Gravity duals of Galilean-invariant quantum critical points

with:

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A. Adams

0804.4053, 0807.1111

also: Son 0804.3972, Rangamani et al 0807.1099, Maldacena et al 0807.1100

work in progress with K. Balasubramanian and with C. McEntee, D. Nickel

Grandiose but brief introduction

Some relativistic CFTs have an effective description at strong coupling in terms of gravity (more generally strings) in AdS space.

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Some laboratory systems have critical points described by relativistic CFTs.

– QCD a little above T_c acts like a CFT

– some quantum-critical condensed matter systems have emergent lightcones

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So, in searching for experiments with which string theory has some interface, it's worth noting that:

non-relativistic CFTs exist.

Goal for today:

Method of the missing box

AdS : relativistic CFT

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Focus on scale-invariant case (sometimes CFT):
partly for guidance, partly because it's the most interesting.

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Note restriction to Gal.-invariance $\partial_t \psi = \vec{\nabla}^2 \psi$
distinct from: Lifshitz-like fixed points $\partial_t^2 \psi = (\vec{\nabla}^2)^2 \psi$
are not relativistic, but have antiparticles.
gravity duals of those: S. Kachru, X. Liu, M. Mulligan, 0808.1725

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hope: useful in the same way as AdS for strongly coupled **relativistic** liquids, such as those made from QCD (i.e. QGP)
If we're not going to get it exactly right, we can at least match the symmetries.

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different hydro: conserved particle number.
$\exists$ proposed QFT counterexamples to η/s-bound conjecture which are nonrelativistic.
cold atoms at unitarity come closer than anything but QGP.

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Galilean scale invariance

$i, j = 1 \ldots d$ spatial dims

Galilean symmetry:

translations P_i, rotations M_{ij}, time translations H, Galilean boosts K_i, number or mass operator N:

$[K_i, P_j] = \delta_{ij} N$ (we're using 'non-relativistic natural units' where $\hbar = M = 1$)

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dilatations D: $[D, P] = -iP$ (D measures length dimensions)

$[D, H] = -izH$ ($z \equiv$ dynamical exponent: $x \to \lambda x, \quad t \to \lambda^z t$)

closure of algebra $\longrightarrow$ $[D, K] = i(z-1)K, \quad [D, N] = i(z-2)N.$

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Schrödinger symmetry:

In the special case $z = 2$, there is an additional conformal generator, $C = ITI$

$$[M_{ij}, C] = 0, \quad [K_i, C] = 0, \quad [D, C] = -2iC, \quad [H, C] = -iD.$$

comments

- there's only *one* special conformal symmetry, not $d + 1$ like in relativistic case.

the systems we discuss will also have a discrete symmetry $\mathcal{CT}$: $\begin{cases} H \to -H \\ \Psi \to \Psi^\dagger \\ \hat{N} \to -\hat{N} \end{cases}$

- [Nishida-Son] irreps of Schrod ($z = 2$) labelled by $\Delta_0, N_0 \equiv \ell$.
- [Tachikawa] unitarity bound: $\Delta \geq \frac{d}{2}$ (independent of spin.)

QFT realization

free fermions (or free bosons) $S_0 = \int dt d^d x \left(\psi^\dagger i \partial_t \psi + \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi \right)$

$$n(\vec{x}) \equiv \psi^\dagger \psi, \;\; \vec{j}(\vec{x}) \equiv -\frac{i}{2}\left(\psi^\dagger \vec{\nabla}\psi - \vec{\nabla}\psi^\dagger \psi \right)$$

$$N = \int d^d x \; n(\vec{x}), \;\; P_i = \int j_i(\vec{x}), \;\; M_{ij} = \int (x_i j_j(\vec{x}) - x_j j_i(\vec{x}))$$

$$K_i = \int x_i n(\vec{x}), \;\; D = \int x_i j_i(\vec{x}), \;\; C = \int \frac{x^2 n(\vec{x})}{2}$$

satisfy all the commutation relations not involving the Hamiltonian.
With $H_0 = \int d\vec{x} \frac{1}{2} \nabla_i \psi^\dagger \nabla_i \psi$, ψ saturates unitarity bound.

