Lattice Gauge Theory for Fractional Chern Insulators

John McGreevy, UCSD

based on:

JM, Brian Swingle, Ky-Anh Tran 1109.1569, Phys. Rev. B 85, 125105

Maissam Barkeshli, JM 1201.4393

Maissam Barkeshli, JM 1206.6530, Phys. Rev. B 86, 075136

For this talk: FQHE $\equiv$ interesting.

It provides experimental realizations of:

- fractionalization in $dim > 1$
- topological order (in the Wen sense)
- emergent gauge theory
- topological field theory
- a relatively well-controlled non-Fermi liquid.

Outline

1. Parton construction as duality
2. Chern insulators summary
3. Application to fractional Chern insulators

   JM, Brian Swingle, Ky-Anh Tran, 1109.1569, Phys. Rev. B 85, 125105

   Also: Ying Ran, Yuan-Ming Lu 1109.0226
4. Other FQH states on the lattice
5. Transitions out of topologically-ordered states

   Maissam Barkeshli, JM, 1201.4393
6. Application to topological non-Fermi liquids

   Maissam Barkeshli, JM 1206.6530, Phys. Rev. B 86, 075136

Outline

Parton construction as duality

Chern insulators summary

Application of parton construction to fractional Chern insulators

Other FQH states on the lattice

Transitions out of topologically-ordered states

Application to topological non-Fermi liquids

Parton construction as duality

...between a model of interacting electrons (or spins or bosons or ...) and a gauge theory of (candidate) 'partons' or 'slave particles' (i.e. a guess for useful low-energy degrees of freedom).

The goal is to describe states *in the same Hilbert space* as the original model, in terms of other (hopefully better!) variables.

Parton construction as duality

...between a model of interacting electrons (or spins or bosons or ...) and a gauge theory of (candidate) 'partons' or 'slave particles' (i.e. a guess for useful low-energy degrees of freedom).

The goal is to describe states *in the same Hilbert space* as the original model, in terms of other (hopefully better!) variables.

Inevitable (?) for fractionalization of quantum numbers (spin-charge separation, fractional charge ...) in $d > 1+1$

Makes possible:

- new mean field ansatze,
- candidate many-body groundstate wavefunctions,
- low-energy effective theory,
- accounting of topological ground-state degeneracy and edge states,
- transitions to nearby states.

Difficulties:

- making contact with microscopic description,
- deciding fate of strongly coupled gauge theories.

Parton construction: step 1 of 2

[Jain, Wen ∼89]

1. Kinematics

Relabel states of many-body $\mathcal{H}$ w/ auxiliary variables. Not all states made by fs are in $\mathcal{H}$.

$$e.g. \quad c = f_1 f_2 f_3 = \frac{1}{3!}\epsilon_{\alpha\beta\gamma} f_\alpha f_\beta f_\gamma$$

fs are complex fermion annihilation ops.

redundancy:

$f_1 \to e^{i\varphi(x)} f_1, f_2 \to e^{-i\varphi(x)} f_2, f_3 \to f_3, \ldots$

in fact, $SU(3)$: $f_\alpha \to U_\alpha^\beta f_\beta, \quad c \to \det U c$

Parton construction: step 1 of 2

[Jain, Wen ~89]

1. Kinematics

Relabel states of many-body $\mathcal{H}$ w/ auxiliary variables. Not all states made by fs are in $\mathcal{H}$.

e.g. $c = f_1 f_2 f_3 = \frac{1}{3!}\epsilon_{\alpha\beta\gamma} f_\alpha f_\beta f_\gamma$

fs are complex fermion annihilation ops.

redundancy:

$f_1 \to e^{i\varphi(x)} f_1, f_2 \to e^{-i\varphi(x)} f_2, f_3 \to f_3, \ldots$

in fact, $SU(3)$: $f_\alpha \to U_\alpha^\beta f_\beta$, $c \to \det U c$

To write covariant action for fs, introduce gauge fields:

a_0: aux variable imposing $f_1^\dagger f_1 = f_2^\dagger f_2 = c^\dagger c =$ number of e^-; $f_2^\dagger f_2 = f_3^\dagger f_3$. a_i: arise from e^- bilinears

OR: demand ETCRs: $1 = \{c, c^\dagger\} = \{f_1 f_2 f_3, f_3^\dagger f_2^\dagger f_1^\dagger\}$ modulo gauge constraints.

Practical viewpoint: constructs possible wavefunctions.

guess weakly interacting partons: $H_{\text{partons}} = -\sum_{ij} t_{ij} f_i^\dagger e^{i a_{ij}} f_j + h.c.$

fill bands of f.

fluctuations of a?

Parton construction: step 2 of 2

2. Dynamics

Such a rewrite is always possible, many possibilities.

Default result: the gauge theory **confines** at low energies.

That is: energy cost to separate partons $\gg$ gap, (lattice spacing)$^{-1}$, chemistry.... $\implies$ back to e^-.

Parton construction: step 2 of 2

2. Dynamics

Such a rewrite is always possible, many possibilities.

Default result: the gauge theory **confines** at low energies.

That is: energy cost to separate partons $\gg$ gap, (lattice spacing)$^{-1}$, chemistry.... $\implies$ back to e^-.

2+1d gauge theory likes to do this

(Maxwell or YM kinetic term is an irrelevant operator.)

[Polyakov 1976]: even compact $U(1)$ gauge theory. $\partial_\mu \sigma \equiv \epsilon_{\mu\nu\rho} \partial_\nu a_\rho$

$$\text{monopoles} \implies V_{\text{eff}} = \Lambda^3 e^{i\sigma} + h.c. \implies \text{mass for } \sigma\text{, area law}$$

Important point: it's *deconfined* states of parton gauge theories that are interesting.

Known exceptions which allow for this:

- partial Higgsing to $\mathbb{Z}_n$.
- lots of charged d.o.f.s at low E
- in 2+1 : CS term $a \wedge da$ is marginal, can gap out gauge dynamics.

e.g. FQHE in continuum, Laughlin states

Interacting e^- $\longrightarrow$ $f_1 f_2 f_3$, f_α charge $1/3$. each f_α

in 2+1 dims, in uniform B at — in same B, suppose free.

$$\frac{1}{3} = \nu_e \equiv \frac{N_e}{N_\Phi(e)} = \frac{N_e}{eBA/(hc)} \qquad \nu_f = \frac{N_f}{N_\Phi(e/3)} = \frac{N_e}{N_\Phi(e/3)} = 3\nu_e = 1$$

partial filling $\implies$ if free, gapless. $\qquad \implies$ gap!

