Uses of string theory

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April 4, 2008

What is String Theory?

Three interpretations of this question:

1. Literally, what is it, *i.e.* "How do we formulate string theory from first principles in a meaningful and non-contingent way?"

2. "What is string theory good for?"

3. "What have string theorists been trying to do?"

String theory is an alien artifact

Interpretation 2: What is string theory for?

This depends on when you ask.

If you asked someone this question in 1970, they would have said it's a model of the strong interactions.

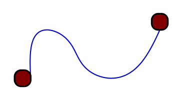
If you asked them again in 1974 they would have said

- there's a better microscopic model (QCD)
- the string model has some real problems:

1. it has an extra massless spin two particle that isn't observed experimentally.

2. it is unstable

3. it lives in the wrong number of dimensions $(26 = 2 \times 12 + 2)$

QCD also contains string-like excitations:  More on this later.

Massless spin two particle?

Well, problem 1 was an illusion. It was soon realized that effects of this particle are observed, and that it's called the graviton.
You stick to the earth is because you are exchanging (spacelike, virtual) gravitons with it.

So, granting that string theory makes sense, it is actually a solution to a long-standing problem (!!):
Find a quantum mechanical theory that predicts and describes gravity.

We still don't know any other such theory that has stationary states that look like smooth flat space.

Why should we want such a thing?

Quantum mechanics and gravity

General relativity (GR) is great: It predicts its own demise.
The Singularity Theorems of Penrose and Hawking (proved using GR) tell us that there are regions of space and time
(in the real world!! – in the very early universe, inside black holes)
that are not described by the GR we understand.

In particular, quantum mechanics is important there.

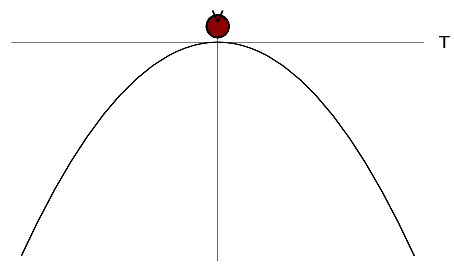
Some successes of string theory in this capacity:

- information paradox Clearly predicts unitarity, not clear how.
- singularity resolution Some spacelike singularities resolved handsomely.

tachyons!

Problem 2 is a real one.

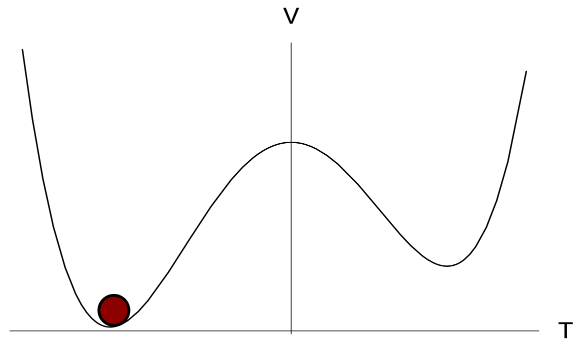
The spectrum did contain a “tachyon”: a mode with negative mass-squared.
This is an instability – the motion off the top of the hill:



It really is unstable to rolling down the hill.

With hindsight: before 1973, they were just studying one particular vacuum of the theory, which we call “the bosonic string in flat space”.
What do I mean by one ’vacuum’? Critical point of effective potential

Really the potential looks more like this:



But we only knew how to describe a small neighborhood of the maximum in this picture.

Soon, people found another vacuum without tachyons.

The vacuum that was found had ten dimensions instead. (closer!)
Actually there are several such vacua ('superstrings'):
IIA, IIB, type I, heterotic $E_8 \times E_8$, heterotic $SO(32)$.

too many dimensions

Problem 3 was actually solved in ∼1922 by Kaluza and Klein:
if dimensions are small compared to your wavelength, you can't see them.

Actually, even the statement that there are 6 small dimensions is not definite.
We know states that have more and states that have fewer dimensions,
they just aren't as symmetric, and are harder to study.

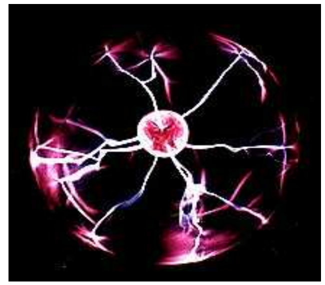
There are many ways to *compactify* the extra dimensions.

Some of the compactified states have particle physics in them,
i.e. interesting gauge theories:
chiral fermion matter particles which interact by exchanging vector bosons. There was a lot of enthusiasm.
It was, however, already clear that there wasn't a unique vacuum.
Parameters describing the size and shape of the extra dimensions and the string coupling gave *massless* moduli fields.

Flux vacua

Most of the vacua of string theory, though, include fluxes.