towards interacting NRCFT

Consider the following Hamiltonian:

$$H = \int d\vec{x}\frac{1}{2}\nabla_i\psi^\dagger\nabla_i\psi + \frac{1}{2}\int d\vec{x}d\vec{y}\psi^\dagger(\vec{x})\psi^\dagger(\vec{y}) \quad V\left(\underbrace{|\vec{x}-\vec{y}|}\right)\psi(\vec{y})\psi(\vec{x})$$

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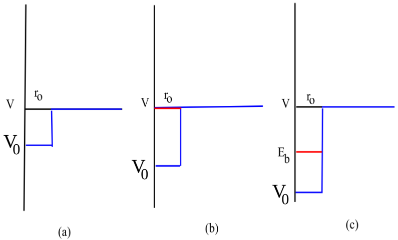
Choose a short-range two-body potential $V(r)$, *e.g.*:



a) $V_0 < 1/mr_0^2$: No bound state
b) $V_0 = 1/mr_0^2$: Bound state with zero energy
c) $V_0 > 1/mr_0^2$: At least one bound state with non-zero energy.

unitarity limit

scattering length $|a| \sim$ size of bound-state wavefunction.
case b corresponds to infinite scattering length.

When atoms collide, they spend a long time considering whether or not to bind. σ saturates bound on scattering cross section from (s-wave) unitarity (*i.e.* this is the strongest possible coupling).

For physics at wavelengths $\gg r_0$, there is no scale in the problem.

dilatations: $a \to \lambda a, r_0 \to \lambda r_0$. $a = \infty, r_0 = 0$ is a fixed point.

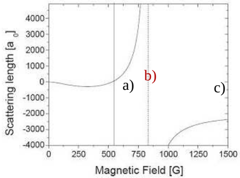
In this limit, the details of the potential are irrelevant, can choose $V = \delta^d(r)$:

$$\mathcal{L} \sim \bar{\psi}_\alpha i\partial_t \psi^\alpha - \bar{\psi}_\alpha \frac{\vec{\nabla}^2}{2M}\psi^\alpha + g\bar{\psi}_\uparrow\psi_\uparrow\bar{\psi}_\downarrow\psi_\downarrow$$

g has a fixed point where $a \gg$ interparticle dist $\gg r_0$

Zeeman effect $\Longrightarrow$ scattering length can be controlled using an external magnetic field).

$a = \infty$ is a crossover point between BCS and BEC groundstates.



some examples of systems realizing this symmetry

a) fermions at unitarity, $2 < d < 4$. (bosons suffer from 'Efimov effect')
b) 2d anyon gas [Jackiw-Pi]
c) DLCQ of relativistic CFT:

$$2p_+p_- - \vec{p}^2 = 0 \Longrightarrow E = \frac{\vec{p}^2}{2M} \quad (E \equiv p_+, M \equiv p_-)$$

schrödinger$_d$ $=$ subgroup of $SO(d+1,2)$ preserving a lightlike direction.
d) something else (see below)

geometric realization

A metric whose isometry group is the schrödinger group:

$$L^{-2} ds^2_{\mathrm{Schr}^z_d} = \frac{2 d\xi dt + \vec{dx}^2 + dr^2}{r^2} - 2\beta^2 \frac{dt^2}{r^{2z}}$$

'schrödinger space'

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'schrödinger space'
Compare to Poincaré *AdS* in light-cone coordinates:

$$ds^2_{AdS_{d+3}} = \frac{-d\tau^2 + dy^2 + \vec{dx}^2 + dr^2}{r^2}$$

$$= \frac{2d\xi dt + \vec{dx}^2 + dr^2}{r^2}$$

without the β^2 term, ∂_t is lightlike.

action of isometries

Galilean symmetry:

