University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 220 Symmetries Fall 2020
Assignment 7

Due 12:30pm Monday, November 23, 2020

Thanks for following the submission guidelines on hw 01. Please ask me by email if you have any trouble.

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1. **Brain-warmers.**

   (a) Show that the *adjoint* representation matrices

   $$\left(T^A\right)_{BC} \equiv -\mathbf{i} f_{ABC}$$

   furnish a dim G-dimensional representation of the Lie algebra

   $$[T^A, T^B] = \mathbf{i} f_{ABC} T^C \quad .$$

   Hint: commutators satisfy the Jacobi identity

   $$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0.$$

   (b) Show that if $(T_A)_{ij}$ are generators of a Lie algebra in some unitary representation R, then so are $-(T_A)^\star_{ij}$. Convince yourselves that these are the generators of the complex conjugate representation $\bar{R}$.

2. $\mathsf{so}(4)$**.**

   Show that $\mathsf{so}(4) = \mathsf{so}(3) \times \mathsf{so}(3)$.

3. **The rest of the Lie algebra in Cartan-Weyl form.**

   (a) Use the Jacobi identity to show that $|[E_\alpha, E_\beta]\rangle$ has weight $\alpha + \beta$, and hence $[E_\alpha, E_\beta] \propto E_{\alpha+\beta}$ for some constant N.

   (b) Can you conclude from this that if α is a root, 2α is not a root?