Loop Group, Ersatz Fermi Liquid and Strange Metal

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Loop group: mathematical definition

Free loop group: the group of continuous maps from a manifold M to a topological group G, $L\,G = \{\gamma : M \to G \,|\, \gamma \in C(M, G)\}$.

For example, $M = S^1$, then for $\theta \in S^1$, $\gamma(\theta) \in G$. Multiplication in $L\,G$ is defined as $(\gamma_1 \cdot \gamma_2)(\theta) = \gamma_1(\theta)\,\gamma_2(\theta)$. Inverse is $\gamma^{-1}(\theta) = \gamma(\theta)^{-1}$. Identity $\gamma(\theta) = e \in G$.

A loop group is any subgroup of $L\,G$.

Fermi liquids: new perspective from loop group

LU(1): Emergent symmetry at the IR fixed point

We will be interested in generic metallic systems with microscopic lattice translation symmetry and charge conservation, i.e. the UV symmetry group is $G_{\text{UV}} = U(1)\times\mathbb{Z}^d$ with d being the spatial dimension of the system. At the IR Fermi liquid (FL) fixed point, we expect the symmetry group to be enlarged, which can be thought of as the G_{UV} being embedded into G_{IR} through some map φ during the RG flow process.

The map $\varphi : G_{\text{UV}} \to G_{\text{IR}}$ is usually difficult if not impossible to find. Here we can make use of some well-known properties of FL to identify the emergent symmetry group G_{IR}. Consider FL system in $d = 2$ without pairing instability, the BCS diagram will be irrelevant without attractive interactions, hence the only relevant interaction is forward scattering, which in Landau quasiparticle picture is $\sum_{k,k'} F_{k,k'}\, n_k\, n_{k'} = \sum_k \left(F_{k,k'}\, n_{k'}\right) n_k = \sum_k \delta\epsilon_k\, n_k$. That is, the forward scattering does not change the particle number at each k-point, but only renormalizes the dispersion in $\sum_k \epsilon_k\, n_k$. Therefore, particle number at each point on the Fermi surface (FS) is separately conserved, giving rise to an emergent symmetry group that is much larger than G_{UV}.

To be mathematically more precise about this G_{IR}, we consider a *closed* FS in 2d parametrized by $\theta \in [0, 2\pi)$. At a particular θ value, if the corresponding particle number N_θ is *integer-valued*, then we can associate a U(1) symmetry to this point, with the symmetry operator given by $e^{-i f_\theta N_\theta}$. For a FS with many such discrete θ points, a generic symmetry operator would be $e^{-i \sum_\theta f_\theta N_\theta}$. It then seems that the total emergent symmetry group would be many copies of U(1). However, there is a caveat. Quantum mechanically, the occupation numbers at each FS point are given by the distribution of wavefunction amplitudes. Therefore, the condition that N_θ being integer-valued at each θ value is not satisfied in general. Instead, we really should work with generic particle number distribution $n(\theta)$ around the FS, and the symmetry operator now takes the form

$$e^{-i \int d\theta\, f(\theta)\, n(\theta)}.$$

In this description, each symmetry operator corresponds to a function $f(\theta)$. We further require $f(\theta)$ to be a smooth function of θ in this continuum limit. Then we have the following maps:

$$\theta \in S^1 \to f(\theta) \in C^\infty \to e^{-i \int d\theta\, f(\theta)\, n(\theta)} \in U(1),$$

i.e. the smooth function f maps S^1 to $U(1)$. The group of all such functions exactly forms the loop group LU(1) defined above. Therefore, we claim $G_{\text{IR}} = \text{LU}(1)$.