$SU(3)$ gauge field projects $\mathcal{H}_f$ to $\mathcal{H}_{e^-}$

integrate out gapped partons: $\int [Df]\, e^{i\int f^\dagger(\partial + a)f} = e^{i\frac{k}{2\pi}\mathrm{CS}(a)}$

e.g. FQHE in continuum, Laughlin states

Interacting e^- $\longrightarrow$ $f_1 f_2 f_3$, f_α charge $1/3$. each f_α

in 2+1 dims, in uniform B at — in same B, suppose free.

$$\frac{1}{3} = \nu_e \equiv \frac{N_e}{N_\Phi(e)} = \frac{N_e}{eBA/(hc)} \qquad \nu_f = \frac{N_f}{N_\Phi(e/3)} = \frac{N_e}{N_\Phi(e/3)} = 3\nu_e = 1$$

partial filling $\implies$ if free, gapless. $\implies$ gap!

$SU(3)$ gauge field projects $\mathcal{H}_f$ to $\mathcal{H}_{e^-}$

integrate out gapped partons: $\int [Df]\, e^{i\int f^\dagger(\partial + a)f} = e^{i\frac{k}{2\pi}\mathrm{CS}(a)}$

$k = 1 \implies$ deconfined.

EFT is $SU(3)_1$ CS theory with gapped fermionic quasiparticles $\simeq U(1)_3$ Laughlin state. (Laughlin qp = hole in f with Wilson line)

$$\Psi = \langle 0| \prod_i c(r_i) |\Phi_{mf}\rangle = \left(\underbrace{\prod_{ij}^{N} z_{ij} e^{-\sum_i^N |z_i|^2/(4\ell_B^2(e/3))}}_{\nu = 1 \text{ Laughlin state with charge } 1/3} \right)^3$$

Outline

Parton construction as duality

Chern insulators summary

Application of parton construction to fractional Chern insulators

Other FQH states on the lattice

Transitions out of topologically-ordered states

Application to topological non-Fermi liquids

Chern insulators

QHE is topological in two senses: $\sigma^{xy} = \frac{p}{q}\frac{e^2}{h}$ $p, q \in \mathbb{Z}$ despite (in fact because of) disorder.

IQHE: $q = 1$, happens for free electrons. $p \in \mathbb{Z}$ because of topology of single-particle orbits (more in a moment).

FQHE: $q > 1$, requires interactions. $q \in \mathbb{Z}$ because of topology of *many-body* wave function.
Electron *fractionalizes*: excitations have charge $1/q$, fractional statistics.

Chern insulators, continued

Although the effect was discovered in 2DEG in large B, [Thouless, Haldane 80s]: for $e-$ in a (tightbinding model of a) solid, external B not necessary or meaningful.

$$H_{\text{kinetic}} = \sum_{ij \in \mathcal{E}} t_{ij} c_i^\dagger c_j + h.c.$$

i, j label sites in a lattice with $\mathcal{E} = \{\text{edges}\}$. $c_i^\dagger$ creates an e^- at site i. perhaps spin labels, omitted.

Tight-binding means that the set of paths is restricted to $\mathcal{E}$, so only the fluxes

$$\prod_{ij \in \text{loop}} t_{ij} = \prod e^{i \int_i^j A(x) dx}$$

matter. Absorb these in phases in the hopping amplitudes.

Chern insulators, continued

Although the effect was discovered in 2DEG in large B, [Thouless, Haldane 80s]: for $e-$ in a (tightbinding model of a) solid, external B not necessary or meaningful.

$$H_{\text{kinetic}} = \sum_{ij \in \mathcal{E}} t_{ij} c_i^\dagger c_j + h.c. = \int_{\text{BZ}} dk \; c_a^\dagger(k) h^{ab}(k) c_b(k) + h.c.$$

i, j label sites in a lattice with $\mathcal{E} = \{\text{edges}\}$. $c_i^\dagger$ creates an e^- at site i. perhaps spin labels, omitted.

$$c_a(k) = \sum_{\text{unit cells}, \vec{R}} c_i e^{i\vec{k}\cdot \underbrace{(\vec{R} + r_a)}_{\text{site } i}} 1/\sqrt{\text{area}}, \; a = 1..\# \text{ of states per unit cell} = \# \text{ of bands}$$

Tight-binding means that the set of paths is restricted to $\mathcal{E}$, so only the fluxes

$$\prod_{ij \in \text{loop}} t_{ij} = \prod e^{i \int_i^j A(x) dx}$$

matter. Absorb these in phases in the hopping amplitudes.

Why 'Chern'?

BZ is a T^d.

N sites/states per unit cell means $a = 1..N$ number of bands.

if we double the unit cell (e.g. some modulation of the ts), we halve the BZ.

$$h^{ab}(k) : T^d \to \{N \times N \ \text{ matrices}\}$$

evals are band energies, evecs are sections of a rank N vector bundle V no level crossing $\implies V = \oplus_{a=1}^{N} \mathcal{L}$

Natural (Berry) connection and curvature:

$$\vec{\mathcal{A}}_a(k) \equiv -i \langle a, k | \vec{\nabla}_k | a, k \rangle, \qquad \mathcal{F} = -i \frac{\partial}{\partial k_x} \mathcal{A}_y - i \frac{\partial}{\partial k_y} \mathcal{A}_x$$

$\boxed{N = 2 \text{ bands:}}$ Any $h_{ab} = \mathbb{1}_{ab} d_0(k) + \vec{d}(k) \cdot \vec{\sigma}_{ab}(k)$

$\epsilon_\pm(k) = d_0 \pm \sqrt{\vec{d} \cdot \vec{d}}$. Bandgap $\leftrightarrow |\vec{d}| > 0 \ \forall k$.

$$\mathcal{F}_\pm = \pm \frac{1}{2} \vec{d} \cdot (\partial_{k_x} \vec{d} \times \partial_{k_y} \vec{d}).$$

$$c_1 = \int_{BZ} \mathcal{F} = \text{winding \# of } \vec{d} : T^2 \to \mathbb{R}^3 - \{0\} \sim S^2 \in \mathbb{Z}.$$

Chern insulators

Physics consequence [TKNN 82]: For a gapped system

$$\sigma^{xy} = \frac{j^x}{E^y} = \lim_{\omega \to 0} \frac{\langle j^x j^y \rangle}{i\omega} = \frac{e^2}{h} \sum_{\text{filled bands, } a} c_1(a)$$

• like QFT anomalies, the effect is indep. of the size of the band gap, $\propto \frac{\Delta}{|\Delta|}$,

comes from *adiabatic* motions, no instantaneous excitations req'd.

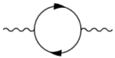
But: empty bands are required by the locality sum rule $\sum_{\text{all bands, } a} c_1(a) = 0$.

