Non-Fermi liquids from Holography

John McGreevy, MIT

based on:

Hong Liu, JM, David Vegh, 0903.2477

Tom Faulkner, HL, JM, DV, 0907.2694

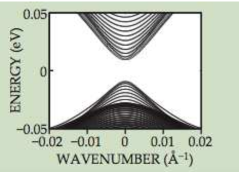
TF, Gary Horowitz, JM, Matthew Roberts, DV, 0911.3402

TF, Nabil Iqbal, HL, JM, DV, 1003.1728 and in progress

see also: Sung-Sik Lee, 0809.3402

Cubrovic, Zaanen, Schalm, 0904.1933

Slightly subjective musical classification of states of matter



insulator

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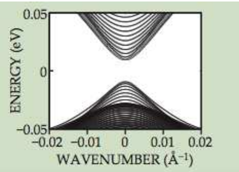

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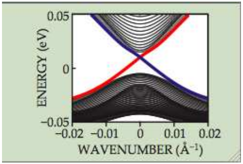

top. insulator

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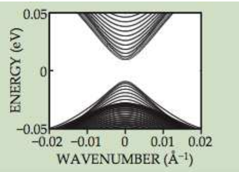

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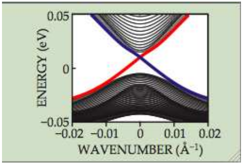

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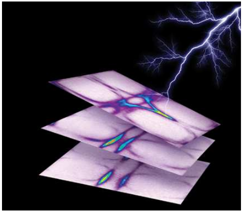


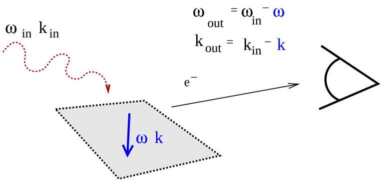
The standard description of metals

The metallic states that we understand well are described by Landau's Fermi liquid theory.

Landau quasiparticles $\rightarrow$ poles in single-fermion Green function G_R

at $k_\perp \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_\star(k_\perp) \sim 0$: $\quad G_R \sim \dfrac{Z}{\omega - v_F k_\perp + i\Gamma}$

Measurable by ARPES (angle-resolved photoemission):



$$\text{Intensity} \propto \text{spectral density}: \quad A(\omega, k) \equiv \mathrm{Im}\, G_R(\omega, k) \overset{k_\perp \to 0}{\longrightarrow} Z\delta(\omega - v_F k_\perp)$$

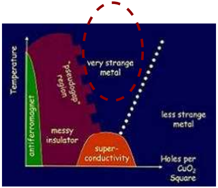
Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_\star^2$.

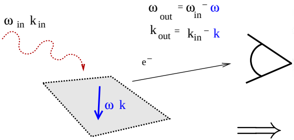
residue Z (overlap with external e^-) is finite on Fermi surface.

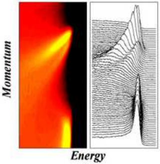
Reliable calculation of thermodynamics and transport relies on this.

Non-Fermi liquids exist, but are mysterious

e.g.: 'normal' phase of optimally-doped cuprates: ('strange metal')







among other anomalies: ARPES shows gapless modes at finite k (FS!)

with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \overset{k_\perp \to 0}{\longrightarrow} 0$.

Working defintion of NFL:

Still a sharp Fermi surface (nonanalyticity of $A(\omega \sim 0, k \sim k_F)$)

but no long-lived quasiparticles.

[Anderson, Senthil] 'critical fermi surface'

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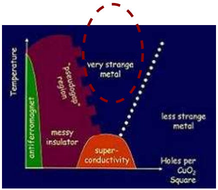

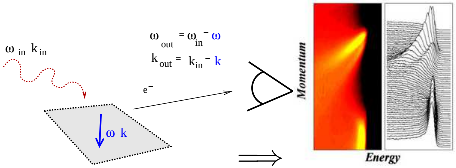

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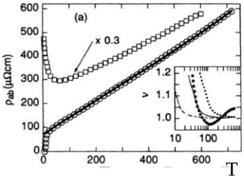
Most prominent

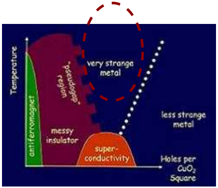
mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, e-phonon: $\rho \sim T^5$,

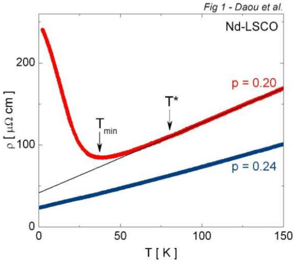
no known robust effective theory: $\rho \sim T$.



Superconductivity is a distraction



Look 'behind' superconducting dome by turning on magnetic field:



Strange metal persists to $T \sim 0$.

Theoretical status of NFL

- Luttinger liquid (1+1-d) $G(k,\omega) \sim (k-\omega)^{2g}$ ✓
- loophole in RG argument:

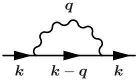
couple a Landau FL perturbatively to a gapless bosonic mode

(magnetic photon, slave-boson gauge field, statistical gauge field, ferromagnetism, SDW, Pomeranchuk order parameter...)

[Holstein et al, Baym et al, ... Halperin-Lee-Read,

Polchinski, Altshuler-Ioffe-Millis, Nayak-Wilczek, Schafer-Schwenzer, Chubukov et al,

Y-B Kim et al, Fradkin et al, Lawler et al, Metzner et al, S-S Lee, Metlitski-Sachdev, Mross et al]



$\rightarrow$ nonanalytic behavior in $G^R(\omega) \equiv \frac{1}{v_F k_\perp + \Sigma(\omega,k)}$ at FS:

$$\Sigma(\omega) \sim \begin{cases} \omega^{2/3} & d = 2+1 \\ \omega \log \omega & d = 3+1 \end{cases} \implies Z \overset{k_\perp \to 0}{\to} 0, \qquad \frac{\Gamma(k_\perp)}{\omega_\star(k_\perp)} \overset{k_\perp \to 0}{\to} \text{const}$$

These NFLs are not strange metals in terms of transport.