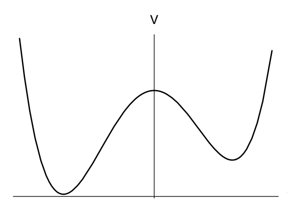
Like electromagnetic fields sourced by charged particles,



these are fields sourced by D-branes.

These fields carry energy $U = \frac{1}{8\pi}\int (E^2 + B^2)$ which depends on the size and shape of the extra dimensions and the string coupling.

For a given value of the fluxes, there is a preferred size and shape.



Some of these vacua look like our world. (So far, you have to squint.)

But: what is it?

Since the Time of Overenthusiasm, we've

a. learned more about the vacua. In particular, most of them don't have massless scalars. We have much more to learn.

b. become much more convinced by a large collection of surprising facts ("Duality") that the alien artifact is not a hoax.

The magic is hard to convey; I'm not really going to try.

(Giant tables of huge numbers computed in totally different ways, predicted to be the same by the theory, turn out to be the same).

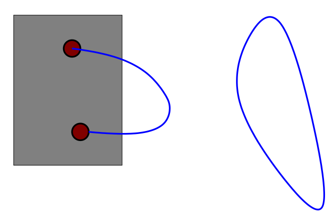
For example, the spectrum of string states in one string theory match to the spectrum of D-branes in another.

Supersymmetry has been an important tool here. We know a lot more about the special solutions which are supersymmetric.

Nonperturbative descriptions

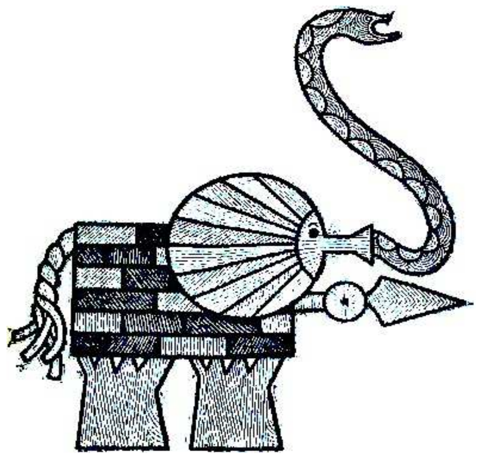
The perspective we have had since 1969 is just the point of view of a single-string
– we set up the background and ask the string what it sees.
This description (summing over histories of the string) is just like first-quantized perturbation theory in QFT (but much more constrained).

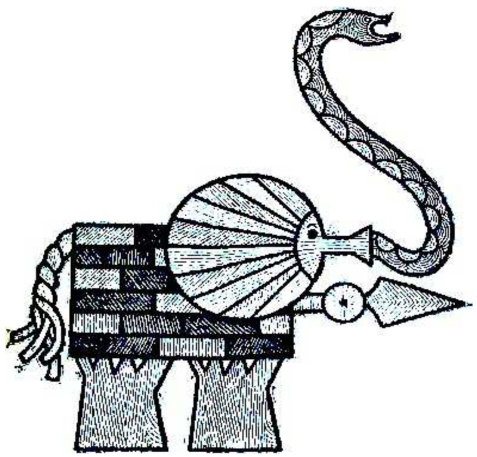
More recently we've learned about other probes of the theory (D-branes) and we know how to ask them what the theory looks like.



We have many approximate descriptions of limits of the theory which agree when they should, and when we can check.

‘Composite sketch’

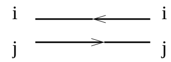


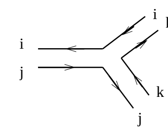


$\xrightarrow[\dots]{}$

“What is string theory for?” attempt no. 2

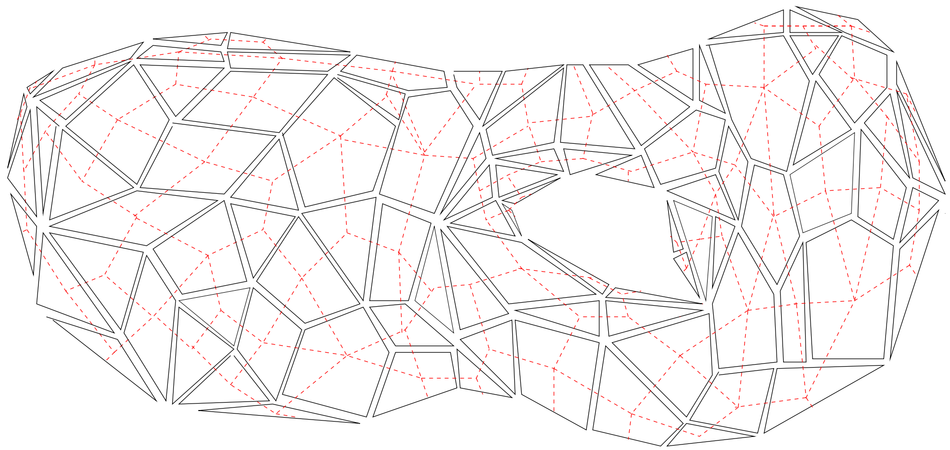
Strongly-coupled large-N field theory

gluon propagator: 

