A talk about Nothing

based on work with:

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What do I mean by ’Nothing’?

A possible phase of quantum gravity where $\langle ds^2 \rangle = 0$. ’unbroken phase’.

An old idea:

• prerequisite for Sakharov’s Machian ’induced gravity’ idea:
  Elasticity of space $M_P^2 \sqrt{g} R$ as a result of quantum fluctuations of matter fields.

• ’The vacuum’ of canonical quantum gravity, CS gravity.

• The inside of Bubbles of Nothing.  \textit{(from Fabinger and Horava)}

This is a nonperturbative (Euclidean QG) instability of the Kaluza-Klein vacuum of GR, and of Scherk-Schwarz vacua of supergravity.  \textit{(Witten)}

• ∃ attempts to construct Nothing in String Field Theory.

\textit{(Horowitz, Lykken, Rohm, Strominger; Yang, Zwiebach)}
• Our best examples are still in \( d = 2 \).

The presence of the Fermi sea spontaneously breaks general covariance; Closed string excitations are ripples. This ’spacetime substance’ is made of D-branes \((\text{JM, H. Verlinde})\).

Other states, different from the perturbative vacuum, have different numbers of fermions, and are described by configurations of the closed-string tachyon.
Why am I talking about it?

- The previous motivations
  unhiggsing restores symmetries, should tell us about microphysics

- It might help with singularity resolution.

Q: how to ask questions about the nothing state?
Attach ‘regions’ of it to regions of normal spacetime.

How?
Localized tachyons.
Does string theory resolve spacelike singularities?
If so, when?

\[ a) \ l_s? \quad b) \ g_\nu^\nu l_s? \quad c) \ l_P? \quad d) \ other \]

Reason to hope it might sometimes be choice a):

In perturbative string theory, the metric is already an emergent quantity
in the sense that the metric is a condensate of string modes
Existence of large dimensions is a result of massless worldsheet bosons
The stiffness of (gedanken-)rulers is a consequence of the rigidity of this condensate.

Given this circumstance, we might imagine that it can be destroyed by the presence of other strings
winding tachyons: strings that want to be there more.
more specific claims:

1. When the matter sector of the worldsheet theory has a mass gap, the theory is in a Nothing phase. In the competition between kinetic terms $G_{\mu\nu}\partial X^\mu \partial X^\nu$ and potential terms $V(X)$, potential wins.

2. There are examples where the perturbative description is self-consistent. 
   \textit{i.e.} such phases can be perturbatively accessible.
   Modes which would back-react are lifted.
   \textit{vs.} The bubble of nothing is a nonperturbative Euclidean QG effect.
strategy

- Take perturbative single-string worldsheet point of view. defined by CFT, $g_s \ll 1$.
- Take seriously the worldsheet mass gap from stringy tachyons.
- Connect the gapped phase to a 'normal' phase and make the whole thing a CFT by Liouville evolution nonlinearly realized conformal symmetry:
  \[
  z \mapsto \lambda z, \quad X \mapsto X - \ln \lambda
  \]

Confession: I won’t include fluctuations of the Liouville field in all examples.
Outline

II. basic example of generating a worldsheet mass gap which can be localized:
review of RG of XY model.

III. localized in space: RS compactification

(hep-th/0502021, with A. Adams, X. Liu, A. Saltman, E. Silverstein)

IV. localized in time: the tachyon at the end of the universe

(hep-th/0506130, with E. Silverstein)

localized in a null direction?

V. comments about other probes
II. XY model

A 2d CFT with a relevant operator whose conformal dimension we can control:

sigma model whose target is $S^1$. $\theta \simeq \theta + 2\pi$.

$$L_{UV} = \frac{L^2}{4\pi l_s^2} \partial \theta \bar{\partial} \theta$$

This model describes superfluid films:

$$\frac{L^2}{4\pi l_s^2} \sim \langle |\Psi|^2 \rangle / T \equiv \rho_s / T,$$

$\Psi = |\Psi|e^{i\theta} \sim$ condensate wavefunction

Phase stiffness is determined by magnitude of condensate.

the main character: $\mathcal{O}_{nm} = e^{i(n\theta + m\tilde{\theta})}$ $\theta = \theta_L + \theta_R$, $\tilde{\theta} = \theta_L - \theta_R$

$\mathcal{O}_{nm}$ makes $\theta$ jump by $2\pi m$ (a disorder operator)

it creates a string with $m$ units of winding around the $S^1$. 