FL killed by gapless boson: small-angle scattering dominates $\implies$

(forward scattering does not degrade current)

'transport lifetime' $\neq$ 'single-particle lifetime'

i.e. in models with $\Gamma(\omega_\star) \sim \omega_\star$, $\rho \sim T^{\alpha > 1}$.

Can string theory be useful here?

It would be valuable to have a non-perturbative description of such states in more than one dimension.

Gravity dual?

Certain strongly-coupled many body systems can be solved using an auxiliary theory of gravity in extra dimensions.

We're not going to look for a gravity dual of the whole material

or of the Hubbard model.

Rather: lessons for principles of "non-Fermi liquid".

Lightning review of holographic duality

Holographic duality (AdS/CFT)

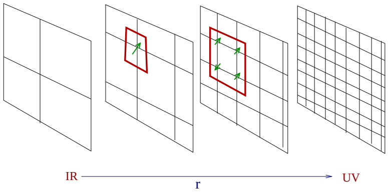
[Maldacena; Witten; Gubser-Klebanov-Polyakov]

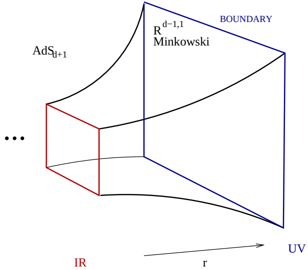
gravity in AdS_{d+1} = d-dimensional Conformal Field Theory

(many generalizations, CFT is best-understood.)

$$AdS: \quad ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + R^2 \frac{dr^2}{r^2}$$

isometries of AdS_{d+1} $\leftrightsquigarrow$ conformal symmetry





The extra ('radial') dimension is the resolution scale.

(The bulk picture is a hologram.)

when is it useful?

$$Z_{QFT}[\text{sources}] \quad = Z_{\text{quantum gravity}}[\text{boundary conditions at } r \to \infty]$$
$$\approx e^{-N^2 S_{\text{bulk}}[\text{boundary conditions at } r\to\infty]}\big|_{\text{extremum of } S_{\text{bulk}}}$$

classical gravity (sharp saddle) $\leftrightsquigarrow$ many degrees of freedom per point, $N^2 \gg 1$

fields in AdS_{d+1} $\leftrightsquigarrow$ operators in CFT
mass $\leftrightsquigarrow$ scaling dimension

boundary conditions on bulk fields $\leftrightsquigarrow$ couplings in field theory

e.g.: boundary value of bulk metric $\lim_{r\to\infty} g_{\mu\nu}$
$=$ source for stress-energy tensor $T^{\mu\nu}$

different couplings in bulk action $\leftrightsquigarrow$ different field theories

large AdS radius R $\leftrightsquigarrow$ strong coupling of QFT

confidence-building measures

- 1. **Many** detailed checks in special examples

  examples: relativistic gauge theories (fields are $N \times N$ matrices), with extra symmetries (conformal invariance, supersymmetry)

  checks: 'BPS quantities,' integrable techniques, some numerics

- 2. sensible answers for physics questions

  rediscoveries of known physical phenomena: *e.g.* color confinement, chiral symmetry breaking, thermo, hydro, thermal screening, entanglement entropy, chiral anomalies, superconductivity, ...

  Gravity limit, when valid, says who are the correct variables.
  Answers questions about thermodynamics, transport, RG flow, ...
  in terms of geometric objects.

- 3. applications to quark-gluon plasma (QGP)

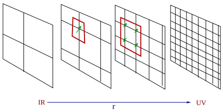
  benchmark for viscosity, hard probes of medium, approach to equilibrium

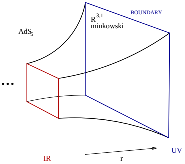
simple pictures for hard problems, an example

Bulk geometry is a spectrograph separating the theory by energy scales.

$$ds^2 = w(r)^2 \left(-dt^2 + d\vec{x}^2\right) + R^2 \frac{dr^2}{r^2}$$

CFT: bulk geometry goes on forever, warp factor $w(r) = \frac{r}{R} \to 0$:





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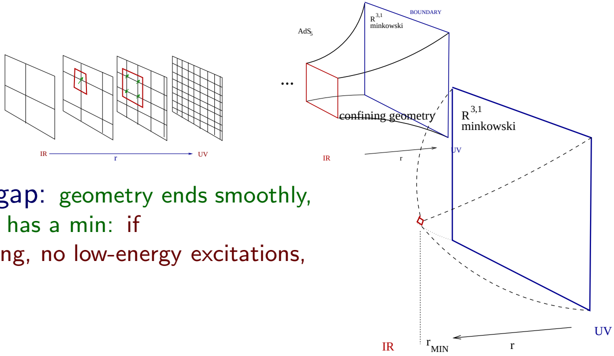
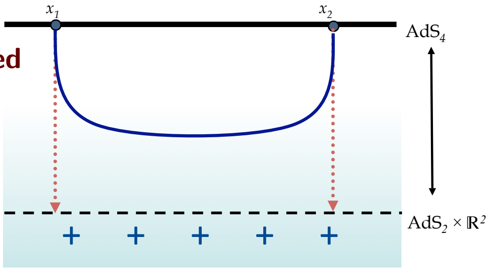


Model with a gap: geometry ends smoothly,
warp factor $w(r)$ has a min: if
IR region is missing, no low-energy excitations,
energy gap.

Strategy to find a holographic Fermi surface

Consider any relativistic CFT with a gravity dual
a conserved $U(1)$ symmetry proxy for fermion number $\longrightarrow A_\mu$
and a charged fermion operator proxy for bare electrons $\longrightarrow \psi$.
Any $d > 1 + 1$, focus on $d = 2 + 1$.

CFT at finite density: **charged** black hole (BH) in AdS.
To find FS: [Sung-Sik Lee 0809.3402]
look for sharp features
in fermion Green functions
at finite momentum
and small frequency.



To compute G_R: solve Dirac equation in charged BH geometry.

