University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215B QFT Winter 2017
Assignment 2

Due 11am Tuesday, January 24, 2017

1. **Brain-warmer.**

   Show that we did the right thing in the numerator of the electron self-energy: use the Clifford algebra to show that

$$\gamma^\mu \left(x\not{p} + m_0 \right) \gamma_\mu = -2x\not{p} + 4m_0 .$$

2. **Bosons have worse UV behavior than fermions.**

   Consider the Yukawa theory

$$S[\phi, \psi] = -\int d^D x \left(\frac{1}{2}\phi \left(\Box + m_\phi \right) \phi + \bar{\psi} \left(-\not{\partial} + m_\psi \right) \psi + y\phi\bar{\psi}\psi + \frac{g}{4!}\phi^4 \right) + \text{counterterms}.$$

   (a) Show that the superficial degree of divergence for a diagram $\mathcal{A}$ with B_E external scalars and F_E external fermions is

$$D_{\mathcal{A}} = D + (D-4) \left(V_g + \frac{1}{2} V_y \right) + B_E \left(\frac{2-D}{2} \right) + F_E \left(\frac{1-D}{2} \right) \qquad (1)$$

   where V_g and v_y are the number of ϕ^4 and $\phi\bar{\psi}\psi$ vertices respectively.

   All the discussion below is about one loop diagrams.

   (b) Draw the diagrams contributing to the self energy of both the scalar and the spinor in the Yukawa theory.

   (c) Find the superficial degree of divergence for the scalar self-energy amplitude and the spinor self-energy amplitude.

   (d) In the case of $D = 3 + 1$ spacetime dimensions, show that the spinor self-energy is actually only logarithmically divergent. (This type of thing is one reason for the adjective 'superficial'.)

   Hint: the amplitude can be parametrized as follows: if the external momentum is p^μ, it is

$$\mathcal{M}(p) = A(p^2)\not{p} + B(p^2).$$

   Show that $B(p^2$ vanishes when $m_\psi = 0$.

3. **Dimension-dependence of dimensions of couplings.**

(a) In what number of space dimensions does a four-fermion interaction such as $G\bar{\psi}\psi\bar{\psi}\psi$ have a chance to be renormalizable? Assume Lorentz invariance.

[optional] Generalize the formula (1) for $D_{\mathcal{A}}$ to include a number V_G of four-fermion vertices.

(b) If we violate Lorentz invariance the story changes. Consider a non-relativistic theory with kinetic terms of the form $\int dtd^dx \left(\psi^\dagger \left(\mathbf{i}\partial_t - \nabla^2\right)\psi\right)$. For what number of space dimensions might the four-fermion coupling be renormalizable?

(c) In the previous example, the scale transformation preserving the kinetic terms acted by $t \to \lambda^2 t, x \to \lambda x$. More generally, the relative scaling of space and time is called the *dynamical exponent* z ($z = 2$ in the previous example). Suppose that the kinetic terms are first order in time and quadratic in the fields. Ignoring difficulties of writing local quadratic spatial kinetic terms, what is the relationship between d and z which gives scale invariant quartic interactions? What if the kinetic terms are second order in time (as for scalar fields)?