Translation in space: $x^i \mapsto x^i + a^i$,

Translation in time: $t \mapsto t + b$

Galilean boosts act linearly:

$$\begin{pmatrix} t \\ \vec{x} \\ \xi \end{pmatrix} \mapsto \begin{pmatrix} t \\ \vec{x} - \vec{v}t \\ \xi + \vec{v}\cdot\vec{x} - \frac{v^2}{2}t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\vec{v} & \mathbb{1} & 0 \\ -\frac{v^2}{2} & -\vec{v} & 1 \end{pmatrix} \begin{pmatrix} t \\ \vec{x} \\ \xi \end{pmatrix} \equiv B_v \begin{pmatrix} t \\ \vec{x} \\ \xi \end{pmatrix}$$

like $\begin{pmatrix} 1 \\ \vec{u} \\ \frac{u^2}{2} \end{pmatrix} \mapsto B_v \begin{pmatrix} 1 \\ \vec{u} \\ \frac{u^2}{2} \end{pmatrix}$ $\quad \begin{pmatrix} \rho \\ \vec{p} \\ \mathcal{E} \end{pmatrix} = \rho \begin{pmatrix} 1 \\ \vec{u} \\ \frac{u^2}{2} \end{pmatrix}$

Dilatations: $(t, \vec{x}, \xi, r) \mapsto (\lambda^z t, \lambda\vec{x}, \lambda^{2-z}\xi, \lambda r)$

$z = 2$ only: Special Conformal Transformation:

$$x^i \to \frac{x^i}{1+ct}, \; t \to \frac{t}{1+ct}, \; \xi \to \xi + \frac{c}{2}\frac{(\vec{x}\cdot\vec{x} + r^2)}{(1+ct)}, \; r \to \frac{r}{1+ct}.$$

$N = -i\partial_\xi$ corresponds to number operator (rest mass).

For N to have a discrete spectrum, $\xi \sim \xi + L_\xi$.

$[\hat{N}, \log\Psi] = i\hbar$ says ξ is the phase of the wavefunction.

comments

1. not possible to realize on a smooth space $d + 2$ dimensions.
   for $D > d + 3$ this is not the only possibility.
2. $\mathrm{Sch}_d^{z=1} = AdS_{d+3} = \lim_{\beta \to 0} \mathrm{Sch}_d^z$ [Goldberger, Barbon-Fuertes]
   compactness of ξ breaks $SO(4,2) \longrightarrow$ schröd
3. if $\xi \in \mathbb{R}$, we can scale away $2\beta^2$ by (remnant of boost) $\begin{cases} t \mapsto \frac{t}{\sqrt{2}\beta} \\ \xi \mapsto \sqrt{2}\beta\xi \end{cases}$
   but discrete spectrum requires compact $\xi \simeq \xi + L_\xi$
   $\frac{\beta}{L_\xi}$ is an invariant parameter.
4. dual to *vacuum* of a gal. inv't field theory (no antiparticles!).
   the ξ-circle is *null*. (light winding modes?)
   (this is the phase of the wavefunction of a state with no particles!)
   at finite temperature or density, not so.
5. all curvature scalars are constant.
6. however, $\exists$ large tidal forces for $z \neq 2$, absent for finite T, μ.
7. this spacetime is conformal to a pp-wave.
   conformal boundary is one-dimensional. nevertheless, we will compute correlators of a CFT with d spatial dims.

What holds it up?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} - \delta^t_\mu \delta^t_\nu g_{tt} \mathcal{E}$$

$\Lambda = -\frac{(d+1)(d+2)}{2L^2}$: CC $\qquad$ $\mathcal{E}$: a constant energy density ('dust')

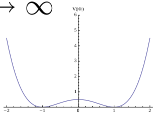
A realization of the dust: metric is sourced by *e.g.* the ground state of an Abelian Higgs model in its broken phase.

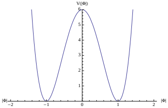
$$S = \int d^{d+3}x \sqrt{g} \left(-\frac{1}{4} F^2 + \frac{1}{2} |D\Phi|^2 - V\left(|\Phi|^2\right) \right)$$

with $D_a \Phi \equiv (\partial_a + ieA_a)\Phi$, with a Mexican-hat potential

$$V\left(|\Phi|^2\right) = g \left(|\Phi|^2 - \frac{z(z+d)}{e^2} \right)^2 + \Lambda$$

extreme type II limit : $g \to \infty \Longrightarrow m_h^2 \to \infty$





$$L_{bulk} = -\frac{1}{4} F^2 - \frac{m^2}{2} A^2 - \Lambda, \qquad m^2 = z(z+d)$$

Holographic dictionary

Basic entry: bulk fields $\leftrightarrow$ operators in dual QFT

Irreps of schrod labelled by $\Delta, \hat{N} = \ell$, so we work at fixed ξ-momentum, ℓ: $\phi(r,t,\vec{x},\xi) = f_{\omega,k,\ell}(r) e^{i(\ell\xi - \omega t + \vec{k}\cdot\vec{x})} \leftrightarrow \mathcal{O}_{\ell,\Delta}(\omega,\vec{k})$ scalar operator.