Chern insulators

Physics consequence [TKNN 82]: For a gapped system

$$\sigma^{xy} = \frac{j^x}{E^y} = \lim_{\omega \to 0} \frac{\langle j^x j^y \rangle}{i\omega} = \frac{e^2}{h} \sum_{\text{filled bands, } a} c_1(a)$$

• like QFT anomalies, the effect is indep. of the size of the band gap, $\propto \frac{\Delta}{|\Delta|}$,
comes from *adiabatic* motions, no instantaneous excitations req'd.
But: empty bands are required by the locality sum rule $\sum_{\text{all bands, } a} c_1(a) = 0$.



• In effective action for B.G. gauge field

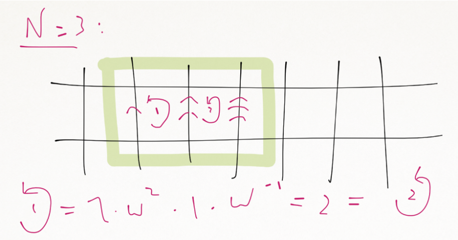
$$\int [dc]\ e^{iS[c] + i \int j_\mu A^\mu} \equiv e^{iS_{\text{eff}}[A]}, \quad j = \text{charge current}$$

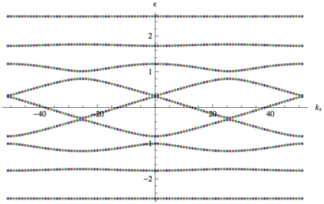
$$S_{\text{eff}} \ni \frac{\sum_{\text{filled } a} c_1(a)}{4\pi} \int_{\text{spacetime}} A \wedge F .$$

"Chern insulator" means $c_1 \neq 0$. An example of a topological insulator.

Simplest topological lattice model

2d square lattice with uniform B with $\int_{\text{square}} B = \frac{2\pi}{N}$.

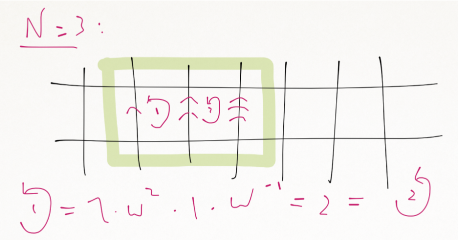


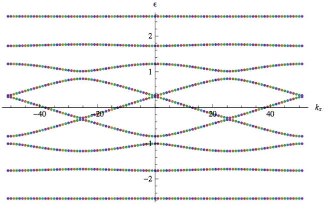


(Like $A_x = 0, A_y = Bx$ gauge.) $N \to \infty, a \to 0$, lowest bands $\to$ LLs.

Simplest topological lattice model

2d square lattice with uniform B with $\int_{\text{square}} B = \frac{2\pi}{N}$.





(Like $A_x = 0, A_y = Bx$ gauge.) $N \to \infty, a \to 0$, lowest bands $\to$ LLs.

Partially fill (e.g. $N_e = N/3$). [context and refs: Roy, Sondhi Physics **4** 36 (2011)]

$$H_{\text{int}} \sim U \sum_{\langle ij \rangle} n_i n_j + \ldots$$

Flat band means small U important in breaking degeneracy.
What state? $c = f_1 f_2 f_3$?
[Vaezi]: Degeneracy of fs doesn't care about charge.

Outline

Parton construction as duality

Chern insulators summary

Application of parton construction to fractional Chern insulators

Other FQH states on the lattice

Transitions out of topologically-ordered states

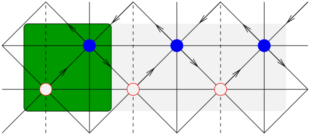
Application to topological non-Fermi liquids

nth root of bandstructure

[JM-Swingle-Tran, Lu-Ran 11]

Each f should experience a phase $\left(e^{i2\pi/N}\right)^{1/3}$.

$\Longrightarrow$ Parton unit cell grows $\times 3$, BZ shrinks $/3$.

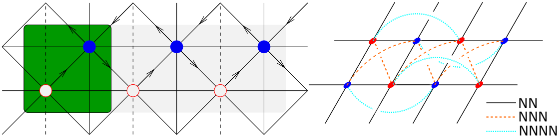


nth root of bandstructure

[JM-Swingle-Tran, Lu-Ran 11]

Each f should experience a phase $\left(e^{i2\pi/N}\right)^{1/3}$.

$\implies$ Parton unit cell grows $\times 3$, BZ shrinks $/3$.

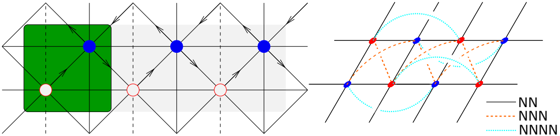


nth root of bandstructure

[JM-Swingle-Tran, Lu-Ran 11]

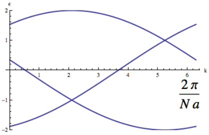
Each f should experience a phase $\left(e^{i2\pi/N}\right)^{1/3}$.

$\Longrightarrow$ Parton unit cell grows $\times 3$, BZ shrinks $/3$.

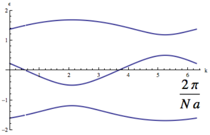


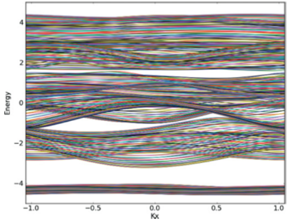
In actual checkerboard model:

Important ambiguity in cube root:



$\xrightarrow{\text{phases}}$





More precisely: parton lattice gauge theory

$$H_{\text{parton gauge theory}} = -\sum_{rr'\in\mathcal{E}} t^f_{rr'} f^\dagger_r V_{rr'} f_{r'} + h.c.$$
$$+ \underbrace{h \sum_{rr\in\mathcal{E}} E^2_{rr'}}_{\text{el. flux costs energy}} - \underbrace{K \sum_{\ell\in\mathcal{L}} \mathrm{tr} \prod_{rr'\in\ell} V_{rr'} + h.c.}_{K>0 \text{ forces smooth gauge configs}}$$

$V_{rr'} = e^{i\int_r^{r'} a}$ is raising op. for E : $[E, V] = V$
with gauge constraint:

$$h(\mathrm{div}E)_r + f^\dagger_{r\alpha} f_{r\alpha} = \#\text{ of } e^- \text{ at } r$$

Key point: strong coupling expansion works w/o confinement because of CS term generated by the filled, gapped topological parton bands. TKNN invariant = lattice CS coupling.

$$\Psi_{\text{candidate}} = (\text{slater det of filled parton bands})^{1/\nu}.$$

For what H_{electron} ?