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- a conserved $U(1)$ current (proxy for fermion number)

$\rightarrow$ gauge field $F = dA$ in the bulk.

An ensemble with finite chemical potential for that current is described by the AdS Reissner-Nordstrom black hole:

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + L^2 \frac{dr^2}{r^2 f}, \qquad A = \mu\left(1 - \left(\frac{r_0}{r}\right)^{d-2}\right) dt$$

$$f(r) = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \quad f(r_0) = 0, \quad \mu = \frac{g_F Q}{c_d L^2 r_0^{d-1}},$$

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- a charged fermion operator $\mathcal{O}_{\mathrm{F}}$ (proxy for bare electrons)

$\rightarrow$ spinor field ψ in the bulk $\mathcal{L}_{d+1} \ni \bar{\psi}\left(D_\mu \Gamma^\mu - m\right)\psi +$ interactions
with $D_\mu \psi = \left(\partial_\mu + \frac{1}{4}\omega_\mu \cdot \Gamma - iqA_\mu\right)\psi$ $\quad$ ($\Delta = \frac{d}{2} \pm mL, \quad q = q$)

'Bulk universality': for two-point functions, the interaction terms don't matter.

Results only depend on q, Δ.

Comments about the strategy

- There are many string theory vacua with these ingredients. In specific examples of dual pairs
  (*e.g.* M2-branes $\leftrightsquigarrow$ M th on $AdS_4 \times S^7$),
  interactions and $\{q, m\}$ are specified.
  Which sets $\{q, m\}$ are possible and what correlations there are is not clear.
- This is a large complicated system ($\rho \sim N^2$), of which we are probing a tiny part ($\rho_\Psi \sim N^0$).
- In general, both bosons and fermions of the dual field theory are charged under the $U(1)$ current: this is a Bose-Fermi mixture.

Notes: frequencies ω below are measured from the chemical potential.

Results are in units of μ.

Computing G_R

Translation invariance in $\vec{x}, t \implies$ ODE in r.
Rotation invariance: $k_i = \delta_i^1 k$
Near the boundary, solutions behave as $(\Gamma^{\underline{r}} = -\sigma^3 \otimes 1)$

$$\psi \stackrel{r\to\infty}{\approx} a_\alpha r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

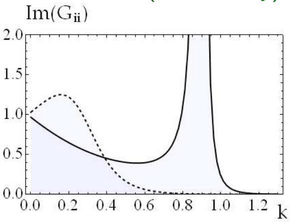
Matrix of Green's functions, has two independent eigenvalues:

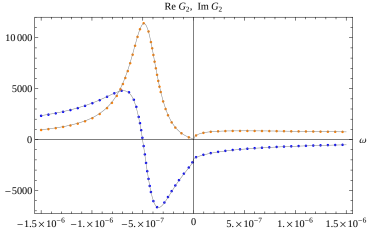
$$G_\alpha(\omega, \vec{k}) = \frac{b_\alpha}{a_\alpha}, \quad \alpha = 1, 2$$

To compute G_R: solve Dirac equation in BH geometry,
impose infalling boundary conditions at horizon [Son-Starinets...Iqbal-Liu].
Like retarded response, falling into the BH is something that *happens*.

Fermi surface!

At $T = 0$, we find (numerically):





'MDC': $G(\omega = -0.001, k)$ 'EDC': $G(\omega, k = 0.9)$

For $q = 1, m = 0$: $k_F \approx 0.918528499$

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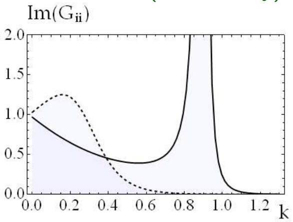

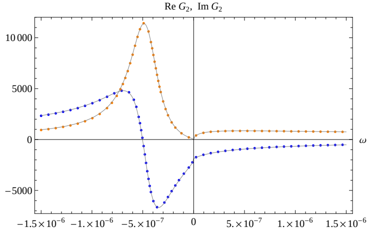

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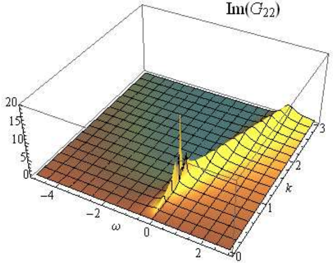


But it's not a Fermi liquid:

The peak moves
with dispersion relation $\omega \sim k_\perp^z$ with

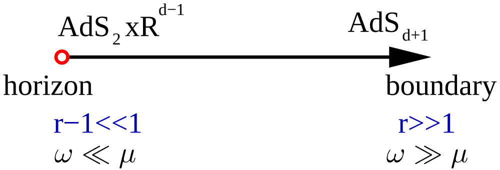
$$z = 2.09 \text{ for } q = 1, \Delta = 3/2.$$
$$z = 5.32 \text{ for } q = 0.6, \Delta = 3/2$$

and the residue vanishes.

Emergent quantum criticality

Whence these exponents?

Near-horizon geometry of black hole is $AdS_2 \times \mathbb{R}^{d-1}$.
The conformal invariance of this metric is **emergent**.
(We broke the microscopic conformal invariance with finite density.)



AdS/CFT $\implies$ the low-energy physics governed by dual **IR CFT**.
The bulk geometry is a picture of the RG flow from the CFT_d to this NRCFT.
Idea for analytic understanding of FS behavior:
solve Dirac equation by matched asymptotic expansions.
In the QFT, this is RG matching between UV and IR CFTs.