$i, j = 1..N$, interaction vertex: 

$\propto \lambda$

A feynman diagram in some gauge theory with N colors:

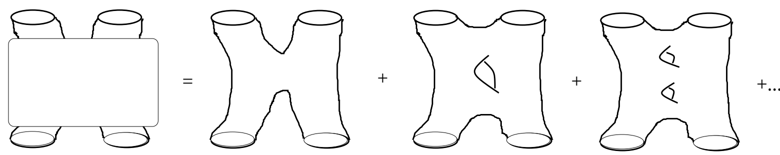


$$\mathcal{A} = \sum_g \left(\frac{1}{N}\right)^{2-2g} f_g(\lambda)$$

λ: coupling constant

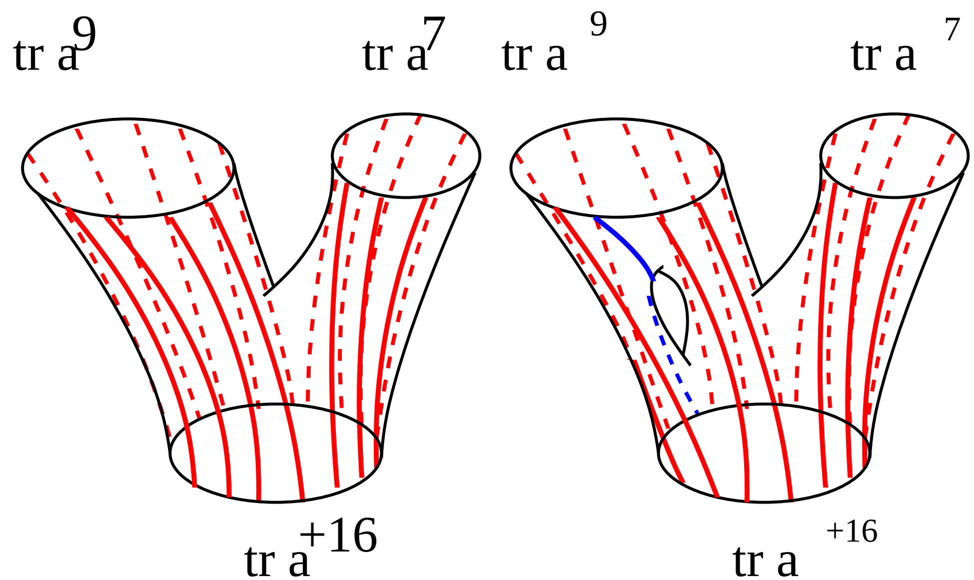
f_g: sum of diagrams which can be drawn on surface with g handles.

When the number of colors is large, this looks like string perturbation theory:



Even free gauge theories have this structure:

$$\langle 0|\mathrm{tr}\ a^9 \mathrm{tr}\ a^7 \mathrm{tr}\ \left(a^\dagger\right)^{16}|0\rangle$$



string/gauge duality

The key insight [Polyakov, Maldacena 97]: the QCD flux tube *is* a fundamental string, just not in the same space.

[string theory in some background X] contains the same info as [a gauge theory on some lower-dim'l space]

X depends on matter content, couplings

weak string coupling —— large N

The strings live in extra, auxiliary dims:
one of these is the resolution scale
– the string theory is a hologram.

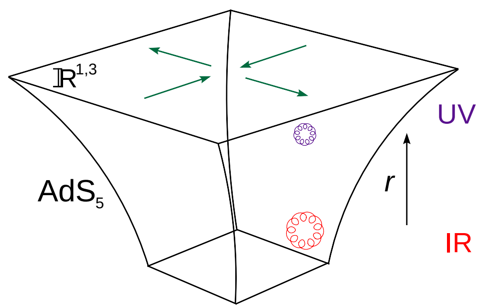
strings —— (non-abelian) faraday flux lines best-understood example:

$$[\text{IIB strings on } AdS_5 \times S^5] = [\mathcal{N} = 4 \text{ super Yang-Mills}]$$

AdS: homogeneous spacetime with negative cosmological constant.

This is a simple example of a 'flux vacuum'.

figure by Lance Dixon



$\mathcal{N} = 4$ SYM: a cousin of QCD. a relativistic 4d conformal field theory.

closed strings $\leftrightarrow$ gauge-invariant operators

basic check: symmetries match. infinitely many other checks. *e.g.* comparison of green's functions

strong/weak duality

$$R^2/\alpha' \sim \sqrt{\lambda}$$

Hard to check, very powerful.

$\lambda \to \infty \implies$ big space $\implies$ classical gravity in AdS.

