

Quantum Mechanics C (130C) Winter 2015 Final exam cover sheet

Please remember to put your name on your exam booklet. This is a closed-book exam. There are **6** problems, each with several parts, of varying levels of difficulty; make sure you try all of the parts. None of the problems require very extensive calculation; if you find yourself involved in a morass of calculation, step back and think. Good luck.

Possibly useful information:

$$\mathbf{U}(t) = e^{-i\mathbf{H}t/\hbar} \text{ satisfies } i\hbar\partial_t\mathbf{U} = [\mathbf{H}, \mathbf{U}].$$

$$\boldsymbol{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\uparrow_{\hat{n}}\rangle = e^{-i\varphi/2} \cos \frac{\theta}{2} |\uparrow_{\hat{z}}\rangle + e^{+i\varphi/2} \sin \frac{\theta}{2} |\downarrow_{\hat{z}}\rangle \quad \text{satisfies} \quad \vec{\boldsymbol{\sigma}} \cdot \hat{n} |\uparrow_{\hat{n}}\rangle = |\uparrow_{\hat{n}}\rangle$$

$$e^{-i\alpha\hat{n}\cdot\vec{\boldsymbol{\sigma}}} = \mathbb{1} \cos \alpha - i\hat{n} \cdot \vec{\boldsymbol{\sigma}} \sin \alpha.$$

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}^{-1} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\mathbf{H}_{\text{SHO}} = \hbar\omega \left(\mathbf{a}^\dagger \mathbf{a} + \frac{1}{2} \right) = \frac{\mathbf{p}^2}{2m} + \frac{1}{2} m\omega^2 \mathbf{q}^2$$

$$\mathbf{q} = \sqrt{\frac{\hbar}{2m\omega}} (\mathbf{a} + \mathbf{a}^\dagger), \quad \mathbf{p} = \frac{1}{i} \sqrt{\frac{\hbar m\omega}{2}} (\mathbf{a} - \mathbf{a}^\dagger); \quad [\mathbf{q}, \mathbf{p}] = i\hbar \implies [\mathbf{a}, \mathbf{a}^\dagger] = 1.$$