Analytic understanding of Fermi surface behavior: results

$$\boxed{G_R(\omega,k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}}$$

The location of the Fermi surface $(a_+^{(0)}(k = k_F) = 0)$ is determined by short-distance physics (analogous to band structure –
$a, b \in \mathbb{R}$ from normalizable sol'n of $\omega = 0$ Dirac equation in full BH)
but the low-frequency scaling behavior near the FS is universal
(determined by near-horizon region – IR CFT $\mathcal{G}$).
$\mathcal{G} = c(k)\omega^{2\nu}$ is the retarded G_R of the op to which $\mathcal{O}_\mathrm{F}$ matches.
its scaling dimension is $\nu + \frac{1}{2}$, with (for $d = 2+1$)

$$\nu \equiv L_2\sqrt{m^2 + k^2 - q^2/2}$$

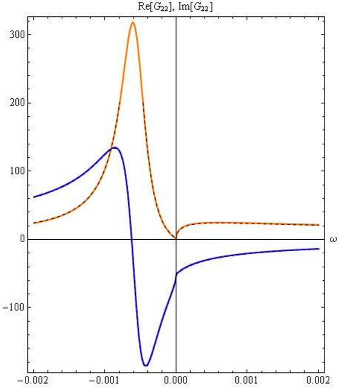
L_2 is the 'AdS radius' of the IR AdS_2.

Consequences for Fermi surface

$$G_R(\omega, k) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k)\omega^{2\nu_{k_F}}}$$

$h_{1,2}, v_F$ real, UV data.
The AdS_2 Green's function
is the self-energy $\Sigma = \mathcal{G} = c(k)\omega^{2\nu}$!

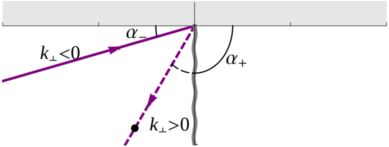
Correctly fits numerics near FS:



$\nu < \frac{1}{2}$: non-Fermi liquid

$$G_R(\omega, k) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - c\omega^{2\nu_{k_F}}}$$

$$\text{if } \nu_{k_F} < \frac{1}{2}, \quad \omega_\star(k) \sim k_\perp^z, \qquad z = \frac{1}{2\nu_{k_F}} > 1$$



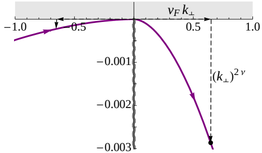
$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp \to 0}{\longrightarrow} \text{const}, \qquad Z \propto k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \overset{k_\perp \to 0}{\longrightarrow} 0.$$

Not a stable quasiparticle.

$\nu > \frac{1}{2}$: Fermi liquid

$$G_R(\omega, k) = \frac{h_1}{k_\perp + \frac{1}{v_F}\omega + c\omega^{2\nu_{k_F}}} \qquad \omega_\star(k) \sim v_F k_\perp$$

c is complex.

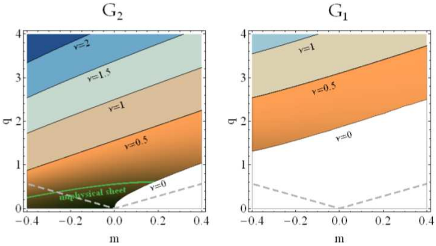


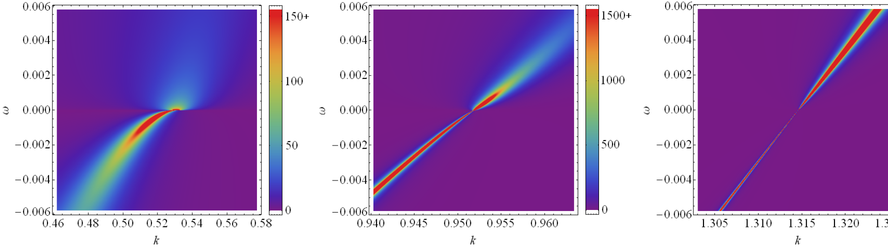
$$\frac{\Gamma(k)}{\omega_\star(k)} \propto k_\perp^{2\nu_{k_F}-1} \overset{k_\perp \to 0}{\longrightarrow} 0 \qquad Z \overset{k_\perp \to 0}{\longrightarrow} h_1 v_F.$$

A stable quasiparticle, but never Landau Fermi liquid.

Summary of spectral properties

Depending on the dimension of the operator $(\nu + \frac{1}{2})$ in the IR CFT, we find Fermi liquid behavior (but not Landau) or non-Fermi liquid behavior:





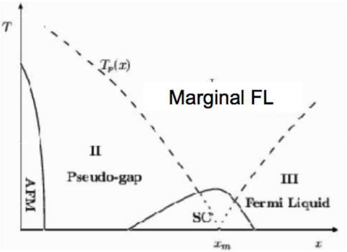
$\nu < \frac{1}{2}$ $\qquad$ $\nu = \frac{1}{2}$ $\qquad$ $\nu > \frac{1}{2}$

$\nu = \frac{1}{2}$: Marginal Fermi liquid

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \ln \omega + c_1 \omega}, \quad \tilde{c}_1 \in \mathbb{R}, \quad c_1 \in \mathbb{C}$$

$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp \to 0}{\longrightarrow} \mathrm{const}, \qquad Z \sim \frac{1}{|\ln \omega_\star|} \overset{k_\perp \to 0}{\longrightarrow} 0.$$

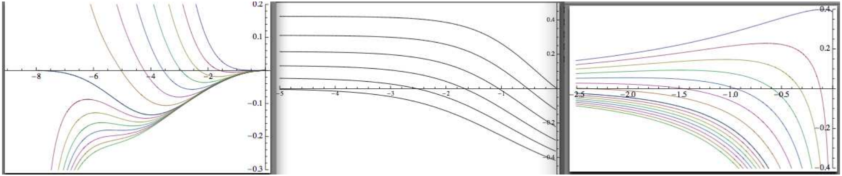
A well-named phenomenological model of high-T_c cuprates near optimal doping



[Varma et al, 1989].

UV data: where are the Fermi sufaces?

Above we supposed $a(k_F)_+^{(0)} = 0$. This happens at k_F: k s.t. $\exists$ normalizable, incoming solution at $\omega = 0$:
The black hole can acquire 'inhomogenous fermionic hair'.



Schrodinger potential $V(\tau)/k^2$ at $\omega = 0$ for $m < 0, m = 0, m > 0$.

τ is the tortoise coordinate   Right ($\tau = 0$) is boundary; left is horizon.

