University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215B QFT Winter 2017
Assignment 4

Due 11am Tuesday, February 7, 2017

1. **Soft gravitons?** Photons are massless, and this means that the cross sections we measure actually include soft ones that we don't detect. If we don't include them we get IR-divergent nonsense.

   Gravitons are also massless. Why don't we need to worry about them in the same way? Here we'll sketch some hints for how to think about this question. Warning: I just wrote these problems and haven't debugged them carefully yet. If something seems questionable to you, please ask me about it.

   (a) Consider the action
   $$S_0[h_{\mu\nu}] = \int d^4x \frac{1}{2} h_{\mu\nu} \Box h^{\mu\nu}.$$
   This is a kinetic term for (too many polarizations of a) two-index symmetric-tensor field $h_{\mu\nu} = h_{\nu\mu}$ (which we'll think of as a small fluctuation of the metric about flat space: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and this is where the coupling below comes from). Like with the photon, we'll rely on the couplings to matter to keep unphysical polarizations from being made. Write the propagator. We still raise and lower indices with $\eta_{\mu\nu}$.

   (b) Couple the graviton to the electron field via
   $$S_G = \int d^4x \; G h^{\mu\nu} T_{\mu\nu}$$
   $$T_{\mu\nu} \equiv \bar{\psi} \left(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu \right) \psi. \tag{1}$$
   What are the engineering dimensions of the coupling constant G? What is the new Feynman rule?

   (c) Draw a (tree level) Feynman diagram which describes the creation of gravitational radiation from an electron as a result of its acceleration from the absorption of a photon ($e\gamma \to eh$). Evaluate it if you dare. Estimate or calculate the cross section (hint: use dimensional analysis).

   (d) Now the main event: study the one-loop diagram by which the graviton corrects the QED vertex. Is it IR divergent? If not, why not?

(e) If you get stuck on the previous part, replace the graviton field by a massless scalar $\pi(x)$. Compare the case of derivative coupling $\lambda\partial_\mu\pi\bar{\psi}\gamma^\mu\psi$ with the more direct Yukawa coupling $y\pi\bar{\psi}\psi$.

(f) Quite a bit about the form of the coupling of gravity to matter is determined by the demand of coordinate invariance. This plays a role like gauge invariance in QED. Acting on the small fluctuation, the transformation is

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \partial_\mu\lambda_\nu(x) + \partial_\nu\lambda_\mu(x).$$

What condition does the invariance under this (infinitesimal) transformation impose on the object $T_{\mu\nu}$ appearing in (1).