

Physics 220 Symmetries Fall 2020 Assignment 8

Due 12:30pm Monday, November 30, 2020

Thanks for following the submission guidelines on hw 01. Please ask me by email if you have any trouble.

1. Prove that any generator in any representation of a compact, semisimple Lie algebra is traceless. (This fact has important implications for grand unification of forces.)
2. **Some subgroups of $SU(3)$.**
 - (a) Consider the subalgebra of $\mathfrak{su}(3)$ generated by T_1, T_2, T_3 (where $T_A \equiv \lambda_A/2$ and λ_A are the Gell-Mann matrices). What algebra is it? How does the $\mathbf{3}$ of $SU(3)$ decompose into representations of this algebra? What about the adjoint of $SU(3)$?
 - (b) Consider the subalgebra of $\mathfrak{su}(3)$ generated by T_2, T_5, T_7 . What algebra is it? How does the $\mathbf{3}$ of $SU(3)$ decompose into representations of this algebra? What about the adjoint of $SU(3)$?

Hint: the adjoint of $SU(3)$ is $\mathbf{3} \otimes \bar{\mathbf{3}}$ minus the singlet. So if you know how the $\mathbf{3}$ transforms under a subgroup, you can figure out how the adjoint transforms, too.