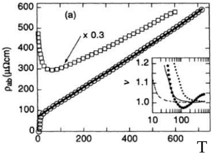
$k > qe_d$: Potential is always positive

$k < k_{osc} \equiv \sqrt{(qe_d)^2 - m^2}$: near the horizon $V(x) = \frac{\alpha}{\tau^2}$, with $\alpha < -\frac{1}{4}$ ("oscillatory region")

$k \in (qe_d, k_{osc})$: the potential develops a potential well, indicating possible existence of a zero energy bound state.

Note: can also exist on asymp. flat BH [Hartman-Song-Strominger 0912]

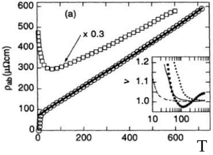
Charge transport



Most prominent mystery $\longrightarrow$
of strange metal phase: $\sigma_{\mathrm{DC}} \sim T^{-1}$
$(j = \sigma E)$

e-e scattering: $\sigma \sim T^{-2}$, e-phonon scattering: $\sigma \sim T^{-5}$, **nothing**: $\sigma \sim T^{-1}$

Charge transport



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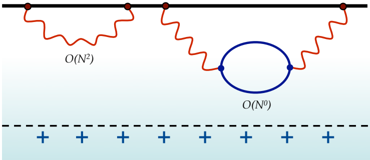
We can compute the contribution
to the conductivity from
the Fermi surface. [Faulkner, Iqbal, Liu, JM, Vegh]

Note: this is not the dominant contribution. $\rightarrow$



$$\sigma_{\rm DC} = \lim_{\omega \to 0} {\rm Im} \frac{1}{\omega} \langle j^x j^x \rangle (\omega, \vec{0}) \sim N^2 \frac{T^2}{\mu^2} + N^0 \left(\sigma_{\rm DC}^{\rm FS} + \ldots \right)$$

Charge transport by holographic non-Fermi liquids

slight complication: gauge field a_x mixes with metric perturbations.

There's a big charge density. Pulling on it with $\vec{E}$ leads to momentum flow.

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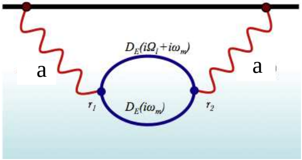
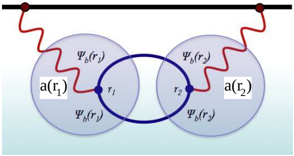


key step: $\mathrm{Im}\, D_{\alpha\beta}(\Omega, k; r_1, r_2) = \frac{\psi^b_\alpha(\Omega,k,r_1)\bar{\psi}^b_\beta(\Omega,k,r_2)}{W_{ab}} A(\Omega, k)$

bulk spectral density $\mathrm{Im}\, D$ ...

1. ... is determined by bdy fermion spectral density, $A(\omega, k) = \mathrm{Im}\, G_R(\omega, k)$
2. ... factorizes on normalizable bulk sol'ns ψ^b



Charge transport by holographic non-Fermi liquids

Like Fermi liquid calculation, $\vec{J} \sim i\psi^\dagger \vec{\nabla}\psi$

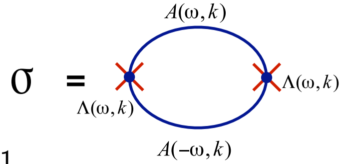
but with extra integrals over r, and no vertex corrections.

$$\sigma_{\mathrm{DC}}^{\mathrm{FS}} = C \int_0^\infty dkk \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{df}{d\omega} \Lambda^2(k,\omega) A^2(\omega,k)$$

$f(\omega) = \frac{1}{e^{\frac{\omega}{T}}+1}$: the Fermi distribution function

Λ: an effective vertex, data analogous to $v_F, h_{1,2}$.

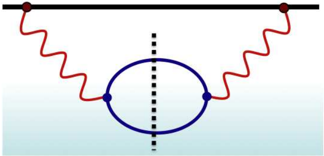
$\Lambda \sim q \int_{r_0}^\infty dr \sqrt{g} g^{xx} a_x(r,0) \frac{\bar\psi^b(r)\Gamma^x\psi^b(r)}{W_{ab}} \sim$ const.

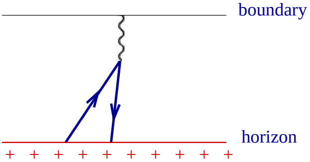


$$\int dkA(k,\omega)^2 \sim \frac{1}{T^{2\nu} g(\omega/T)}$$

scale out T-dependence $\implies \sigma^{DC} \sim T^{-2\nu}$.

Dissipation mechanism

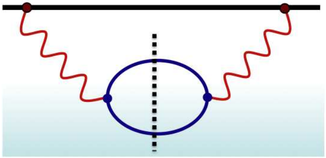


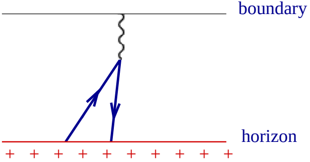


$\sigma_{\mathrm{DC}} \propto \mathrm{Im}\,\langle jj \rangle$ comes from fermions falling into the horizon.
dissipation of current is controlled by the decay of the fermions into the AdS_2 DoFs.
$\Longrightarrow$ single-particle lifetime controls transport.

marginal Fermi liquid: $\nu = \frac{1}{2} \Longrightarrow \boxed{\rho_{FS} = \left(\sigma^{DC}\right)^{-1} \sim T}$.

Dissipation mechanism





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marginal Fermi liquid: $\nu = \frac{1}{2} \Longrightarrow$ $\boxed{\rho_{FS} = \left(\sigma^{DC}\right)^{-1} \sim T}$.

The optical conductivity $\sigma(\Omega)$ can distinguish the existence of quasiparticles ($\nu > \frac{1}{2}$) through the presence of a transport peak.