Consider a probe scalar field:

$S[\phi] = -\int d^{d+1}x\sqrt{g}\left((\partial\phi)^2 + m^2\phi^2\right).$

or: δg^x_y also satisfies this equation

Scalar wave equation in this background:

$$\left(-r^{d+3}\partial_r\left(\frac{1}{r^{d+1}}\partial_r\right) + r^2(2l\omega + \vec{k}^2) + r^{4-2z}l^2 + m^2\right) f_{\omega,\vec{k},l}(r) = 0.$$

For $z \le 2$, the behavior of the solution near the boundary ($r \sim 0$) is:

$$f \propto r^{\Delta}, \qquad \Delta_{\pm} = 1 + \frac{d}{2} \pm \sqrt{\left(1+\frac{d}{2}\right)^2 + m^2 + \delta_{z,2}l^2}.$$

For $z > 2$, not power law. (??!)

some basic checks (focus on $z = 2$)

1) $\Delta_+ + \Delta_- = d + 2$ matches dimensional analysis on

$$S_{bdy} \ni \int dt d^d x \ \phi_0 \mathcal{O}$$

(ϕ_0 is the source for $\mathcal{O}$)

$$[x] = -1, [t] = -2, [\phi_0] = \Delta_-, [\mathcal{O}] = \Delta_+.$$

2) unitarity bound $\Delta \geq \frac{d}{2}$ matches requirement on m to prevent bulk tachyon instability (analog of BF-bound).

GKPW

$$\langle e^{-\int \phi_0 \mathcal{O}} \rangle \simeq e^{-S[\phi_0]}\big|_{EOM} , \qquad S[\phi_0] \equiv S[\phi|\phi \overset{r\to\infty}{\to} \phi_0]$$

$$f_{\omega,\vec{k},l}(r) \sim r^{\frac{d+2}{2}} K_\nu(\kappa r), \;\; \nu = \sqrt{\left(\frac{d+2}{2}\right)^2 + l^2 + m^2}, \; \kappa^2 = 2l\omega + \vec{k}^2$$

The on-shell action to order ϕ_0^2 is

$$S[\phi_0] = \frac{1}{2} \int d\omega dk \phi_0(-\omega, -k) \mathcal{F}(\kappa, \epsilon) \phi_0(\omega, k)$$

where the ‘flux factor’ is

$$\mathcal{F}(\kappa, \epsilon) = \lim_{r\to\epsilon} \sqrt{g} g^{rr} f_\kappa(r) \partial_r f_\kappa(r) = \sqrt{g} g^{rr} \partial_r \left(r^{1+\frac{d}{2}} \ln K_\nu(\kappa r) \right)_{r=\epsilon}$$

$$\to \langle \mathcal{O}_1(x,t) \mathcal{O}_2(0,0) \rangle \propto \frac{\Gamma(1-\nu)}{\Gamma(\nu)} \delta_{\Delta_1,\Delta_2} \theta(t) \frac{1}{|\epsilon^2 t|^\Delta} e^{-ilx^2/2|t|}$$

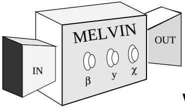
consistent with (in fact, determined by [NS]) NR conformal Ward identities.

Is it possible to embed this geometry into string theory?

Answering this question will pay off in two ways:

1. A hint about which NRCFTs we are describing.
2. A way to find finite temperature solutions.

"Null Melvin Twist"

is a machine 

which generates new type II SUGRA solutions from old Ganor et al, Gimon et al. (with different asymptotics)

Previous work: dials set to 'highly non-commutative'.

