University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215B QFT Winter 2019
Assignment 4

Due 12:30pm Wednesday, February 6, 2019

1. **Brain-warmer.** Verify the form of the propagator for the massive vector field by plugging in the mode expansion and using the completeness relation for the polarization vectors.

2. **Scalar particle scattering cross sections.**

   Let's consider again the example of a complex scalar field Φ interacting with a real scalar field ϕ with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^\star \partial^\mu \Phi - \frac{1}{2}m^2 \Phi^\star \Phi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}M^2\phi^2 + \mathcal{L}_I \tag{1}$$

$$\text{with} \quad \mathcal{L}_I = -g\Phi^\star\Phi\phi.$$

   We can call the Φ particles are 'snucleons,' since they are scalar analogs of nucleons.

   What is the leading-order differential cross-section $\frac{d\sigma}{d\Omega}$ for $2 \to 2$ snucleon-snucleon scattering in $d = 3$ space dimensions in the center-of-mass frame? Draw the relevant Feynman diagram(s).

   What is the total cross section in the limit that the snucleons are massless?

3. **Decay of a scalar particle.**.

   Consider the following Lagrangian, involving two real scalar fields Φ and ϕ:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi \partial^\mu \Phi - M^2\Phi^2 + \partial_\mu\phi\partial^\mu\phi - m^2\phi^2\right) - \mu\Phi\phi^2.$$

   The last term is an interaction that allows a Φ particle to decay into two ϕs, if the kinematics allow it. Calculate the lifetime of the Φ particle to lowest order in μ. Draw the relevant Feynman diagram(s). What is the condition on the masses for a finite lifetime?

4. **Meson scattering.**

   Now consider the Yukawa theory with fermions, with

$$\mathcal{L} = \bar{\Psi}\left(\mathbf{i}\partial\!\!\!/ - m\right)\Psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 + \mathcal{L}_{\text{int}}$$

   and $\mathcal{L}_{\text{int}} = g\bar{\Psi}\Psi\phi$.

(a) Draw the Feynman diagram which gives the leading contribution to the process $\phi\phi \to \phi\phi$.

(b) Derive the correct sign of the amplitude by considering the relevant matrix elements of powers of the interaction hamiltonian. Compare with the Feynman rules for fermions stated in lecture.

(c) Evaluate the diagram in terms of a spinor trace and a momentum integral. Do not do the momentum integral. Suppose that the integral is cutoff at large k by some cutoff Λ. Estimate the dependence on Λ.

5. **Ward identity in scalar QED.** We noted in lecture that scalar QED is different from the usual spinor QED in that the coupling to the gauge field is not just $j^\mu A_\mu$ where j^μ is a current independent of A. In this problem we'll see how this changes the proof of the Ward identity.

(a) Consider a Green's function in scalar QED of the form $G \equiv \langle 0|\mathcal{T}\mathcal{O}_1 \cdots \mathcal{O}_n|0\rangle$ where $\mathcal{O}_i \equiv \mathcal{O}_i(x_i)$ has charge Q_i under the transformation

$$\Phi(x) \to e^{\mathbf{i}\alpha(x)}\Phi(x), A_\mu \to A_\mu. \tag{2}$$

The phrase "$\mathcal{O}$ has charge Q" means $\mathcal{O}(x) \mapsto e^{\mathbf{i}Q\alpha(x)}\mathcal{O}(x)$. (Notice that (2) is *not* the gauge transformation, since A does not transform.) Derive the Schwinger-Dyson equation which follows from demanding that the path integral is invariant under the change of variables (2) to first order in α. (We are supposed to call the result a Ward-Takahashi identity.)

(b) Consider an amplitude in scalar QED with an external photon of polarization ϵ: $\mathcal{M} = \epsilon^\mu \mathcal{M}_\mu$. Using the LSZ reduction formula, show the result of the previous part implies the Ward identity $p^\mu \mathcal{M}_\mu = 0$. Conclude that the longitudinal photons decouple in scalar QED as well.