{ |\omega| \geq |\vec{k}| \} \).

Interaction is kinematically forbidden.

[Landau: minimum damping velocity in superfluid;
Gubser-Yarom; Faulkner-Horowitz-JM-Roberts-Vegh]

- When we include gravitational backreaction [in progress with Andrea Allais]
  (dual to effects of FS on gauge theory dynamics)
  the IR geometry will be different from AdS.

Optimism: A quantum electron star is a happy medium between
\( AdS_2 \) (no fermions) and classical electron star (heavy fermions).
Concluding remarks
Two driving questions about holographic duality

1. What physics is contained in the simplest classical gravity version of the correspondence (large $N$, strong coupling)?
   
   In principle, any QFT has a quantum string theory dual.
   
   Not yet a practical description.
   
   So far the gravity limit encompasses: color confinement, relativistic gauge theory plasma, non-BCS superconductivity...
   
   here: it includes ‘strange metal’
   
   (the most mysterious phase of high-$T_c$ superconductors).

2. Which quantum systems admit such a description?
   ‘AdS/CFT’ is a bad name. Holographic duality is much more general. e.g.
   
   ▶ relevant deformations of CFT
   
   ▶ here: microscopically non-relativistic systems.
Lessons for gravity from many-body physics

- Violation of no-hair expectations for AdS black holes.
- Information is not lost in BH evaporation.
- How does space emerge from QM?
  Entanglement RG [G. Vidal]:
  a real space RG which keeps track of entanglement builds an extra dimension
  \[ ds^2 = dS^2 \]  
  [Swingle 0905.1317, Raamsdonk 0907.2939]

- Basic facts about QM forbid traversable wormholes in AdS
  (information can’t propagate between decoupled theories)  
  [Swingle, to appear]
  even at finite \( N \), small \( \lambda \) \( \Rightarrow \) ∃ “quantum horizons”

- Weak evidence for weak gravity conjecture [Arkani et al] from studies of holographic superconductors  
  [Denef-Hartnoll, 0901]
Are there new strongly-coupled states of matter?

Theoretical prediction of liquid phase? [Weisskopf]
of color confinement (70s), of superconductivity (1911 or 1956),
of FQHE (1982), of strange metal (80s)…
An old strongly-coupled state of matter from holography

If we didn’t happen to be made from the excitations of a confining gauge theory (QCD), we would have predicted color confinement using AdS/CFT via this cartoon:
Are there new strongly-coupled states of matter?

Theoretical prediction of liquid phase? [Weisskopf]
of color confinement (70s), of superconductivity (1911 or 1956),
of FQHE (1982), of strange metal (80s)...

Our ability to imagine possibilities for states of matter
so far has been limited by our weak coupling descriptions
and by our ability to build things.

For some model systems, the RG picture satisfies a nice equation
(Einstein’s equation!).
Perhaps this will help us to answer...

What other states of interacting matter may still be hidden?
The end.

Thanks to my collaborators:

Allan Adams
Andrea Allais
Koushik Balasubramanian
Maissam Barkeshli
Tom Faulkner
Ben Freivogel

Dave Guarerra
Gary Horowitz
Nabil Iqbal
Hong Liu
David Mross
Andrew Potter
Matt Roberts

T. Senthil
A. Sever
S. Josephine Suh
Brian Swingle
Ky-Anh Tran
David Vegh
The end.

Thanks to my collaborators:

Allan Adams
Andrea Allais
Koushik Balasubramanian
Maissam Barkeshli
Tom Faulkner
Ben Freivogel

Dave Guarerra
Gary Horowitz
Nabil Iqbal
Hong Liu
David Mross
Andrew Potter
Matt Roberts

T. Senthil
A. Sever
S. Josephine Suh
Brian Swingle
Ky-Anh Tran
David Vegh

Thanks for listening.