Strong coupling expansion

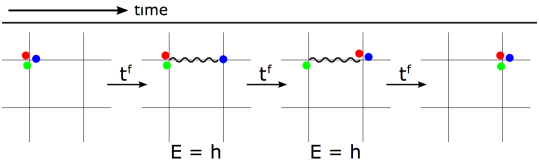
Let $h \gg K, |t^f|$ (we'll see that this is good.)

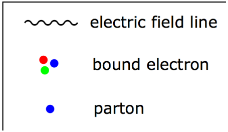
(Has a bad rep in particle physics since continuum limit is $e_{bare} = h \to 0$. here we don't need to take this limit.)

At $h \to \infty$, color singlets win. Excitations are "baryons" $(= e^-)$.

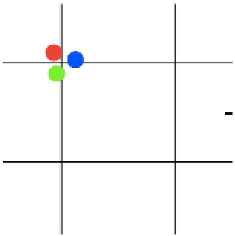
Big degeneracy between any arrangement of color singlets.

For $h < \infty$,
hopping breaks
degeneracy:

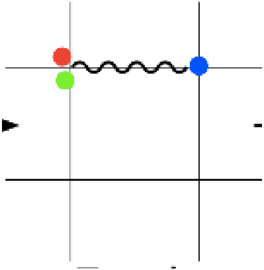




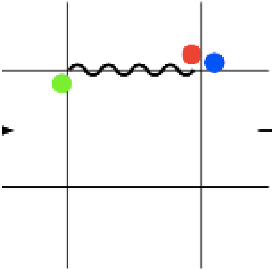
electron hopping



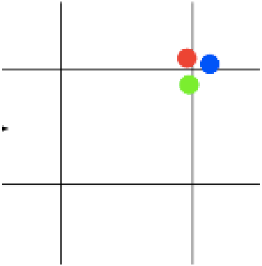
electron hopping



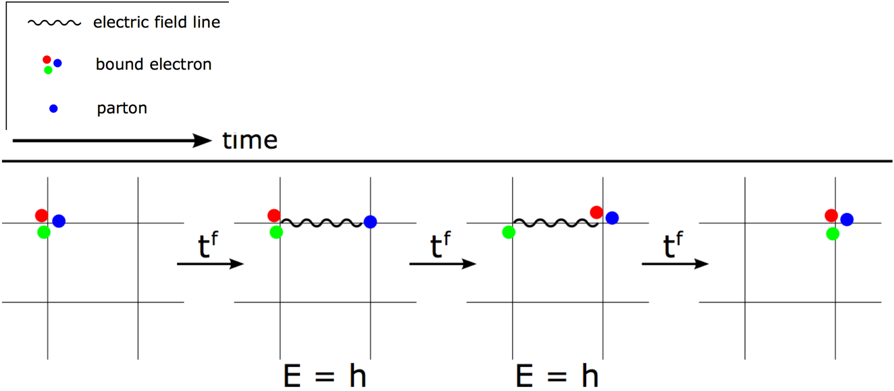
electron hopping



electron hopping



Hopping via strong coupling expansion

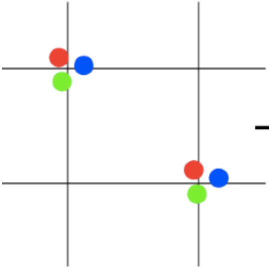


$\boxed{t^e = (t^f)^3/h^2}$ including phases.

(hence the name.) In units of some particular t_0^e:

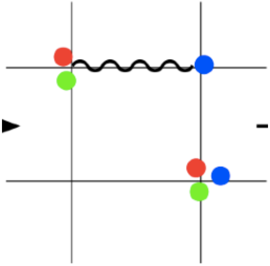
$$\frac{t^e_{rr'}}{t^e_0} = \left(\frac{t^f_{rr'}}{t^f_0}\right)^3$$

electron-electron interactions

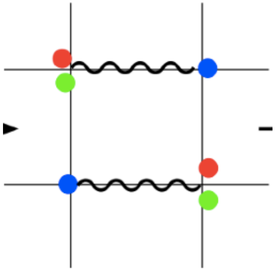


$-$

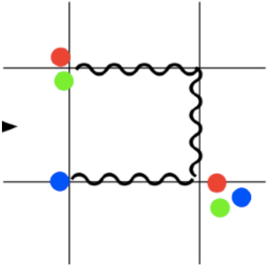
electron-electron interactions



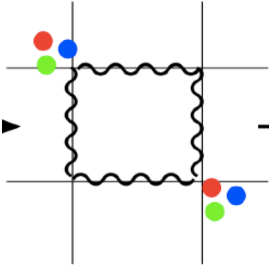
electron-electron interactions



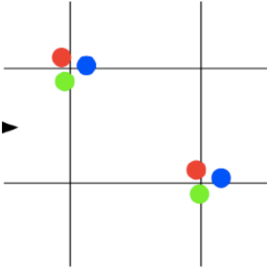
electron-electron interactions



electron-electron interactions

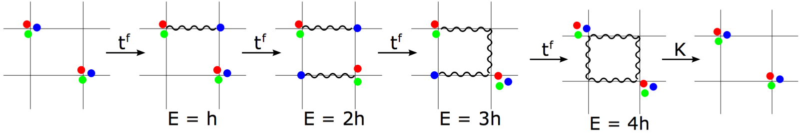


electron-electron interactions



ee interactions

- from parton exchange

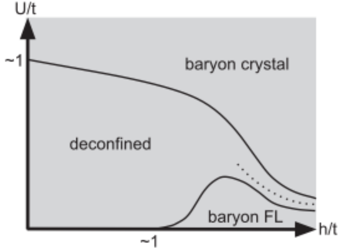


short-ranged, $\sim Kt_f^4/h^4 \ll t^e$, repulsive.

- ring exchange (from K) is smaller.
- parton exclusion force if two baryons are next to each other, they block one virtual state that would have contributed to hopping.

$\implies$ nn repulsion, $U \sim t_f^2/h$.

So: include bare e^-e^- interaction to cancel the t^2/h.
[Note: this is not fine-tuning from the point of view of the electrons: the hopping terms in H_e is also of this order.]



nth root of bandstructure

• If parton fluxes are not uniform in enlarged cell, the e^- wave function isn't translation invariant. (CDW *and* FQH).

We can find another wave function which is trans invariant by breaking the $SU(3)$ gauge symmetry:

$$h(k) = \sum_{colors,\alpha=1}^{3} h^\alpha c^\dagger_\alpha c_\alpha, \qquad h^2 \equiv \hat{T}_{\text{one subcell}} h^1, h^3 \equiv \hat{T}_{\text{one subcell}} h^2$$

breaks $SU(3)_1 \to U(1)_2 \times U(1)_4$, same physics.

nth root of bandstructure

- If parton fluxes are not uniform in enlarged cell, the e^- wave function isn't translation invariant. (CDW *and* FQH).