Note: there is evidence for the stronger statement at finite N, λ

This is a two-way street:

$\leftarrow$ a definition of string theory in some background which does not rely on string perturbation theory.

$\rightarrow$ analytic results on a class of strongly coupled field theories.

[Hong, Krishna, Pavel Kovtun,...]

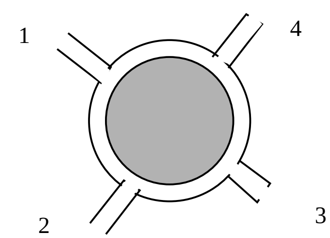
Parton scattering at strong coupling

$\exists$ huge literature computing scattering amplitudes (*e.g.* of gluons) in nonabelian gauge theories
e.g. for QCD 'backgrounds' to new physics at colliders.

These amplitudes are IR divergent and require regularization.

Regulator-dependence cancels after summing over indistinguishable final states (in any physical measurement)

[Bloch-Nordsieck-Kinoshita-Lee-Nauenberg-Sterman-Weinberg...]

large N: 

$$\mathcal{A}_{MHV}^{planar} = \sum_{\pi} \mathrm{tr}\ \left(T^{\pi(1)}...T^{\pi(n)}\right) \mathcal{A}_n(\mathbf{k}_{\pi(1)}, \mathbf{k}_{\pi(2)}...\mathbf{k}_{\pi(n)})$$

Scattering amplitudes of supersymmetric theories are useful

- they share many qualitative properties with QCD amplitudes
- they are simpler, useful for understanding structure
- they are a testing ground for calculation techniques
- they can be used as building blocks

Askable Q:

$$\boxed{\textbf{What does the S-matrix look like at strong coupling?}}$$

For planar amplitudes with special helicities (MHV), BDS [Bern-Dixon-Smirnov] guess (motivated by form of summable IR singularities, magic at low-orders in pert. theory):

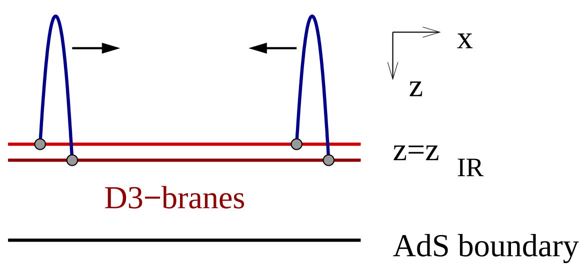
$f(\lambda) =$ 'cusp anomalous dimension'

Holographic description

[Alday-Maldacena, 07]

Introduce, as an IR regulator, n D3-branes into the AdS space. The gluons we will scatter are the lowest oscillation modes of the strings stretching between them.

The string here is the color-electric flux tube (cloud of virtual gluons) that the gluon must drag around with it wherever it goes.



Tunneling

The amplitude is dominated by a semiclassical process:

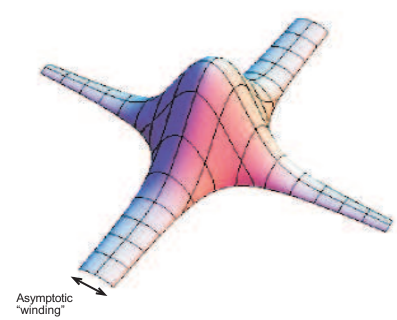
$$\mathcal{A} \sim e^{-S_{classical}}$$

like high-energy, fixed-angle string scattering in flat space. [Gross-Mende]

From string point of view: tunneling.

From field theory point of view: Sudakov suppression.

The probability for a colored particle not to radiate is very small especially at strong coupling.

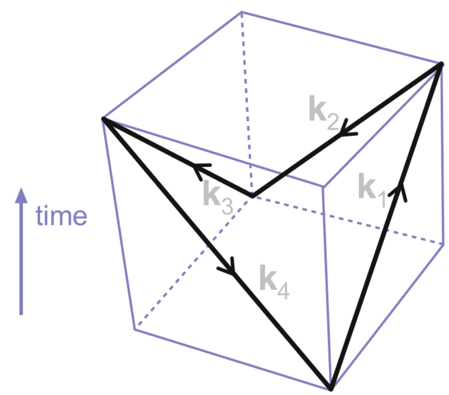
The dominant process looks like 

Finding the saddle

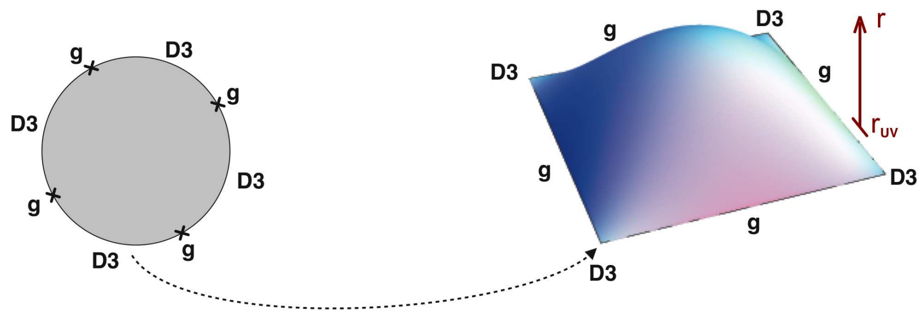
To find dominant trajectory, a useful change of variables:
'T-duality' exchanges momentum and winding of string
in fact, a fourier transformation in loop space. [JM, A. Sever]