Questions regarding the stability of this state

Charged AdS black holes and frustration

Entropy density of black hole:

$$s(T=0) = \frac{1}{V_{d-1}}\frac{A}{4G_N} = 2\pi e_d \rho. \qquad \left(e_d \equiv \frac{g_F}{\sqrt{2d(d-1)}}\right)$$

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not supersymmetric ... lifted at finite N

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pessimism: $S(T=0) \neq 0$ violates third law of thermodynamics, unphysical, weird string-theorist nonsense.

optimism:

we're describing the state where the SC instability is removed by hand (**here:** don't include charged scalars, **expt:** large $\vec{B}$).

[Hartnoll-Polchinski-Silverstein-Tong, 0912.]: bulk density of fermions modifies extreme near-horizon region (out to $\delta r \sim e^{-N^2}$), removes residual entropy. (Removes non-analyticity in $\Sigma(\omega)$ for $\omega < e^{-N^2}\mu$)

Stability of the groundstate

Charged bosons: In many explicit constructions, $\exists$ charged scalars.

• At small T, they can condense spontaneously breaking the $U(1)$ symmetry, changing the background [Gubser, Hartnoll-Herzog-Horowitz...].

spinor: $G_R(\omega)$ has poles only in LHP of ω [Faulkner-Liu-JM-Vegh, 0907]

scalar: $\exists$ poles in UHP $\longrightarrow$ $\langle \mathcal{O}(t) \rangle \sim e^{i\omega_\star t} \propto e^{+\mathrm{Im}\,\omega_\star t}$

$\Longrightarrow$ growing modes of charged operator: holographic superconductor

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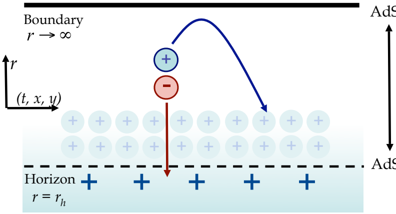
why: black hole *spontaneously* emits charged particles [Starobinsky, Unruh, Hawking].

AdS is like a box: they can't escape.

Fermi: negative energy states get filled.

Bose: the created particles then cause *stimulated emission* (superradiance).

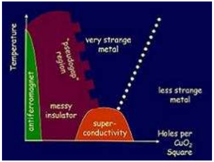


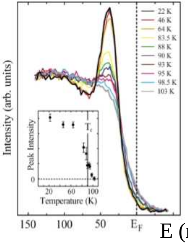
A holographic superconductor is a "black hole laser".

Photoemission ‘exp’ts’ on holographic superconductors

So far: a model of
some features of the normal state.

In SC state: a sharp peak forms in $A(k, \omega)$.





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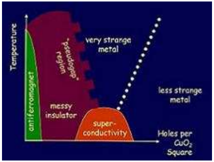
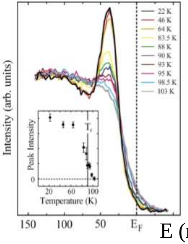




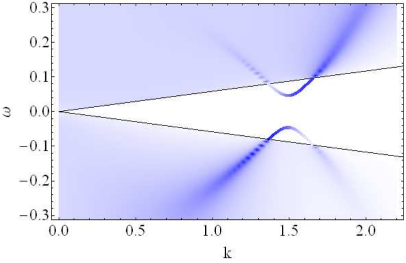
With a suitable coupling between ψ and φ,
the superconducting condensate
opens a gap in the fermion spectrum.

[Faulkner, Horowitz, JM, Roberts, Vegh]

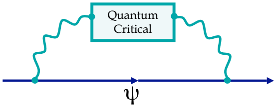
if $q_\varphi = 2q_\psi$ we can have

$$L_{\text{bulk}} \ni \eta_5 \varphi \bar{\psi} \mathcal{C} \Gamma^5 \bar{\psi}^T + \text{h.c}$$

The (gapped) quasiparticles
are exactly stable in a certain
kinematical regime
(outside the lightcone of the IR CFT) –
the condensate lifts the IR CFT modes
into which they decay.



Framework for non-Fermi liquid

a cartoon of the mechanism: 

a similar picture has been advocated by [Varma et al]

Comparison of ways of killing a FL

- a Fermi surface coupled to a critical boson field

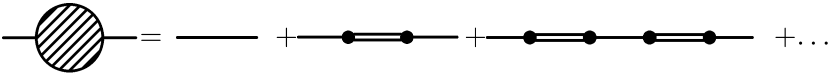
$$L = \bar{\psi}\left(\omega - v_F k\right)\psi + \bar{\psi}\psi a + L(a)$$

small-angle scattering dominates.

- a Fermi surface mixing with a bath of critical fermionic fluctuations with large dynamical exponent [FLMV 0907.2694, Faulkner-Polchinski 1001.5049, FLMV+Iqbal 1003.1728]

$$L = \bar{\psi}\left(\omega - v_F k\right)\psi + \bar{\psi}\chi + \psi\bar{\chi} + \bar{\chi}\mathcal{G}^{-1}\chi$$

χ: IR CFT operator



$$\langle\bar{\psi}\psi\rangle = \frac{1}{\omega - v_F k - \mathcal{G}} \qquad \mathcal{G} = \langle\bar{\chi}\chi\rangle = c(k)\omega^{2\nu}$$

$\nu \leq \frac{1}{2}$: $\bar{\psi}\chi$ coupling is a relevant perturbation.

Concluding remarks

1. The green's function near the FS is of the form ('local quantum criticality', analytic in k.) found previously in perturbative calculations, but the nonanalyticity can be order one.
2. This is an *input* of many studies. (Dynamical Mean Field Theory)
3. [Denef-Hartnoll-Sachdev, Hartnoll-Hofman] The leading N^{-1} contribution to the free energy exhibits quantum oscillations in a magnetic field.
4. Main challenge: step away from large N. So far:
   - Fermi surface is a small part of a big system.
   - Fermi surface does not back-react on IR CFT.
   - IR CFT has $z = \infty$.

The end.