We can find another wave function which is trans invariant by breaking the $SU(3)$ gauge symmetry:

$$h(k) = \sum_{colors,\alpha=1}^{3} h^\alpha c_\alpha^\dagger c_\alpha, \qquad h^2 \equiv \hat{T}_{\text{one subcell}} h^1, h^3 \equiv \hat{T}_{\text{one subcell}} h^2$$

breaks $SU(3)_1 \to U(1)_2 \times U(1)_4$, same physics.
Think of the parton band structure as variational parameters.

nth root of bandstructure

• If parton fluxes are not uniform in enlarged cell, the e^- wave function isn't translation invariant. (CDW *and* FQH).

We can find another wave function which is trans invariant by breaking the $SU(3)$ gauge symmetry:

$$h(k) = \sum_{colors,\alpha=1}^{3} h^\alpha c_\alpha^\dagger c_\alpha, \qquad h^2 \equiv \hat{T}_{\text{one subcell}} h^1, h^3 \equiv \hat{T}_{\text{one subcell}} h^2$$

breaks $SU(3)_1 \to U(1)_2 \times U(1)_4$, same physics.

Think of the parton band structure as variational parameters.

• If $\sum_{\text{filled}} c_1 > 0$ we get non-abelian states. e.g. $SU(3)_2$. this happens.

in exact diagonalization: [D Sheng et al]

• Can make T-invariant examples by doubling.

• In 3d: no CS term. Easiest possibility for deconfinement is $\mathbb{Z}_n$ or S_n phases.

Outline

Parton construction as duality

Chern insulators summary

Application of parton construction to fractional Chern insulators

Other FQH states on the lattice

Transitions out of topologically-ordered states

Application to topological non-Fermi liquids

Other FQH states on the lattice

Composite fermion description is popular because of phenomenology/numerology of GaAs.

Could be different for different microscopics (*e.g.* graphene, FCI).

Attempts to port composite fermion picture: Murthy-Shankar

[Other wavefunctions: X-L Qi, Qi-Barkeshli]

We know how to do parton construction on lattice now.
$\exists$ parton constructions of all known FQH states.

Composite Fermi Liquid (brief review)

Interesting e.g. : e^- at $\nu = 1/2$. (spin polarized)
compressible composite Fermi liquid [Halperin-Lee-Read, ~93]

$$\begin{aligned}\mathcal{L} &= f^\dagger\left(\partial_\tau - ia_0 - \mu\right) f \\ &+ -\frac{1}{2m_b} f\left(\nabla - iA - ia\right)^2 f \\ &+ \frac{1}{2}\int_{r'} V_{rr'} n_f(r) n_f(r') \\ &+ \frac{1}{4\pi(2m)} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho\end{aligned}$$

$\nu = \frac{N_e}{N_\Phi} = \frac{1}{2m}$
$V(r) \sim 1/r^\eta$ can be long-ranged, if no screening.

$$c(r) = \hat{M}^{2m}(r) f(r)$$

$\hat{M} \equiv e^{i\sigma}$ creates 2π flux of a.

Mean field ansatz: $\langle a\rangle = -A \implies$ fs see no B-field.

Lots of successful phenomenology.
Gauge fluctuations lead to non-Fermi liquid behavior:

- $C_v \sim T^{2/3}$ or $T \log T$ depending on range of V (short or long ($\eta = 1$))
- $G_e(r,t)$ exponentially decays in bulk, electronic quasiparticles short-lived.

Projective construction of HLR

$\boxed{c = fb}$

[MPA Fisher, Alicea et al 2009]

f electrically neutral, charge $+1$ under a .

b electric charge 1, charge -1 under a .

$$H_{\text{partons}} = \sum_{ij} t^b_{ij} b^\dagger_i e^{i(A_{ij}+a_{ij})} b_j + t^f_{ij} f^\dagger_i e^{-ia_{ij}} f_j + h.c$$
$$- \quad (\mu - A^i_0) n^b_i + a^i_0 (n^b_i - n^f_i) + V_{ij} n^b_i n^b_j \ .$$

Assume free partons. fs neutral, finite density $\to$ FL.

bs are bosons at $\nu = 1/2$, can form Laughlin state:

$$b = d_1 d_2, \qquad d_\alpha \text{ at } \nu = 1.$$

- a is very deconfined: Fermi surface **and** Chern-Simons term.
- Recover composite fermions: integrate out gapped $\nu = 1/2$ boson qps, producing a CS term for a, attaches 2 units of flux to f.

Below $\nu = 1/2$ Laughlin gap: $b = \hat{M}^{2m}$.

- $$\Psi(\{r\}) = \langle 0 | \prod_i c(r_i) | \Phi_{mf} \rangle = \mathcal{P}_{LLL} \left(\prod_{i<j} z^2_{ij} \det_{ij} e^{i \vec{k}_i \cdot \vec{r}_j} \right)$$ [Read-Rezayi]

- $\exists$ a similar story for any composite fermion state.

Remarks

1. **lattice HLR**:
   "factorization of bandstructure" $\boxed{t^e = t^f t^b / h}$
   Analog of parton charge assignment: put phases of t^e in t^b.
2. Why this is progress:
   - We can put it on the lattice.
   - It repairs a gauge anomaly in CFL theory on $g > 1$ surfaces.
   - The parton theory includes more dofs, can access nearby states.

Remarks

1. **lattice HLR**:
   "factorization of bandstructure" $\boxed{t^e = t^f t^b / h}$
   Analog of parton charge assignment: put phases of t^e in t^b.
2. Why this is progress:
   - We can put it on the lattice.
   - It repairs a gauge anomaly in CFL theory on $g > 1$ surfaces.
   - The parton theory includes more dofs, can access nearby states.
3. Any incompressible boson $\nu = 1/2$ state would give the same outcome. $\implies \exists$ many HLR states.
   Some of these states may have topological order in the sense of towers of states which don't mix. "topological non-fermi liquid".
4. In spite of boson gap, this state is compressible!
   to add c, add $f \implies$ 2 units of flux of a (gapless!) changes LL degeneracy for b, can add b for free.