The problem reduces to finding the extremal-area surface ending on a polygon ('plateau problem in AdS');
edges of polygon are specified by the *momenta* of the particles ordered according to color-ordering.

for four gluons:



In the momentum-space variables, the string worldsheet (euclidean) history looks like

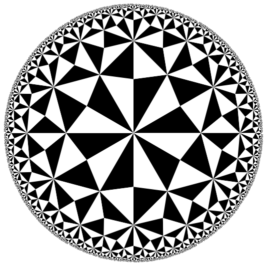


For a general curve C, this is the same AdS prescription for calculating Wilson loop expectation values at strong coupling: $W[C] = \mathrm{tr}\; P\; e^{i \oint_C A}$.

$$\mathcal{A} = \langle W[\Pi] \rangle \sim e^{-\frac{\sqrt{\lambda}}{2\pi}(\mathrm{Area})}$$

Sudakov factors from string theory

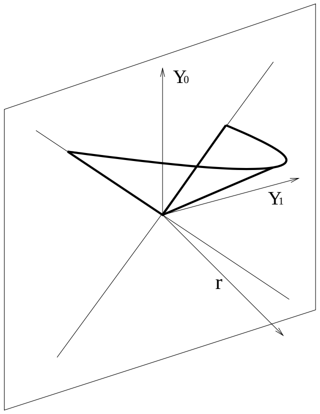
A divergence is associated to each corner of the polygon ('cusp')

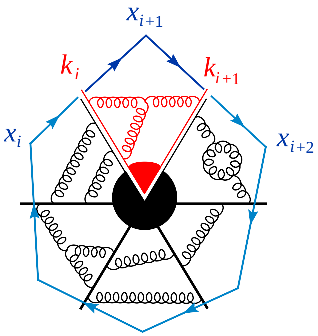


from the infinite volume of AdS near the boundary:

$$\mathcal{A}_{\rm div} \sim f(\lambda) \ln^2 \frac{s_{i,i+1}}{\mu^2}, \quad s_{i,i+1} = -(\mathbf{k}_i + \mathbf{k}_{i+1})^2$$

This matches the Sudakov IR divergences of planar perturbation theory: (LHS: string calculation of the 'cusp anomalous dimension'.)

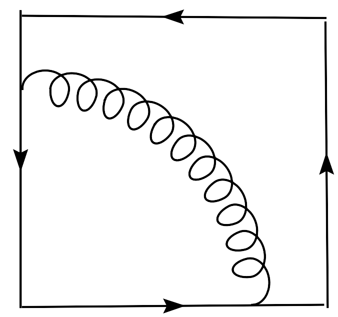




Finite parts

For four gluons, the area is equal [AM] to the 'self-inductance' of the loop:

$$\mathcal{A} = \oint_\Pi \oint_\Pi \frac{d\mathbf{y} \cdot d\mathbf{y}'}{(\mathbf{y} - \mathbf{y}')^2}$$



= Log of the expectation value of the Wilson loop in abelian gauge theory.

This is the BDS ansatz (!), when dimensionally regularized:

$$\mu^{2\epsilon} \oint_\Pi \oint_\Pi \frac{d\mathbf{y} \cdot d\mathbf{y}'}{[-(\mathbf{y} - \mathbf{y}')^2]^{1+\epsilon}} \; .$$

Comment about self-inductance

$$\mathcal{A} = \oint_{\Pi} \oint_{\Pi} \frac{d\mathbf{y} \cdot d\mathbf{y}'}{(\mathbf{y} - \mathbf{y}')^2}$$

Compare to formula for mutual inductance [Purcell, section 7.7]:

$$M_{12} = \frac{1}{c^2} \oint_{C_1} \oint_{C_2} \frac{dr_1 \cdot dr_2}{|r_1 - r_2|}$$

Self-inductance of infinitely thin wire is infinite.
In a real wire, this is 'regularized' by the thickness of the wire.

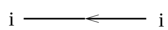
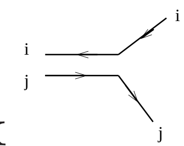
To compare these divergent quantities, we need to match regulators.

Conjecture (w/ C. McEntee): the regulator which is natural in AdS ('anchor brane' at z_{IR}) is to give the 'wire' some finite thickness.

