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Quantum Mechanics C (Physics 130C) Winter 2015
Worksheet 6

Announcements

- The 130C web site is:

  http://physics.ucsd.edu/~mcgreevy/w15/ .

  Please check it regularly! It contains relevant course information!

- Office hours are 2:30-3:30PM but I'm available upon request. Grab your homework!

Problems

1. **C-NOT Evil**

   Recall from worksheet 4 we defined the operator

   $$U_{CNOT} \equiv |0_A\rangle\langle 0_A| \otimes \mathbb{1}_B + |1_A\rangle\langle 1_A| \otimes \sigma^x_B \tag{1}$$

   From the previous homework we know that a unitary operator which factorizes $U = U_A \otimes U_B$ cannot generate entanglement. However notice that 1 *does not* factorize. Therefore we might be able to create entanglement with it. Let's do it.

   Construct an input state $|\text{In}\rangle$ which has no entanglement but where $|\text{Out}\rangle \equiv U_{CNOT}|\text{In}\rangle$ is *maximally* entangled.

2. **With All Your ρ's Combined**

   Quantum states are said to form a convex set with extremal points corresponding to pure states. What does that mean?

   (a) Prove that if ρ_1 and ρ_2 are density matrices then $\sigma \equiv q\rho_1 + (1-q)\rho_2$ is as well where q is a probability.

   This implies that for every pair of points in the set the straight line that connects them is also contained in the object. For example a disk is a convex set but an annulus is not.

   (b) A point is said to be extremal if it can't be written as a (non-trivial) linear combination of other states.

   Prove the claim that pure states are extremal in this set.

(c) Define $S(\rho) = -\mathrm{Tr}\,(\rho \log \rho)$ to be the von Neumann entropy. I'd like to prove something about $S(\sigma)$. Assume that ρ_1 and ρ_2 have mutually orthogonal support.[1] Show that in this case $S(\sigma) = qS(\rho_1) + (1-q)S(\rho_2) - (q \log q + (1-q)\log(1-q))$

[1] Eigenvectors of ρ_1 with non-zero eigenvalue are all orthogonal to the similarly defined eigenvectors of ρ_2