Outline

Parton construction as duality

Chern insulators summary

Application of parton construction to fractional Chern insulators

Other FQH states on the lattice

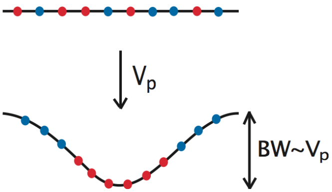
Transitions out of topologically-ordered states

Application to topological non-Fermi liquids

Transitions out of FQH states

To realize FQH or FCI, we wanted :

bandwidth $\ll$ interaction energy, U_e $\ll$ gap between bands



Consider deforming the bands, e.g. by applying pressure

(or applying a periodic potential to the continuum FQH state)

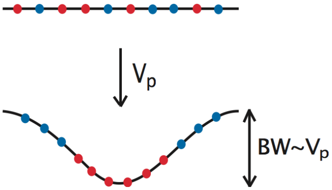
Transitions out of FQH states

To realize FQH or FCI, we wanted :

bandwidth $\ll$ interaction energy, U_e $\ll$ gap between bands

Consider deforming the bands, e.g. by applying pressure

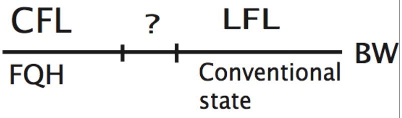
(or applying a periodic potential to the continuum FQH state)



eventually:

interaction energy, U_e $\ll$ bandwidth $\ll$ gap between bands

Expect conventional states: insulator, Fermi liquid, superfluid



Questions:

- Which of these transitions can be continuous?
- What are the resulting critical theories?
- Can anything else happen in between these extremes?

Transitions out of topologically ordered states: preview

Bosons:

$\nu = 1/2$ Laughlin FQH state (topologically ordered state, no local order parameter)	$\leftrightarrow$	Superfluid (spontaneous symmetry breaking, local order parameter)

Fermions:

(HLR) Composite Fermi Liquid (non-Fermi liquid state with emergent gauge symmetry and emergent Fermi surface)	$\leftrightarrow$	Landau Fermi liquid (conventional metal)

Previous work: Wen 2000, Barkeshli-Wen 2008

Wu-Wen, Chen-Fisher-Wu 1993

Bosons at half-filling

Required: an effective description in which we can interpolate.

$$\boxed{b = d_1 d_2}$$

A state with $n_b(r) = 1$ has $n_{d_1}(r) = 1$ AND $n_{d_2}(r) = 1$. This constraint is imposed by a $U(1)$ gauge symmetry under which $d_{1,2}$ have opposite charge.

Mean-field states:

d_1 fills a band with Chern number $c^1 = 1$,
and
d_2 fills a band with Chern number $c^2 = 1, 0, -1$.

$$\ln \int [Dd_1 Dd_2] e^{iS[d_a, a]} = \ldots + \frac{c^1 + c^2}{4\pi} i \int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho .$$

[Wen-Wu, Chen-Fisher-Wu 93, Ran-Vishwanath-Lee 0806.2321, Barkeshli, JM]

Boson states at $\nu = 1/2$

$\boxed{(c_1, c_2) = (1, 1):}$ $U(1)_2$ CS theory. $\nu = 1/2$ Laughlin FQH.

$\boxed{(c_1, c_2) = (1, 0):}$ $U(1)_1$ CS theory. Mott insulator of b.

Boson states at $\nu = 1/2$

$\boxed{(c_1, c_2) = (1, 1) :}$ $U(1)_2$ CS theory. $\nu = 1/2$ Laughlin FQH.

$\boxed{(c_1, c_2) = (1, 0) :}$ $U(1)_1$ CS theory. Mott insulator of b.

$\boxed{(c_1, c_2) = (0, 0) :}$ gauge theory confines $\implies$?

$\boxed{(c_1, c_2) = (1, -1) :}$ This state is a superfluid (SF)! [Affleck-Harvey-Witten 82, Lee-Ran-Vishwanath 08, MB-JM 12, Lu-Lee 12, Grover-Vishwanath 12]

cheap argument: integrate out gapped ds:

$$\mathcal{L}_{\text{eff}}[a, A] = \frac{1}{2\pi} A_\mu \partial_\nu a_\rho \epsilon_{\mu\nu\rho} + \frac{1}{g^2} (\partial a)^2 \overset{\text{integrate out } A}{\simeq} A^2$$

Boson states at $\nu = 1/2$

$\boxed{(c_1, c_2) = (1, 1):}$ $U(1)_2$ CS theory. $\nu = 1/2$ Laughlin FQH.

$\boxed{(c_1, c_2) = (1, 0):}$ $U(1)_1$ CS theory. Mott insulator of b.

$\boxed{(c_1, c_2) = (0, 0):}$ gauge theory confines $\implies$?

$\boxed{(c_1, c_2) = (1, -1):}$ This state is a superfluid (SF)! [Affleck-Harvey-Witten 82, Lee-Ran-Vishwanath 08, MB-JM 12, Lu-Lee 12, Grover-Vishwanath 12]

cheap argument: integrate out gapped ds:

$$\mathcal{L}_{\text{eff}}[a, A] = \frac{1}{2\pi} A_\mu \partial_\nu a_\rho \epsilon_{\mu\nu\rho} + \frac{1}{g^2}(\partial a)^2 \overset{\text{integrate out } A}{\simeq} A^2$$

better argument: compact $U(1)$ gauge theory, no CS term.

flux of a carries electric charge, because of monopole fermion zeromodes.

[Polyakov, Unsal]: $\quad V_{\text{eff}} \sim \Lambda^3 e^{i\sigma} d_1 d_2 + h.c. = \Lambda^3 e^{i\sigma} b + h.c.$

$\implies$ b condenses. Note: only $e^{i\sigma} b$ appears.

The dual photon σ is the goldstone boson!

(For this gauge theory, Coulomb phase = Higgs phase!)

(And the partons are the SF vortices (fermionic).)

A little more on $\nu = 1/2$ states from partons

Consider $(c_1, c_2) = (1, C)$ and integrate out d_1:

$$\mathcal{L}[a, f_2] = \mathcal{L}_{\text{kinetic}}[a, f_2] + \frac{1}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

$$b = \hat{M} d_2, \text{ where } \hat{M} \text{ attaches } 2\pi \text{ flux of } a.$$

Couple to external EM field, integrate out d_2 also:

$$\mathcal{L}[a, A] = \epsilon^{\mu\nu\rho}\left(\frac{C+1}{4\pi} a_\mu \partial_\nu a_\rho + \frac{C}{4\pi} A_\mu \partial_\nu A_\rho + \frac{C}{2\pi} A_\mu \partial_\nu a_\rho\right)$$

$C = 1$: gives correct σ^{xy} and topology-dependent degeneracies of $\nu = 1/2$ **Laughlin**.