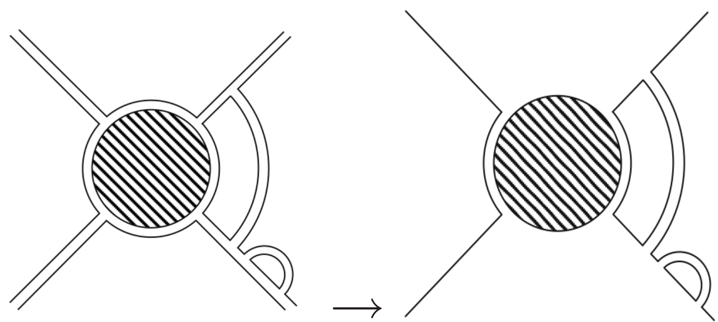
Generalizing string prescription to include quarks

[JM, A. Sever]

Field theory: Add to the $\mathcal{N}=4$ theory $N_f \ll N_c$ fields in the fundamental representation of the gauge group ('quarks').

quark propagator  quark-antiquark-gluon vertex 

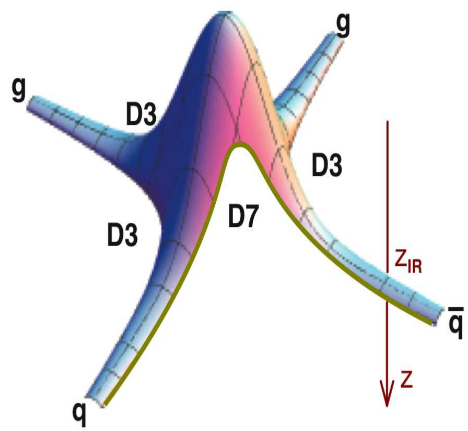
Adds boundaries to 't Hooft diagrams:



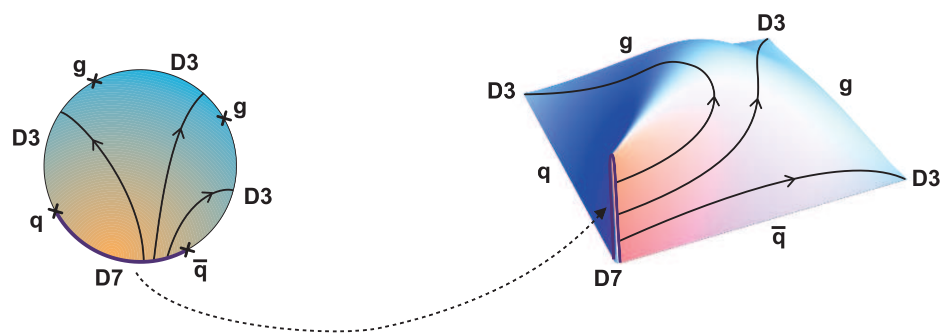
String theory: Add new D-branes ('D7-branes') which extend in the 'radial direction of AdS' (the strings with one end on the new branes and one end on the boundary have the charges of quarks.) massless

The process is still dominated by a saddle point,
but with different boundary conditions on dominant worldsheet trajectory.

real space:



momentum space:

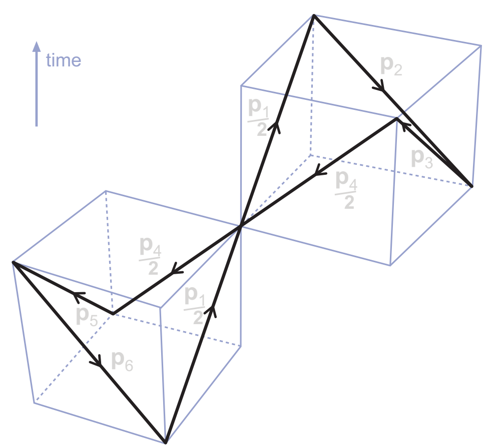


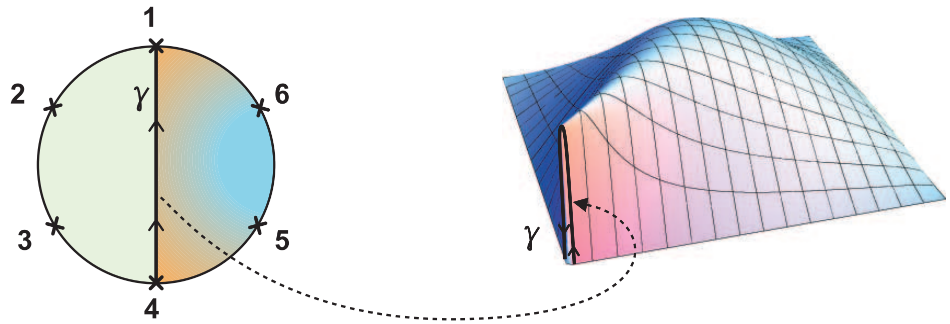
A new relation between field theories

Consider the (a priori unrelated) polygon associated to 6-gluon scattering with color-ordered momenta satisfying $\mathbf{p}_6 = \mathbf{p}_2$, $\mathbf{p}_5 = \mathbf{p}_3$. Momentum conservation