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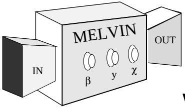
is a machine 

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Choose two killing vectors $(\partial_y, \partial_\chi)$ and:

1. Boost along y with boost parameter γ
2. T-dualize along y.
3. Twist: replace $\chi \to \chi + \alpha y$, α constant
4. T-dualize back along y
5. Boost back by $-\gamma$ along y
6. Scaling limit: $\gamma \to \infty$, $\alpha \to 0$ keeping $\beta = \frac{1}{2}\alpha e^{\gamma}$ fixed.

Schrödinger spacetime in string theory

Input solution of type IIB supergravity: $AdS_5 \times S_5$

$$ds^2 = \frac{-d\tau^2 + dy^2 + d\vec{x}^2 + dr^2}{r^2} + ds^2_{S_5} \qquad \vec{x} \equiv (x^1, x^2).$$

$ds^2_{S_5} = ds^2_{\mathbb{P}^2} + \eta^2$. $\eta \equiv d\chi + \mathcal{A} =$ vertical one-form of Hopf fibration

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Feeding this to the melvinizer gives:

$$ds^2 = \frac{1}{r^2}\left(-\left(1+\frac{\beta^2}{r^2}\right)d\tau^2 + \left(1-\frac{\beta^2}{r^2}\right)dy^2 + 2\frac{\beta^2}{r^2}d\tau dy + d\vec{x}^2 + dr^2\right) + ds^2_{S_5}$$

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Defining $\xi \equiv \frac{1}{2\beta}(y - \tau)$, $t \equiv \beta(\tau + y)$, and reducing on the 5-sphere:

$$\longrightarrow ds^2 = \frac{2d\xi dt + d\vec{x}^2 + dr^2}{r^2} - \frac{dt^2}{r^4} \qquad (\mathrm{Schr}^{z=2}_{d=2})$$

The ten-dimensional metric is sourced by

$$B = \beta r^{-2}\eta \wedge (d\tau + dy), \;\; F_5 = (1 + \star)\mathrm{Vol}(S^5) \;\; \xrightarrow{5d} A = r^{-2} dt, \;\; m^2 = 8, \;\; \Lambda.$$

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– No higgs field, alas.

– This can be done for $S^5 \to$ any Sasaki-Einstein 5-manifold.

Emblackening

If we feed the AdS planar black hole
(dual of 4d relativistic CFT at finite T) to the melvinizer, we get

$$ds^2 = \frac{1}{r^2 K}\left(-\frac{f}{r^2}dt^2 - 2d\xi dt - \frac{g}{4}\left(\frac{dt}{2\beta} - \beta\xi\right)^2 + Kd\vec{x}^2 + \frac{Kdr^2}{f}\right) + \frac{1}{K}\eta^2 + ds^2_{\mathbb{P}^2}$$

where $f \equiv 1 + g \equiv 1 - \frac{r^4}{r_H^4}$ and $K = 1 + \beta^2 \frac{r^2}{r_H^4}$

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where $f \equiv 1 + g \equiv 1 - \frac{r^4}{r_H^4}$ and $K = 1 + \beta^2 \frac{r^2}{r_H^4}$
The ten-dimensional metric is sourced by:

$$B = \beta r^{-2}\eta \wedge \left(\frac{1+f}{2}dt + 2(1-f)\beta^2 d\xi\right)$$
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Melvinization preserves lovely Rindler horizon at $r = r_H$.

5d reduction

$$ds^2 = \frac{K^{-2/3}}{r^2}\left(-\frac{f}{r^2}dt^2 - 2d\xi dt - \frac{g}{4}\left(\frac{dt}{2\beta} - \beta\xi\right)^2 + Kd\vec{x}^2 + \frac{Kdr^2}{f}\right)$$

The 5-dimensional metric is sourced by a massive gauge field, scalars:

$$A = \beta r^{-2}\left(\frac{1+f}{2}dt + 2(1-f)\beta^2 d\xi\right), \; e^{-2\Phi} = K$$

An effective action ($8\pi G = 1$):

$$S = \frac{1}{2}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\Phi)^2 - \frac{1}{4}F^2 - 4A^2 - V(\Phi)\right)$$

where $V(\Phi) = 4e^{2\Phi/3}(e^{2\Phi} - 4)$.