Winding tachyon

\[ \Delta_{nm} = \left( \frac{n}{L} \right)^2 + (mL)^2 \]

in \(2\pi l_s^2 = 1\) units \(\implies\) For \(L < L_c = \sqrt{2}l_s\), \(\Delta_{0,\pm1} = L^2 < 1\) are relevant.

**Q**: What happens when a gas of such insertions condenses?

Vortex condensation \(\delta L = \mu \cos \tilde{\theta}\) destroys long range correlations of the \(\theta\) variable:

- **when** \(\mu = 0\), correlations are algebraic:
  \[
  \langle e^{ip\theta(z)}e^{-ip\theta(w)} \rangle \sim \frac{1}{|z-w|^{l_s^2p^2}}
  \]

  For \(\mu \neq 0\),

  \[
  \langle e^{ip\theta(z)}e^{-ip\theta(w)} \rangle \sim e^{-m|z-w|}
  \]

  To see this: fermionize.
The lines don’t go straight up. The tachyon exerts a force on the radius.
Universal jump in phase stiffness.

Claim: supersymmetric sine-gordon is qualitatively identical with antiperiodic boundary conditions.

now let’s make a string theory with this.
III. Riemann surface compactification

Make $\theta$ the coordinate along a one-cycle of a RS $\Sigma_h$

Consider IIA on $\Sigma_h$.

In large-volume (‘$\alpha' \to 0$’) limit, worldsheet beta functions agree with supergravity: $\beta_{\mu\nu} = R_{\mu\nu}$

constant negative curvature is a local minimum.

classically, complex structure moduli are flat directions.
tadpole for volume $V_\Sigma$:

$$V_{\text{eff,8d}} \propto \left( \frac{g_s}{V_\Sigma} \right)^{2/3} (2h - 2)$$

rolls towards $V_\Sigma \to 0, g_s \to 0$, slowly if $V_\Sigma \gg l_s^2$.

There are $2^{2h}$ choices of spin structure in the target space.

Periodic BCs for the target fermions project out winding tachyons.
Consider a neighborhood of a handle that has antiperiodic boundary conditions (APBCs).
If the curvature $l_s^2/V_\Sigma$ is small enough,

$$ds^2 \sim dx^2 + (L_0^2 + O(1/V_\Sigma))d\theta^2 + ...$$

XY model varying adiabatically with $x$ and $t$.

Spectrum of wound strings is as in flat space plus perturbations.

$$\alpha' m^2 = -1 + L_0^2/2l_s^2 + p^2 + \text{osc...}$$

If complex structure moduli are such that the length of the minimal geodesic on the A-cycle has $L_0 < L_c$, there’s a winding tachyon.

It is localized to the region where $L < L_c$.

Note that the restriction to $\frac{dL}{dx} \ll 1$ is important: e.g. flat space in polar coordinates.

**Q**: what happens when it condenses?

**Claim**: The handle pinches off, leaving Nothing in its place.
This is why I emphasized the ’universal jump’ in

\[
\frac{\rho_s}{T} = \frac{L^2}{4\pi l_s^2}.
\]
Some disclaimers:

1. The proximate result of the tachyon gets only part of the way to $\langle ds^2 \rangle = 0$.
There’s a region of 8d type 0 (plus radiation) with bulk tachyons which peacefully condense....

2.

’Pseudopods’ of excess positive curvature, shrink back to constant negative curvature.
3. If $h$ changes, the worldsheet Witten index

$$\text{tr}_{\text{ws}}(-1)^F = \chi(\Sigma_h) = 2 - 2h$$

seems to jump!

resolution: some vacua are left behind in the Nothing region. ’dust’.

I will give evidence for each of these from LSM.

Note: two possibilities
lose a handle.

disconnect.

$$\begin{array}{c}
\text{a)} \\
\text{b)}
\end{array}$$

$$\begin{array}{c}
\text{b)} \\
\text{b)}
\end{array}$$
Consistency checks

In oriented string theory on a RS, there are $2h$ massless vector fields from $A_\gamma = \int_{\gamma \in H_1(\Sigma_h)} B_{\text{NSNS}}$.

Changing $h$ changes this number. How?

A-cycle is easy:
The winding tachyon around $A$ is charged under $A_A$. $\implies$ Higgsed.