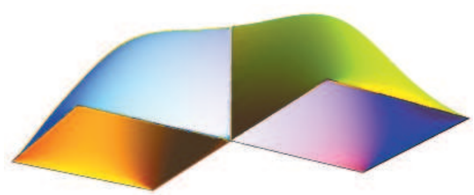
$$\Longrightarrow \qquad \frac{1}{2}\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \frac{1}{2}\mathbf{p}_4 = 0 \ .$$

implies a self-crossing of the polygon:





Symmetry implies that as we cross γ, the image must cross itself ("Whitney umbrella"), the worldsheet satisfies the D7-boundary conditions.



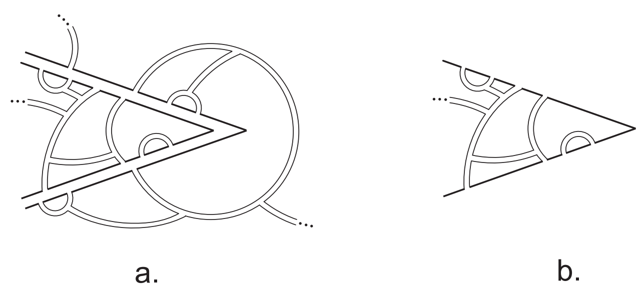
Area is additive $\Longrightarrow$ at strong coupling:

$$\ln \mathcal{A}_{qgg\bar{q}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \frac{1}{2} \ln \mathcal{A}_{6g}(2\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, 2\mathbf{k}_4, \mathbf{k}_3, \mathbf{k}_2)$$

BDS ansatz for 6-gluons gives an explicit prediction.

Checks

- Correct IR divergences
- The tension of the flux tube attached to a quark is half as big:



The image of the LHS picture is folded back on itself $\Longrightarrow$ twice the tension.

- Checks with Regge limits (where perturbation theory is summable).

Can generalize to other amplitudes.

Positive outcomes of string description

- another check of AdS/CFT
- debugging BDS (in progress)
- new relationship between field theories
- surprising relation between scattering amplitudes and Wilson loops

$$\mathcal{A}(k_1 ... k_n) = \langle W[\Pi] \rangle \equiv \langle \text{tr } P e^{i \oint_\Pi A} \rangle$$

We [JM, A. Sever] have derived a similar relation in planar gauge theory.

cartoon version Polyakov, by analogy with string theory:

$$\mathcal{A}_n(\mathbf{k}_i, \mathbf{e}_i) = \int \prod_{i=1}^{n} ds_i \int [D\mathbf{x}(s)] \langle W[\mathbf{x}(s)] \rangle \prod_{i=1}^{n} \mathbf{e}_i \cdot \mathbf{V}_{\mathbf{k}_i}(\mathbf{x}(s_i)) \ .$$

Loop space fourier transform of this gives AM prescription.

Reproduces heavy quark EFT formulae, eikonal approx. in appropriate limits.

A complaint

New project which may fail utterly

I don't know any (exact, quantum) relativistic CFTs in nature (with $d > 2$) (like $\mathcal{N} = 4$ SYM)

but: $\exists$ many interesting fixed points with Galilean invariance.

even in particle physics [Mehen-Stewart-Wise]

e.g. fermions at unitarity [Nishida-Son]

Some may have string duals, or classical gravity duals,
BUT: not in AdS.

To find the right universality class, we should at least get the symmetries right.

We [K. Balasubramanian, JM] have a guess for an analog of AdS_5 with at least some of the right properties.

The last slide

We don't actually know what string theory is.

AdS/CFT is our best definition, but it only works for asymptotically-AdS spaces.

Assigning a purpose to a object we don't understand is dangerous.

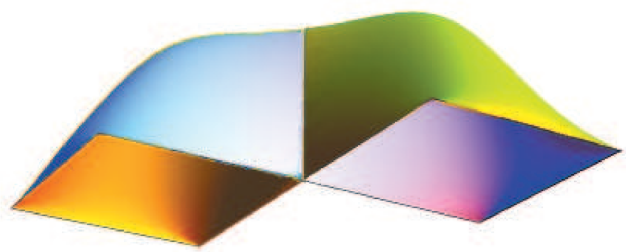
String theory is a quantum gravity. These are hard to come by.

