University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 239/139 Fall 2019
Assignment 11 ("Why?")

Due never

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1. **Simple stabilizer codes.**

   (a) Consider the Hamiltonian on two qbits

   $$-H = X_1 X_2 + Z_1 Z_2 .$$

   Show that the terms commute and that the groundstate is

   $$\frac{|00\rangle + |11\rangle}{\sqrt{2}} .$$

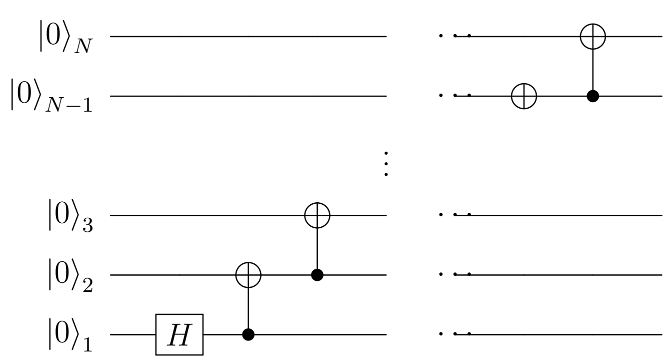
   (b) Consider the (non-local) Hamiltonian on N qbits

   $$H_{\text{GHZ}} = -X_1 \cdots X_N - \sum_{i=1}^{N-1} Z_i Z_{i+1} . \qquad (1)$$

   Show that all the terms commute. Show that the groundstate is (the GHZ state)

   $$\frac{|00...0\rangle + |11...1\rangle}{\sqrt{2}} .$$

   (c) Show that the following circuit U produces the GHZ state from the product state $|0\rangle^{\otimes N}$.

   (d) What state does U produce from $|1\rangle_1 \otimes |0\rangle^{\otimes N-1}$?

(e) Find the result of feeding the Hamiltonian $-\sum_i Z_i$ (whose groundstate is the product state $|0\rangle^{\otimes N}$) through the circuit, *i.e.* what is

$$U\left(-\sum_i Z_i\right)U^\dagger \quad ?$$

Hint: use the rules for the action of CX by conjugation given in lecture.

2. **Algebraic condition for stabilizer code.** We can represent a Hamiltonian on q qbits, where each term is a product of Xs and Zs, by a $2q \times T$ matrix σ, where T is the number of terms in the hamiltonian. (This is the transpose of the object I wrote in lecture.) Each column represents a term in the Hamiltonian. The top q rows indicate where the Zs are and the bottom q rows indicate where the Xs are. Think of it as a map from the set of stabilizers (terms in H) to the set of Pauli operators.

   For example, the matrix for the example in problem 1a is

$$\sigma_{1a} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \quad .$$

   Convince yourself that the condition for all the terms to commute is that

$$\sigma^t \lambda \sigma = 0 \text{ mod } 2$$

   where

$$\lambda \equiv \begin{pmatrix} 0 & 1\!\!1_{q\times q} \\ 1\!\!1_{q\times q} & 0 \end{pmatrix} .$$

   Check that this is the case for the examples above.

   For a beautiful elaboration of this machinery which incorporates translation invariance, see Haah's thesis.