B-cycle:
$$\int_{10} H \wedge \star H = \ldots + \int d^8 x \frac{1}{\tau_2} |F_A + \tau F_B|^2$$
$$\tau_2 \propto g_{\theta \theta} \implies g_{YM}^B \to \infty$$

"classical confinement" \footnote{Kogut, Susskind, PRD9, 3501(1974)}

(familiar from COM $U(1)$ of unstable branes) A pair of strings wound around $B$ oppositely develops a flux line between them in $R^7$

along which $\langle T \rangle = 0$.

Microscopically, This is what happens to a string on the B-cycle:
If you put RR flux through $A$: $q = \int_{A \times Q} F^{(1+q)}$

$Q$ is some $q$-sphere in the other dims

D-brane sources appear (if there’s enough energy for the process to happen).
Why?

1. Stringy modes don’t penetrate the handle. localized mass gap = big potential barrier.

other probes?

2. GLSM: \( \{ w^2 - |\phi|^2 = \xi, \ w \in R \} \)

note: FI term is usually a kahler modulus...
\( \xi \) actually determines both volume and tachyon.
why the FI parameter controls the vortex density

Recall Buscher trick: the dual circle coordinate is a dynamical theta angle.

\[ S = \int d^2 z \left( L^2 (\partial \theta + A)^2 + \tilde{\theta} F + \frac{1}{e^2 F^2} \right) \]

gauge symmetry acts as \( A \mapsto A - d\lambda, \theta \mapsto \theta + \lambda \).

At long distance, vortex configuration has \( F \sim \delta(z - z_0) \)
its contribution is \( e^{-S_{cl}} e^{i\tilde{\theta}(z_0)} \)

with \((2,2)\) susy, this is \( e^{-t}, \quad t \equiv \xi + i\tilde{\theta} \).

this is why this LSM is better than \( W = P(XY - \mu) \).
first attempt

one $U(1)$ with chirals $\phi_+, \eta_+, \phi_-, P_-$

$$D = |\phi_|^2 + |\eta_+|^2 - 2|P_-|^2 - 2|\phi_-|^2 - \xi$$

$$\beta_\xi = \sum_i Q_i \equiv Q_T = -2$$

$\xi \to +\infty$ in IR. add $W = mP_-\phi_+\eta_+$ take $m \sim e$ large, mass of fluctuations off vacuum manifold

large $|\xi|$ semiclassical.

at $\xi \to +\infty$ (IR): either $\phi_+$ or $\eta_+$ must be nonzero branches of $\phi_+\eta_+ = 0$ are disconnected

if $\phi_+ \neq 0$, use $U(1)$ to set $\phi_+ = w \in R_+$. $w^2 - 2|\phi_-|^2 = \xi \quad \longrightarrow \text{a cap.}$
at $\xi \to -\infty$ (UV):, either $\phi_{-2}$ or $P_{-2}$ must be zero
if $P_{-2} = 0$, $\phi_+ \eta_+ = 0$, if not, $\phi_+ = \eta_+ = 0$.
if $t \equiv \xi + i\theta \neq 0$, still nonsingular
An extra $\mathbb{P}^1$ is attached at $\phi_+ = \eta_+ = 0$.

Claim: This weird UV phase is near the narrow handle universality
class.

technicality: winding tachyon and ’deformation’ not mutually \((2, 2)\).

\[
\delta L = q_+ q_- (-\mu)(P_2 \bar{P}_2 + \phi_+ \bar{\eta}_+ + \bar{\phi}_+ \eta_+)
\]

\(q_\pm \equiv \frac{1}{\sqrt{2}} (Q_\pm + \bar{Q}_\pm)\) are the preserved supercharges.

Claim: the fact that we’ve broken the worldsheet supersymmetry \((2, 2) \longrightarrow (1, 1)\) doesn’t disturb the usual GLSM RG flow, for small \(\mu\).

the off-vacuum field space of the LSM (the embedding space) provides coords on the Nothing region.
Dust vacua!

So far, we’ve talked about the ’higgs branch’ of the vacuum manifold.

\[ \Sigma = \sigma + \theta \lambda + \theta^2 (F + iD) + ... \]

consider region of large \( \sigma \).
\[ L \ni -|\phi|^2 |\sigma|^2 \quad \Rightarrow \quad \Phi \text{s are massive, integrate out.} \]

\[ \tilde{W} = t\Sigma + Q_T \Sigma \ln \Sigma \]
\[ L \ni \int d\theta^+ d\bar{\theta}^- \tilde{W} + \text{h.c.} \]