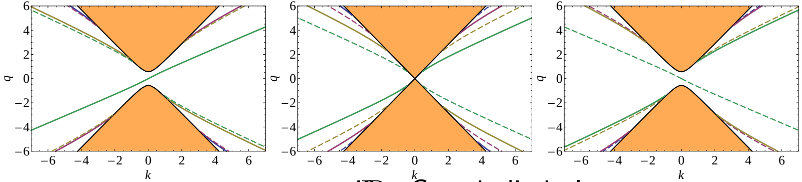
Thanks for listening.

Public service announcement

Please practice holography responsibly.

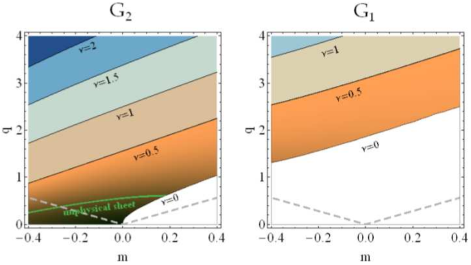
Where are the Fermi sufaces?

$m = -0.4,\ 0,\ 0.4$:

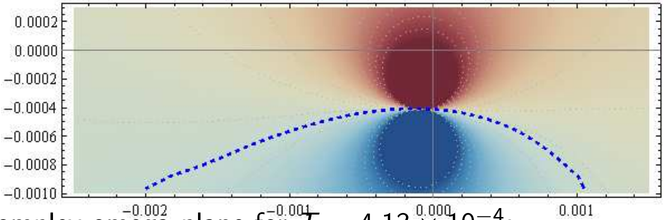


orange: 'oscillatory region': $\nu \in i\mathbb{R}$, G periodic in $\log \omega$

$$\delta_k = \frac{1}{2} + \nu_k, \qquad \nu_k = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - q^2/2}$$



Finite temperature



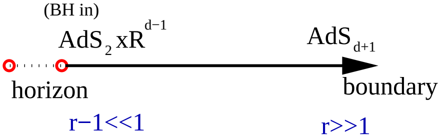
The complex omega plane for $T = 4.13 \times 10^{-4}$:

dashed line: trajectory of the pole between $k = 0.87$(left)...0.93(right).

$\min_k(\mathrm{Im}\,\omega_c) \simeq T$ (up to 1% accuracy).

In background: density plot for $\mathrm{Im}\,G_{22}(\omega)$ at $k = 0.90$

near-horizon geometry is a BH in AdS_2



$$\Sigma(\omega, T) = T^{2\nu} g(\omega/T) = (2\pi T)^{2\nu} \frac{\Gamma\left(\frac{1}{2} + \nu - \frac{i\omega}{2\pi T} + iqe_d\right)}{\Gamma\left(\frac{1}{2} - \nu - \frac{i\omega}{2\pi T} + iqe_d\right)} \xrightarrow{T \to 0} c_k \omega^{2\nu}$$

Fermion poles always in LHP!

$$\arg c_k = \arg\left(e^{2\pi i\nu} \pm e^{-2\pi q e_d}\right) \qquad \mathcal{G} = c_k \omega^{2\nu}$$

$\pm$ for boson/fermion.

$$\omega_c^{2\nu} = \mathrm{real}\cdot\left(e^{-2\pi i\nu} - e^{-2\pi q e_d}\right).$$

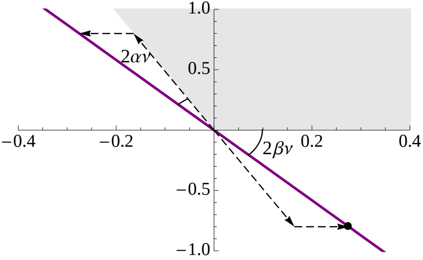


Figure: A geometric argument that poles of the fermion Green function always appear in the lower-half ω-plane: Depicted here is the $\omega^{2\nu}$ covering space on which the Green function is single-valued. The shaded region is the image of the upper-half ω-plane of the physical sheet.

Fermi velocity

Think of $\omega = 0$ Dirac eqn as Schrödinger problem.

Like Feynman-Hellmann theorem: $\partial_k \langle H \rangle = \langle \partial_k H \rangle$
we can derive a formula for v_F in terms of expectation values in the bound-state wavefunction $\Phi^+_{(0)}$.

Let:

$$\langle \mathcal{O} \rangle \equiv \int_{r_\star}^{\infty} dr \sqrt{g_{rr}} \mathcal{O} \ ,$$

$$J^\mu \equiv \bar{\Phi}^+_{(0)} \ \partial_{k_\mu} \not{D}_{0,k_F} \Phi^+_{(0)} = \bar{\Phi}^+_{(0)} \Gamma^\mu \Phi^+_{(0)}$$

is the bulk particle-number current.

$$v_F = \frac{\langle J^1 \rangle}{\langle J^0 \rangle} = \frac{\int dr \sqrt{g_{rr} g^{ii}} \left(|y|^2 - |z|^2 \right)}{\int dr \sqrt{g_{rr}(-g^{tt})} \left(|y|^2 + |z|^2 \right)}.$$

$$\Phi = \begin{pmatrix} y \\ z \end{pmatrix}$$

Note: $\frac{g^{ii}}{-g^{tt}} = f(r) \leq 1$ implies that $v_F \leq c$.

Fermi velocity

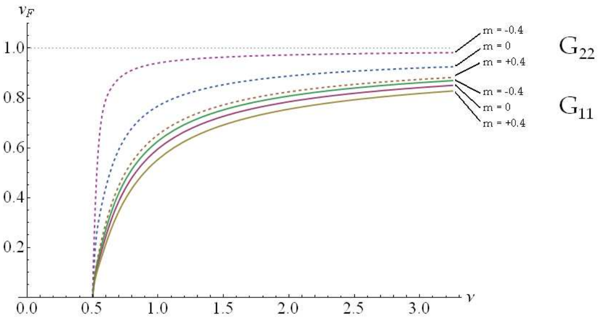
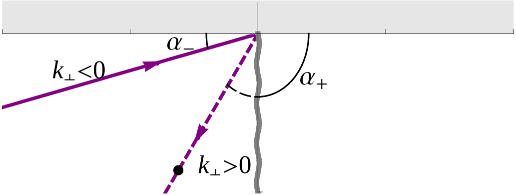


Figure: The Fermi velocity of the primary Fermi surface of various components as a function of $2\nu > 1$ for various values of m.

