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Quantum Mechanics C (Physics 130C) Winter 2015
Worksheet 5

Announcements

- The 130C web site is:

  http://physics.ucsd.edu/~mcgreevy/w15/ .

  Please check it regularly! It contains relevant course information!

- Office hours are 2:30-3:30PM but I'm available upon request. Grab your homework!

Problems

1. **Try it out!**

   Consider the following operators: $\rho_a = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} \end{pmatrix}$ $\rho_b = \begin{pmatrix} \frac{1}{7} & -\frac{2}{7} \\ -\frac{2}{7} & \frac{4}{7} \end{pmatrix}$ $\rho_c = \begin{pmatrix} \frac{1}{2} & \mathbf{i}\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{2} \end{pmatrix}$

   Explain why each can't represent a physical state.

   Consider the following operators: $\rho_1 = \begin{pmatrix} \frac{1}{4} & \frac{\mathbf{i}\sqrt{3}}{4} \\ \frac{-\mathbf{i}\sqrt{3}}{4} & \frac{3}{4} \end{pmatrix}$ $\rho_2 = \begin{pmatrix} \frac{2}{7} & 0 \\ 0 & \frac{5}{7} \end{pmatrix}$ $\rho_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

   Which of these can possibly represent a pure state?

   Hint: If ρ is pure it must be a projector onto some state.

2. **Tracing**

   Recall the trace of an operator $\text{Tr}\,[A] = \sum_m \langle m|A|m\rangle$ for the some basis set $\{|m\rangle\}$

   Prove that this definition is independent of basis.

   Prove the cycle property: $\text{Tr}\,[ABC] = \text{Tr}\,[BCA] = \text{Tr}\,[CAB]$

3. **Purity**

   Define again the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{\mathbf{i}\phi}|1\rangle)$ as well as $\rho_\beta = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$

   (a) Write the density matrix ρ_ψ associated with $|\psi\rangle$

   (b) Show that for both the states $\langle Z\rangle = 0$

   (c) Define the *purity* of a state as $\text{Tr}\,[\rho^2]$. Prove that this equal to 1 if ρ is pure. Compute it for both ρ_ψ and ρ_β.

   (d) Compute $\langle X\rangle$ with the above density matrices.