R-symmetries:
\[ \theta^+ \mapsto e^{i\alpha} \theta^+, \quad \theta^- \mapsto e^{i\alpha} \theta^- \]
\[ \tilde{W} \propto \Sigma \quad \Rightarrow \quad \Sigma \mapsto e^{i(\alpha^+ - \alpha^-)} \Sigma \]

The second term reflects the anomaly in the axial R-symmetry.

\[ Q_T = -2 \in 2\mathbb{Z} \quad \Rightarrow \]
there is a non-anomalous $\mathbb{Z}_2 = \langle g \rangle \subset U(1)_{\text{axial}}$ by which the chiral GSO acts.

vacua appear at $0 = \frac{\partial \tilde{W}}{\partial \sigma}$

$$\sigma_{\pm} = \pm e^{t/2}$$

Reliable at large $t > 0$.

$$g : \sigma_+ \mapsto \sigma_-$$

$$\{\text{two vacua, } \sigma_{\pm}\}/\text{GSO} = \text{point}/\text{diagGSO}$$

8d type zero.
The tachyon at the end of the universe

Attempt to turn the previous picture sideways.

Consider the FRW-like:

\[ ds^2 = -dt^2 + L^2(t)d\theta^2 + ds^2_\perp \]

with APBCs on \( \theta \)

Like inside of the BTZ black hole.

classically: \( \ddot{L} = 0 \).
demand \( \dot{L} \ll 1 \)
take \( \dot{L} > 0 \) (bang).

Note: \( \exists \) witten bubble
Consider evolving towards the past. What does a single particle probe see? in GR or with periodic BCs

$$Z \sim \int [dX] e^{i \frac{4 \pi l_s^2}{s}} \int d^2 z G_{\mu \nu} \partial X^\mu \bar{\partial} X^\nu$$

when $G_{\theta \theta} < l_s^2$, fluctuations are unsuppressed.
Now, if $\dot{L} \ll 1$, $\alpha'm^2_{\text{winding}} = -1 + L^2/l_s^2$: winding tachyon if $L < L_c$

$$\rightarrow Z \sim \int [dX] \exp \left( \frac{i}{4\pi l_s^2} \int d^2z \left[ G_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu - \hat{T}(X^0) \cos \tilde{\theta} \right] \right)$$
an attempt

\[ \hat{T}(X^0) \sim \mu e^{-\kappa X^0} \]

where \( \kappa \) is the ’tachyon mass at the onset time’ determined by nonlinear dynamics.

Like sine-Liouville.

Use this integral to try to define amplitudes.

(like Strominger-Takayanagi, Schomerus)

This specifies a particular state.
Note: like in Liouville, we only know the asymptotic behavior away from $\langle T \rangle$ of $\langle T \rangle, \langle \Phi \rangle$...

Claim: results are insensitive to the behavior under the barrier.

On the worldline, $T$ is a potential (position-dependent mass).

What do these amplitudes compute?
Coefficients of the wavefunction in the free-string basis.
sample calculation

Old trick \((Gupta-Trivedi-Wise)\): \(X^0 = X_0^0 + \hat{X}^0\).

\[
\frac{\partial}{\partial \mu} Z_{T^2} = \int dX_0^0 \int [d\hat{X}^0] \int [dX_\perp] e^{iS_{kin}} \frac{C}{\mu} e^{-\kappa X_0^0} e^{-Ce^{-\kappa X_0^0}}
\]

here \(C \equiv \int d^2\sigma \mu e^{-\kappa \hat{X}^0} \hat{T}\) is the nonzeromode part of \(T\).

\[
= \int [d\hat{X}^0] \int [dX_\perp] \frac{C}{\kappa \mu} \left( \int_0^\infty dy e^{-Cy} \right) e^{iS_{kin}}
\]

\[
Z_{T^2} = -\frac{\ln \mu/\mu_*}{\kappa} \hat{Z}_{T^2} = (X_*^0 - X_\mu^0) \hat{Z}_{T^2}
\]

\(\mu_* = e^{\kappa X_*^0}\) IR cutoff in the free region, \(\mu \equiv e^{\kappa X_\mu^0}\).

Compare:

\[
Z_{T^2} \text{(no tachyon)} = T \hat{Z}_{T^2}
\]

\(T = \delta(0) = \int_{-\infty}^\infty dX_0^0\)
Some final comments

0. **suppression of back-reaction**: if indeed formerly-light string modes are made heavy by tachyon condensate, their back-reaction to the time-dependence will be suppressed.

1. **reversing the process**: some amount of radiation comes out. by making some agreement with someone far away, and sending in exactly the time-reversal of the radiation that comes out (specific correlations), you could (in principle!) create such a wormhole.