There's some big collection of vacua, some of which are much MORE different from ours than just varying the masses of the neutrinos:

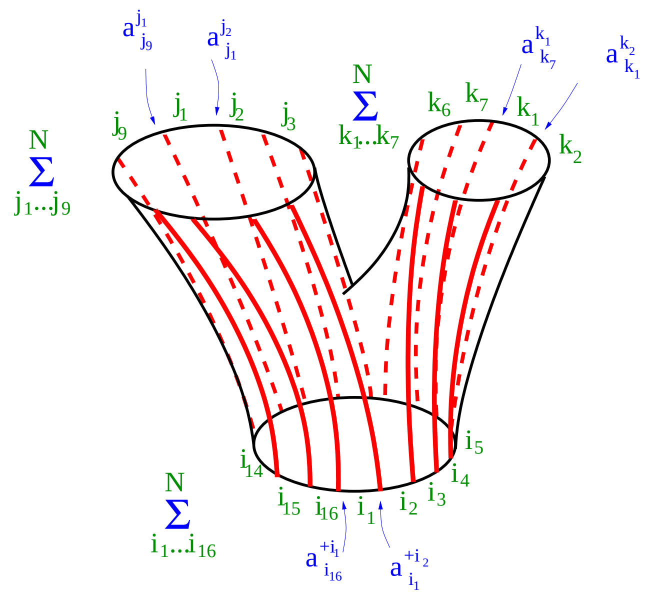
- some of them are ten-dimensional and nothing ever happens, they have nothing in them, and the vacuum is pristine and simple
- some of them contain nothing in a much more drastic sense.
- some of them are useful for realistic physics in very different way than was anticipated in 1985.

The End

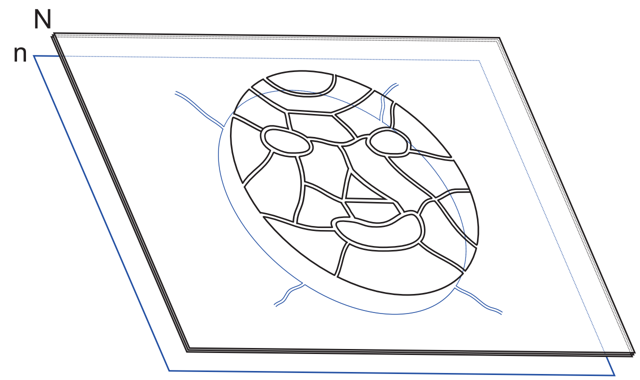
Thanks for listening.



What I meant by that diagram



Which amplitudes are contained in the Polyakov formula



Version 2: What do string theorists do?

One real goal at the moment is to understand the configuration space – how are different vacua connected to each other by not-vacua?

Like a particle moving on rolling hills, it can't sit on the side of the hill.

A word about language:

People used to speak of different string theories.
We now think it's better to call them different vacua (”vacuums”) of string theory.

A conjecture which we are testing is:

All the vacua are connected to each other by going uphill and then down again.

Every tachyon is an opportunity

If you go up and down again, at some point you were at the top.

One way in which we can access this is by studying theories with "tachyons"

like the bosonic string.

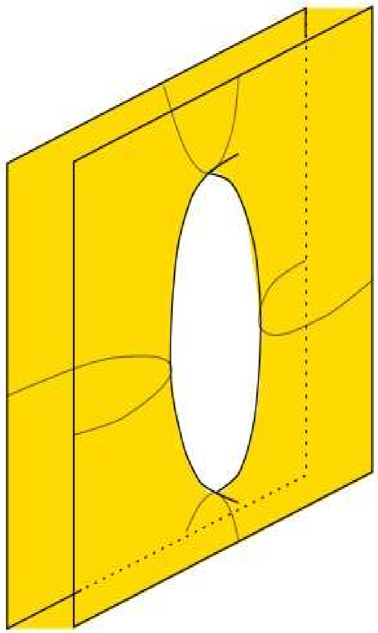
To what are the unstable vacua unstable?

Often they go to other vacua like the ones we've discussed.

Not always.

In string theory, the metric, our ruler and our clock, is made up of a coherent Bose-Einstein condensate of closed strings. (this is also not controversial, just an unfamiliar way to think about it.)

These condensates can be hard to maintain, they can go poof. What would it be like if this happened to the metric?



From Fabinger and Horava.

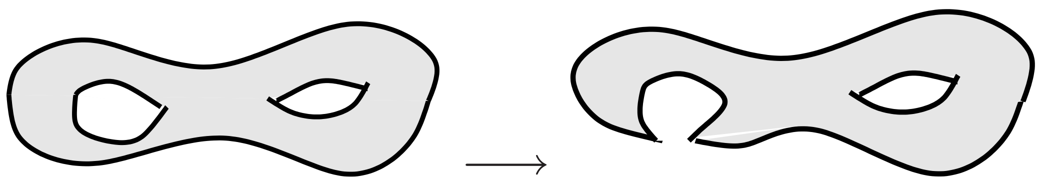
The "nothing" inside this bubble is much more extremely nothing than the vacuum of empty space.

Topology can change as a result of condensation of winding string tachyons.

Two possibilities

lose a handle.

a)



disconnect!