An explanation for the particle-hole symmetry



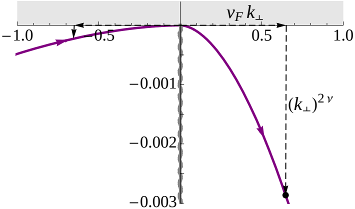


Figure: Left: Motion of poles in the $\nu < \frac{1}{2}$ regime. As k varies towards k_F, the pole moves in a straight line (hence $\Gamma \sim \omega_c$), and hits the branch point at the origin at $k = k_F$. After that, depending on $\gamma(k_F)$, it may move to another Riemann sheet of the ω-plane, as depicted here. In that case, no resonance will be visible in the spectral weight for $k > k_F$. Right: Motion of poles in the $\nu > \frac{1}{2}$ regime, which is more like a Fermi liquid in that the dispersion is linear in $k_\perp$; the lifetime is still never of the Landau form.

Note: the location of the branch cut is determined by physics:

at $T > 0$, it is resolved to a line of poles.

Oscillatory region

Above we assumed $\quad \nu = R_2\sqrt{m^2 + k^2 - (qe_d)^2} \in \mathbb{R}$

$$\nu = i\lambda \quad \Leftrightarrow \text{Oscillatory region.}$$

This is when particle production occurs in AdS_2. [Pioline-Troost]
Effective mass below BF bound *in* AdS_2. [Hartnoll-Herzog-Horowitz]
$\mathrm{Re}\,\omega^{i2\lambda} = \sin 2\lambda \log \omega \implies$ periodic in $\log \omega$ with period $\frac{\pi}{|\nu|}$.
comments about boson case:
Net flux into the outer region $> 0 =$ superradiance of AdS RN black hole (rotating brane solution in 10d)
Classical equations know quantum statistics!
like: statistics functions in greybody factors
Required for consistency of AdS/CFT!
boson: particles emitted from near-horizon region, bounce off AdS_{d+1} boundary and return, causing further stimulated emission.
spinor: there is particle production in AdS_2 region, but net flux into the outer region is negative ('no superradiance for spinors').

Oscillatory region and log-periodicity

When $\nu(k)$ is imaginary, $\mathcal{G} \sim \omega^{\nu}$ is periodic in $\log \omega$.

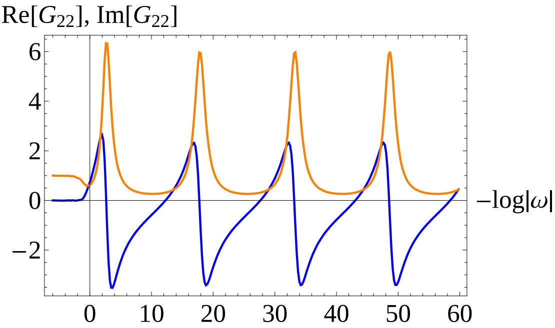


Figure: Both $\mathrm{Re}\, G_{22}(\omega, k = 0.5)$ (blue curve) and $\mathrm{Im}\, G_{22}(\omega, k = 0.5)$ (orange) are periodic in $\log \omega$ as $\omega \to 0$.

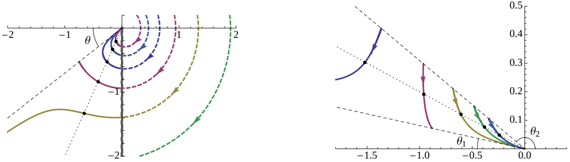
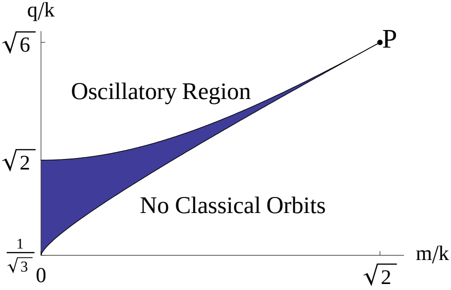


Figure: The motion of poles of the Green functions of spinors (left) and scalars (right) in the complex frequency plane. Both plots are for parameter values in the oscillatory region ($q = 1, m = 0$). In order to give a better global picture, the coordinate used on the complex frequency plane is $s = |\omega|^{\frac{1}{20}} \exp(i \arg(\omega))$. The dotted line intersects the locations of the poles at $k = k_0 = ...$, and its angle with respect to the real axis is determined by $\mathcal{G}(k, \omega)$. The dashed lines in the left figure indicate the motion of poles on another sheet of the complex frequency plane at smaller values of $k < k_0$. As k approaches the boundary of the oscillatory region, most of the poles join the branch cut. It seems that one pole that becomes the Fermi surface actually manages to stay in place. These plots are only to be trusted near $\omega = 0$.



Information from WKB. At large q, m, the primary Fermi momentum is given by the WKB quantization formula: $k_F \int_{s_-}^{s^+} ds \sqrt{V(s;\alpha,\beta)} = \pi$, where $\alpha \equiv \frac{q}{k}, \beta \equiv \frac{m}{k}$, s is the tortoise coordinate, and $s_\pm$ are turning points surrounding the classically-allowed region. For $k < q/\sqrt{3}$, the potential is everywhere positive, and hence there is no zero-energy boundstate. This line intersects the boundary of the oscillatory region at $k^2 + m^2 = q^2/2$ at the point $P = (\alpha, \beta) = (\sqrt{6}, \sqrt{2})$. Hence, only in the shaded (blue) region is there a Fermi surface. The exponent $\nu(k_F)$ is then given by $\nu(k_F) = \frac{\pi\sqrt{1+\beta^2-\alpha^2/2}}{\int ds \sqrt{V(s;\alpha,\beta)}}$. This becomes ill-defined at the point P, and interpolates between $\nu = 0$ at the boundary of the oscillatory region, and $\nu = \infty$ at $k = q/\sqrt{3}$.