Assignment 1

Posted January 4, 2013  Due 2pm Friday, January 11, 2013

Please remember to put your name at the top of your homework.

Announcements

• The 130C web site is:
  
  http://physics.ucsd.edu/~mcgreevy/w13/ .

  Please check it regularly for announcements and updates to the lecture notes and other fun things.

• Recitation sections begin Tuesday January 8.

Readings

• The first 130C reading assignment is Chapter 2 of Preskill’s Quantum Information Notes, Chapter 2. Chapter 1 of 130C will begin by following the logic this discussion, with occasional simplifications.

• You should read Chapter 1 of Le Bellac quickly. I’m going to assume you have heard most of these things before. Read chapter 2 of Le Bellac. Here, too, I hope the ideas are somewhat familiar.

Problem Set 1

1. I am curious about this. What is currently your favorite field of physics? Possibilities might include: experimental or theoretical particle physics, string theory, condensed-matter physics (soft or hard, experiment or theory), astrophysics, atomic physics...
2. Quick, I hope, exercises in linear algebra definitions.

(a) Show that the determinant of a unitary matrix is a phase.

[Hint: use the fact that \( \det(AB) = \det A \det B \).]

(b) Consider some operators acting on a Hilbert space with a resolution of the identity of the form

\[
\mathbb{1} = \sum_n |n\rangle \langle n| .
\]

Recall that the matrix representation of an operator in this basis is \( A_{nm} = \langle n| \hat{A} |m\rangle \). Using Dirac notation, show that the matrix representation of a product of operators \( (\hat{A} \hat{B})_{nr} \) is given by the matrix product of the associated matrices \( \sum_m A_{nm} B_{mr} \).

For the purposes of the following problems, recall that a **spectral representation** of an operator \( \hat{H} \) on an \( N \)-dimensional vector space is an expression of the form

\[
\hat{H} = \sum_{i=1}^{N} \nu_i \hat{P}_i
\]

where \( \hat{P}_i \) is a set of \( N \) orthogonal projection operators.

3. **Do it by hand if you have to.** Find the spectral representation for the operator \( \hat{M} \) with the following matrix representation:

\[
M = \begin{pmatrix}
3 & \sqrt{2} & -1 \\
\sqrt{2} & 2 & \sqrt{2} \\
-1 & \sqrt{2} & 3
\end{pmatrix},
\]

Relate \( \hat{P}_i \) to the \( i \)th eigenvector of \( \hat{M} \) and use Dirac notation.

4. **Real and imaginary parts of operators.** You are given two hermitian operators \( \hat{H} \) and \( \hat{G} \). If they commute, i.e. \( 0 = [\hat{H}, \hat{G}] \equiv \hat{H}\hat{G} - \hat{G}\hat{H} \), show that their spectral representations can be given in terms of a common set of projection operators \( \{\hat{P}_i\} \), that is:

\[
\hat{H} = \sum_{i=1}^{N} \nu_i \hat{P}_i, \quad \hat{G} = \sum_{i=1}^{N} \mu_i \hat{P}_i .
\]

5. **Normal matrices.**

An operator (or matrix) \( \hat{A} \) is **normal** if it satisfies the condition \([\hat{A}, \hat{A}^\dagger] = 0\).

(a) Show that real symmetric, hermitian, real orthogonal and unitary operators are normal.
(b) Show that any operator can be written as $\hat{A} = \hat{H} + i\hat{G}$ where $\hat{H}, \hat{G}$ are Hermitian. [Hint: consider the combinations $\hat{A} + \hat{A}^\dagger, \hat{A} - \hat{A}^\dagger$.] Show that $\hat{A}$ is normal if and only if $[\hat{H}, \hat{G}] = 0$.

(c) Show that a normal operator $\hat{A}$ admits a spectral representation

$$\hat{A} = \sum_{i=1}^{N} \lambda_i \hat{P}_i$$

for a set of projectors $\hat{P}_i$. 