$C = 0$: unique groundstate on all surfaces, $\sigma^{xy} = 0$, all excitations are bosons. $\implies$ **Mott inslator**.

$$\Psi = \prod_{ij} \underbrace{z_{ij}}_{d_1} \underbrace{z_{ij}}_{d_2} = \underbrace{\prod_{ij} \underbrace{z_{ij}}_{\text{attach 1 flux unit}} \underbrace{z_{ij}}_{CF}}_{\text{composite fermion picture}} = \underbrace{\prod_{ij} \underbrace{z_{ij}^2}_{\text{attach 2 flux units}} \cdot \underbrace{1}_{\langle\phi\rangle\neq 0}}_{\text{composite boson picture}}$$

Composite boson picture of $C = 0$: $\langle\phi\rangle = 0$.

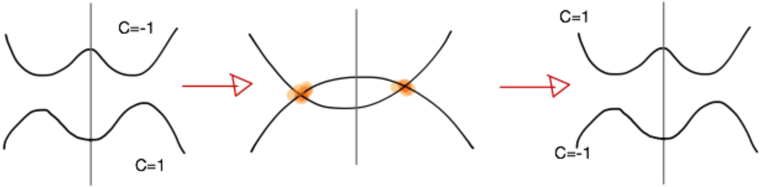
The critical theories

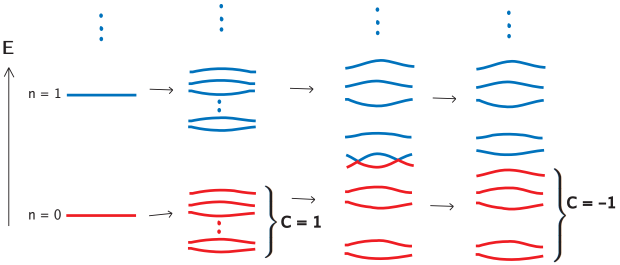
Chern-number-changing transitions of the d bandstructure. [above refs + Sachdev 98]

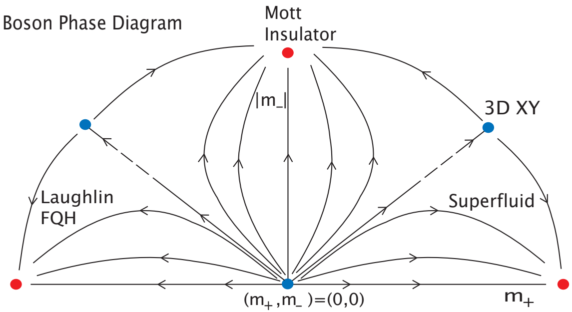
FQH-MI : $(N_f, k) = (1, 3/2)$
MI-SF : $(N_f, k) = (1, 1/2)$
FQH-SF : $(N_f, k) = (\mathbf{2}, 3/2)$

$$\mathcal{L}_{N_f,k} = \frac{N_f k}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + \sum_{i=1}^{N_f} \left(\bar{\psi}_i \gamma^\mu D_\mu \psi_i + m\bar{\psi}_i\psi_i\right)$$

Direct FQH-SF transition requires *two* Dirac points.
Requires lattice symmetry (like in graphene).




E
n = 1
n = 0
C = 1
C = −1


Boson Phase Diagram
Mott
Insulator
|m_|
3D XY
Laughlin
FQH
Superfluid
(m+ ,m_)=(0,0)
m+

Outline

Parton construction as duality

Chern insulators summary

Application of parton construction to fractional Chern insulators

Other FQH states on the lattice

Transitions out of topologically-ordered states

Application to topological non-Fermi liquids

Leverage this for electronic transitions

Boson sector	$\longrightarrow$	**Electron state**
1. $\nu = 1/2$ Laughlin FQH	$\longrightarrow$	1. HLR Composite Fermi liquid
2. Mott insulator	$\longrightarrow$	2. Gapless Mott insulator w/ emergent gauge field and FS
3. Superfluid	$\longrightarrow$	3. Landau Fermi liquid

Study transitions of e^- system using the above boson transitions.
Generalizes [Senthil, 0804.1555]:
continuous Mott transitions between LFL and $U(1)$ spin liquid.

Electron transitions

Claim: If we ignore the gauge fluctuations, the FS decouples from the boson critical theory.

Direct b-f couplings: $\delta\mathcal{L} \propto \int_{k,q} \mathcal{O}_q f_k^\dagger f_{k-q}$ where $\mathcal{O}$ is some operator from the boson sector.

Particle-hole fluctuations of fs: $v \int_q \frac{\omega}{|\vec{q}|} |\mathcal{O}_q|^2$

Leading contribution from $\mathcal{O} = |b|^2$. [Sachdev-Morinari 2002]

Electron transitions

Claim: If we ignore the gauge fluctuations, the FS decouples from the boson critical theory.

Direct b-f couplings: $\delta\mathcal{L} \propto \int_{k,q} \mathcal{O}_q f_k^\dagger f_{k-q}$ where $\mathcal{O}$ is some operator from the boson sector.

Particle-hole fluctuations of fs: $v \int_q \frac{\omega}{|\vec{q}|} |\mathcal{O}_q|^2$

Leading contribution from $\mathcal{O} = |b|^2$. [Sachdev-Morinari 2002]

Conclusion:

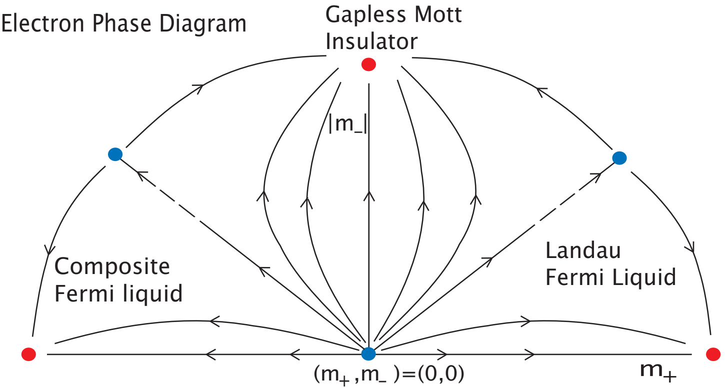
v is irrelevant for MI-SF [Senthil 2008],

and for FQH-SF ($N_f = 2$).

For FQH-MI ($N_f = 1$), we won't trust the $1/N_f$ expansion.

(Fermi surface shape deformations may allow continuous transition [Nandkashore-Metlitski-Senthil 2012].)

Schematic phase diagram for composite FL



- Direct transition from CFL to FL requires spatial symmetry.

Weirdly, this QCP between compressible states is *incompressible*.