$\exists$ consistent truncation of IIB SUGRA w/ massive vector and 3 scalars (!) [MMT]

The Lifshitz ($T = 0$) spacetime [KLM] is also a solution of this system. BH is not.

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$$it \simeq it + \tfrac{n}{T}, \xi \simeq \xi + L_\xi \mu n \qquad \Longrightarrow \qquad T = \frac{\kappa}{2\pi} = \frac{1}{\pi\beta r_H}, \mu = -\frac{1}{2\beta^2}$$

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We got finite density for free. Which is good because $S_{BH} \neq 0$, but no antiparticles.

Entropy: $S_{BH} = \dfrac{1}{4G_N}\dfrac{L_y}{r_H^3} = VL_\xi \dfrac{\pi^2 N^2 T^3}{16\mu^2}$

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Mystery: we are forced to add *extrinsic* boundary terms for the massive gauge field: $S_{\mathrm{bdy}} \ni \int n^\mu A_\nu F^{\mu\nu}$

The required coefficient is exactly the one that changes the boundary conditions on A_μ from Dirichlet to Neumann.

Boundary stress tensor

$$S_{\rm bdy} = \int \sqrt{\gamma}\left(\Theta + c_0 + c_1\Phi + c_2\Phi^2 + n^\mu A^\mu F_{\mu\nu}\left(c_5 + c_6\Phi\right)\right)$$

Vary metric at boundary:

$$T^\mu_\nu = -\frac{2}{\sqrt{\gamma}}\frac{\delta S_{\rm onshell}}{\delta\gamma^\nu_\mu} = \Theta^\mu_\nu - \delta^\mu_\nu\Theta - {\rm c.t.}|_{bdy} \qquad \Theta = \text{extrinsic curvature}$$

Fix counterterm coeffs w/

–Ward identity: $2E = dP$ = residual bulk gauge symmetries

–first law of thermodynamics: $(E + P = TS + \mu N)$

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$$\longrightarrow \quad \mathcal{E} = \frac{E}{V} = -\int \sqrt{\gamma}\, T^t_t = \frac{1}{16\pi G r_H^4} = \frac{\pi^2 N^2}{64} L_\xi \frac{T^4}{\mu^2}$$

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Who is T^ξ_t? Just as T^χ_μ is the R-charge current,

Density: $$\rho = \int \sqrt{\gamma}\, T^\xi_t = \frac{\beta^2}{16\pi G r_H^4} = \frac{\pi^2 N^2 T^4}{32\mu^3} L_\xi$$

Note: $T^\xi_\xi, T^t_\xi = \infty$ with naive falloffs on $\delta_{\mu\nu}$. We don't care about these anyway.

comments about the result:

Scale symmetry symmetry demands that $F(T, \mu) = T^2 f\left(\frac{T}{\mu}\right)$

[Landau-Lifshitz]

– unitary fermions: $f(x)$ has a kink at the superfluid transition.

– for some reason, we find: $f(x) = x^2$.

– the reason [MMTv5]: a) if solution arises from DLCQ, an extra (boost) symmetry: $t \to \alpha t, \xi \to \alpha^{-1}\xi \quad \Longrightarrow T \to \frac{T}{\alpha}, \mu \to \frac{\mu}{\alpha^2}, F \to F \Longrightarrow F(T, \mu) = g\left(\frac{\mu}{T^2}\right)$

b) melvin twist doesn't change planar amplitudes

(bulk explanation: symmetry of tree-level string theory

boundary explanation: 'non-commutative phases' cancel)

Viscosity

Kubo Formula: $\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle T^x_y T^x_y \rangle$

T^x_y couples to h^y_x in the bulk. h^y_x solves the scalar wave equation.
The field theory stress tensor is an operator with particle number zero:
source is $h^y_x(\ell = 0)$. $\implies \quad \eta = \frac{\pi L_\xi N^2}{32} \frac{T^3}{\mu^2}$