In the case of disconnected components, this is quite strange.

2. Restoration of symmetries hidden by nonlinear dynamics.

3. **Q**: What happens in the tails of the tachyon wavefunctions? these are less localized than APS.

4. Effective field theory description of disconnection process?
5. Q: Do D-brane probes agree?

Polyakov (hep-th/9304146) suggests a probe of the Nothingness (diffusion dimension).

\[ d_{\text{eff}} = \frac{d}{d \ln \tau} \left( \frac{\int R(x, x, \tau)}{\int 1} \right) \]

\( R(x, x', \tau) = \) probability of propagating from \( x \) to \( x' \) in worldline time \( \tau \).

\[ d_{\text{eff}} = \begin{cases} 
  d, & \text{flat space} \\
  0, & \text{nothing} 
\end{cases} \]

In closed string theory, this is an annulus amplitude.

Between what branes? see Hikida, Tai hep-th/0510129
The End.
details about (1,1) vacuum manifold

\[ F_{P_{-2}} = m\phi_+\phi_- - \mu\bar{P}_{-2} \quad (1) \]
\[ F_{\phi_+} = mP_{-2}\eta_+ - \mu\bar{\eta}_+ \quad (2) \]
\[ F_{\eta_+} = mP_{-2}\phi_+ - \mu\bar{\phi}_+ \quad (3) \]

\[ (2) + (3) \implies P_{-2} = \frac{\mu}{m} \frac{\phi_+ + \eta_+}{\phi_+ + \eta_+} \]

determines \( P_{-2} \)

\[ \implies |P_{-2}| = \frac{\mu}{m} \]

\[ (2)/(3) \implies \frac{\phi_+}{\eta_+} = \frac{\bar{\phi}_+}{\bar{\eta}_+} \equiv x \in \mathbb{R} \]

\( \eta_+ = x\phi_+ \) determines \( \eta_+ \)

\[ (1) \implies x|\phi_+|^2 = \left( \frac{\mu}{m} \right)^2 \implies x > 0 \]
determines \( |\phi_+| \neq 0 \), fix \( U(1) \) with \( \phi_+ \in R_+ \). D-term

\[
(x + 1/x) \left( \frac{\mu}{m} \right)^2 = \xi + 2 \left( \frac{\mu}{m} \right)^2 + 2|\phi_-|^2
\]

\[
\left( \sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 \left( \frac{\mu}{m} \right)^2 = \xi + 2|\phi_-|^2
\]

This says

\[
2|\phi_-|^2 = w^2 - \xi,
\]

\( w \equiv \sqrt{x} - \frac{1}{\sqrt{x}} \).

Claim: for small enough \( \mu \), (2, 2) RG is preserved.
Aside about field theory dual

In a dual gauge theory, winding tachyon on $\gamma \leftrightarrow$ Wilson loop operator $W$ on $\gamma$

$\langle W \rangle \neq 0 \Leftrightarrow \gamma$ is contractible.

If $\gamma$ is the Euclidean time circle, this is the argument Barbon-Rabinovici, Aharony et al that shows that

a vev for the Polyakov-Susskind loop

$\Leftrightarrow$

the dual geometry contains a BH horizon.
A slide about minisuperspace

Minisuperspace worldline theory:

\( H = 0 \) is a Schrödinger equation with a rapidly falling potential.

If \( V(x) \) grows faster than \(-x^2\), \textit{e.g.} \( V \sim e^{\kappa x} \)

\( x(\tau) \) reaches \( x = \infty \) at finite parameter time \( \tau_\infty \).

\( H \) isn’t self-adjoint.

reparametrization BRST anomaly.

No on-shell poles in Green’s functions.

Required: a prescription for ’bouncing off the future’.

\textbf{Warning:} In field theory, such a prescription is different than local Hamiltonian evolution.
WKB wavefunctions

for bang case wavefunctions look like

\[ u_k(t \to \infty) \sim \frac{1}{\omega(t)} e^{\pm i \int_{0}^{t} dt' \omega(t')} + ... \]

with \( \omega^2(t) = k^2 + m_0^2 + \mu e^{-\kappa t} \).

Shrinking and rapidly oscillating.

A family of choices of ”self-adjoint extensions” arises if we restrict to

\[ u'_k(t \to \infty) \sim \frac{1}{\omega(t)} \cos \left( \int_{0}^{t} dt' \omega(t') + \nu \right) + ... \]