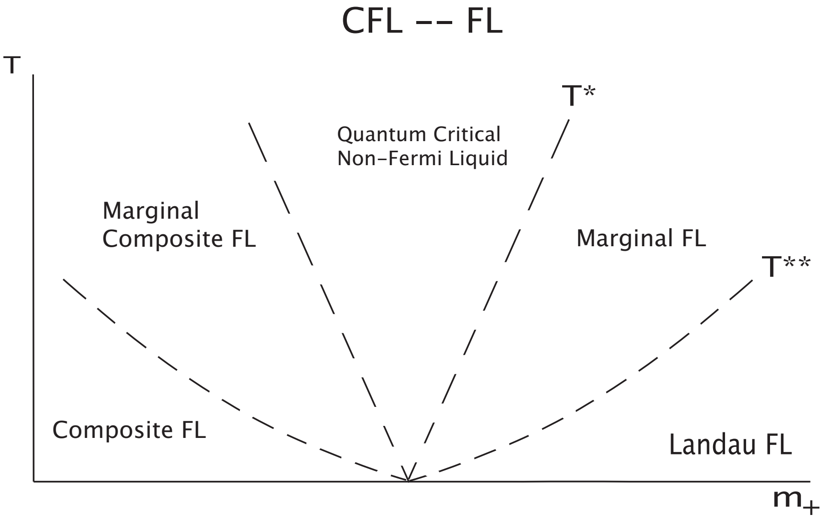
If no lattice symmetry: exotic gapless Mott insulator phase intervenes.

Ioffe-Larkin formula:

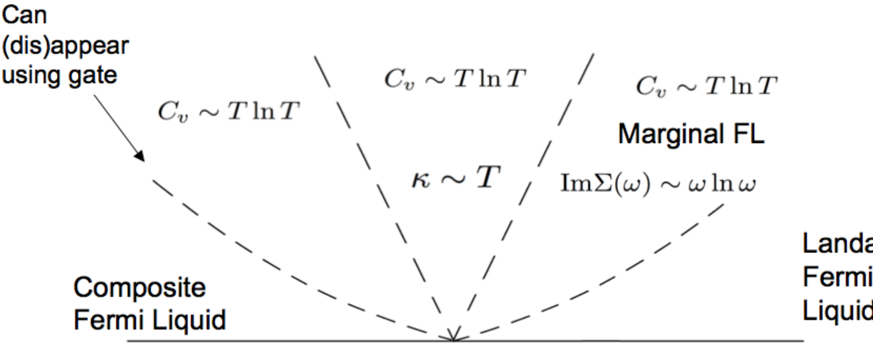
$$\Pi_e^{-1} = \Pi_b^{-1} + \Pi_f^{-1} \quad \Longrightarrow$$

- Two crossover temperature scales.
- Resistivity jumps

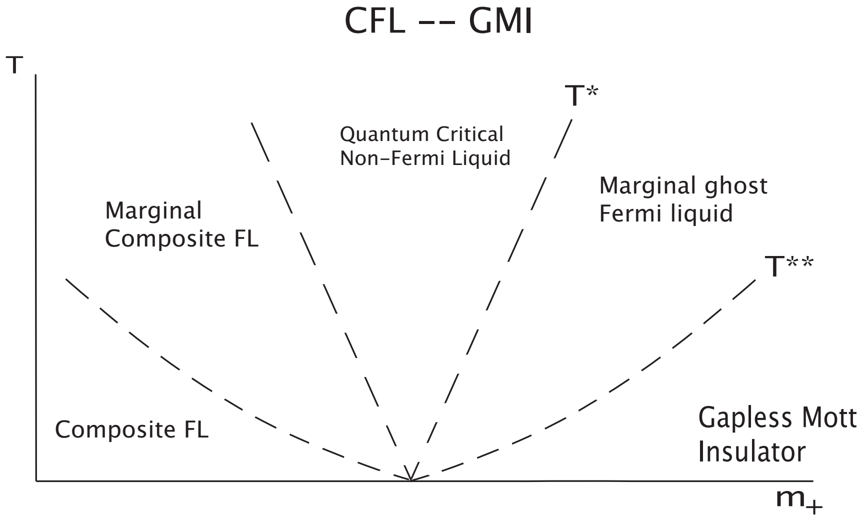
CFL to FL, including gauge fluctuations



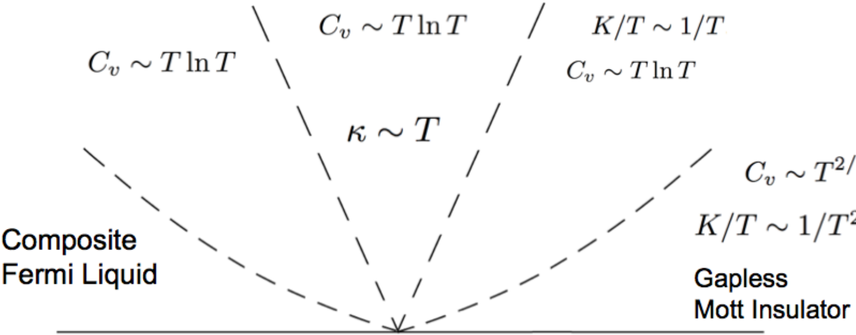
CFL to FL, including gauge fluctuations



CFL to gapless Mott Insulator

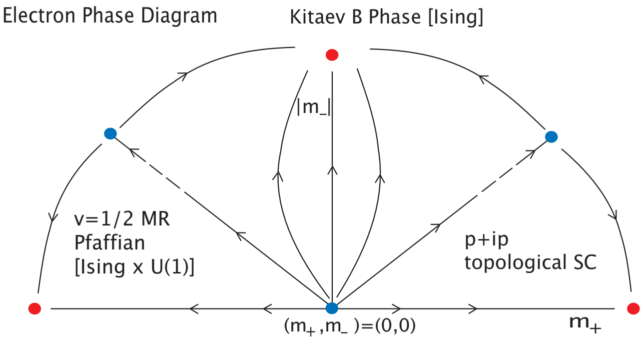


CFL to gapless Mott Insulator



Pairing

Pairing of composite fermions leads to Moore-Read state, gapped.
In the presence of this pairing, critical theories are the boson ones!



[Transition from HLR to Moore-Read is very interesting and under study by Metlitski, Mross, Sachdev, Senthil]

Concluding comments, related to experiments

- Boson Mott-SF transition has been very well-studied. [Bloch, Greiner, ...]
  With T-breaking perturbations (and some lattice symmetry) bosons at half-filling can be pushed from SF to $\nu = 1/2$ FQH Laughlin via an exotic quantum critical point.

- Deformations of FQH states by periodic potential offers a new route to a U(1) gapless "orbital liquid" Mott insulator state.

The end.

Thanks for listening.