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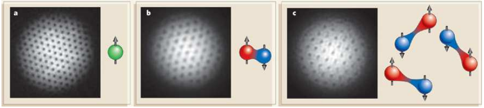
- schr BH $\in$
$\left\{ \text{spacetimes for which the argument of [Iqbal-Liu] shows that } \frac{\eta}{s} = \frac{1}{4\pi} \right\}$
- [C. McEntee, JM, D. Nickel]: confirmed the Kubo result for η by finding diffusion pole for transverse momentum:

$$\omega = Dk^2, \quad D = \frac{\eta}{\rho}$$

Final remarks

- Not unitary fermions, so far. ('Bertsch parameter' $\frac{E(T=0)}{E_0(T=0)} = 0$.)
- Most pressing: how to modify to remove lightcone inheritance, change $F(T, \mu)$, describe $\mu > 0$.
- Our string theory embedding for gravity duals of Galilean invariant CFTs with $z = 2$.
  which z arise in string theory? (Hartnoll-Yoshida: integer $z \geq 2$ at $T = 0$)
- We have found a black solution which asymptotes to the NR metric for $d = 2, z = 2$. (Kovtun-Nickel: arbitrary d in a toy model)
  It would be nice to find black hole solutions for other values of z.
- Fluctuations in sound channel McEntee-JM-Nickel see also [Rangamani et al 0811.2049]
- Superfluid? 

  Should break ξ-isometry (like Gregory-Laflamme), cut off IR geometry.
- Spectrum of $\hat{N}$ needn't be $\mathbb{Z}$: *e.g.* multiple species.
  inhomogeneous ground states (LOFF)? most mysterious near unitarity point. so far: we've found a vacuum solution.

The end.

multiple species

$$L_{\rm bulk} = R + \Lambda - \frac{1}{4}F_1^2 - \frac{1}{2}m_1^2 A_1^2 - \frac{1}{4}F_2^2 - \frac{1}{2}m_2^2 A_2^2$$

with (for $d = 2$)
$m_1^2 = 4z, \quad m_2^2 = -4(z-2), \quad \Lambda = \left(26 - 7z + z^2\right)$ .The z-dependence of Λ is a new development.
The solution is

$$ds^2 = -r^{-2z}dt^2 + r^{-2}(-2d\xi_+ dt + d\vec{x}^2 + dr^2) + d\xi_-^2 r^{2z-4}$$

with

$$A_1 = \Omega_1 r^{-z} dt, \quad A_2 = \Omega_2 r^{z-2} d\xi_- .$$

Interestingly, for $z = 2$, the $g_{\xi_- \xi_-}$ coefficient is 1.

$$[K_i, P_j] = i\delta_{ij}\hat{N}, \quad \hat{N} = i\partial_{\xi_+}$$

So, if we set $\xi_\pm = \xi^1 \pm \xi^2$ and compactify

$$\xi_1 \simeq \xi_1 + L_1, \quad \xi_2 \simeq \xi_2 + L_2$$

then the spectrum of $\hat{N}$ is

$$\left\{ \frac{n_1}{L_1} + \frac{n_2}{L_2} | n_{1,2} \in \mathbb{Z} \right\};$$

in particular $\frac{L_1}{L_2}$ needn't be rational.

We can think of $i\partial_{\xi_1}$ and $i\partial_{\xi_2}$ as the conserved particle numbers of the individual particle species; only their sum appears in the schrodinger algebra.

We also know how to construct an example which allows transitions between species, *i.e.* spectrum of $\hat{N} \neq \mathbb{Z}$ but there is only one conserved particle number.

The end.

algebra

$$[M_{ij},\ N] = [M_{ij},\ D] = 0, [M_{ij},\ P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i),$$

$$[P_i,\ P_j] = [K_i,\ K_j] = 0, [M_{ij},\ K_k] = i(\delta_{ik}K_j - \delta_{jk}K_i)$$

$$[M_{ij},\ M_{kl}] = i(\delta_{ik}M_{jk} - \delta_{jk}M_{il} + \delta_{il}M_{kj} - \delta_{jl}M_{ki})$$

$$[D,\ P_i] = iP_i, [D,\ K_i] = (1-z)iK_i, [K_i,\ P_j] = i\delta_{ij}N,$$

$$[H,\ K_i] = -iP_i, [D,\ H] = ziH, [D,\ N] = i(2-z)N,$$

$$[H,\ N] = [H,\ P_i] = [H,\ M_{ij}] = 0.$$