Furthermore, we see that an overall shift of $f(\theta)$ by 2π gives an extra term $e^{-i 2\pi N_{\text{total}}} = 1$. This is the only U(1) subgroup contained in $G_{\text{IR}} = \text{LU}(1)$, which gives the overall conservation of integer particle number in the system. Intuitively, LU(1) is much bigger than $U(1)$ because other than the $U(1)$ phase factor we have the freedom to choose the smooth function $f(\theta)$ among the infinitely many possibilities.

t' Hooft anomaly of LU(1)

In order to look at the t' Hooft anomaly of LU(1), we need to identify the gauge field associated with this group. Starting from the more familiar $U(1)$ gauge field A_μ, we can define an $A_\mu(\theta)$ for each point on the FS with gauge transformation given by $A_\mu(\theta) \to A_\mu(\theta) + \partial_\mu \lambda(\theta)$, again point-wise. Physically, A_μ gives rise to Berry phase when charged particles move around in real space. However, it is well-known that moving around in momentum space also produces Berry phase. Therefore, to complete the picture, we can extend the gauge field by one more dimension to be $A_\mu = (A_0, A_x, A_y, A_\theta)$, which is now defined on the spacetime manifold $(\mathbb{R}^2 \times S^1) \times \mathbb{R}^1$, where $(x, y) \in \mathbb{R}^2$, $\theta \in S^1$ and $t \in \mathbb{R}^1$. From this perspective, the LU(1) gauge field on spacetime manifold M is equivalent to $U(1)$ gauge field on $M \times S^1$.

The t' Hooft anomaly can be calculated by putting the system on the boundary of a bulk SPT phase. The bulk topological term from the background gauge field will give the anomaly. We put the original $(2+1)$d system on the boundary $\partial \mathcal{M}$ of a $(3+1)$d system $\mathcal{M}$. Then the LU(1) gauge field configuration will be equivalent to a $U(1)$ problem defined on $(4+1)$d bulk $\mathcal{M} \times S^1$ with $(3+1)$d boundary $\partial \mathcal{M} \times S^1$. Then the anomaly is given by the Chern-Simons (CS) term in the 5d bulk:

$$S[A] = \frac{m}{24\pi^2} \int_{\mathcal{M} \times S^1} A \wedge dA \wedge dA, \; m \in \mathbb{Z}.$$

Kac-Moody algebra: consequence of the t' Hooft anomaly

Continue from the CS term derived above, we can calculate the current non-conservation on the boundary to be

$$\partial_\mu j^\mu = \partial_\mu \left(\frac{\delta S}{\delta A_\mu} \right) = \frac{m}{8\pi^2} \epsilon^{\lambda\sigma\tau\kappa} \partial_\lambda A_\sigma \partial_\tau A_\kappa.$$

Without loss of essential physics, we take $A_{x,y} = A_{x,y}(x, y)$, $A_{t,\theta} = A_{t,\theta}(t, \theta)$. Then let's see what happens if we compactify along the x, y directions, so hopefully the physic along the θ direction (i.e. along the FS) will be clearer. Integrating the anomaly equation over x and y:

$$\text{LHS} = \int dxdy \left(\partial_t j^t + \partial_x j^x + \partial_y j^y + \partial_\theta j^\theta \right) = \int dxdy \left(\partial_t j^t + \partial_\theta j^\theta \right) = \int dxdy\, \partial_\mu j^\mu \equiv \partial_\mu \tilde{j}^\mu,$$

where the index $\mu = t, \theta$, and we have used the fact that the current vanishes at infinity.

$$\text{RHS} = \frac{m}{8\pi^2} \int dxdy\, \epsilon^{\lambda\sigma\tau\kappa} \partial_\lambda A_\sigma \partial_\tau A_\kappa = \frac{m}{8\pi^2} \int dxdy\, 2 \left(\partial_t A_\theta \partial_x A_y - \partial_t A_\theta \partial_y A_x - \partial_\theta A_t \partial_x A_y - \partial_\theta A_t \partial_y A_x \right)$$

$$= \frac{m}{4\pi^2} \int dxdy \left(\partial_t A_\theta - \partial_\theta A_t \right) \left(\partial_x A_y - \partial_y A_x \right) = \frac{m}{2\pi} \left(\partial_t A_\theta - \partial_\theta A_t \right) \frac{1}{2\pi} \int dxdy \left(\partial_x A_y - \partial_y A_x \right).$$

If we further assume there is one flux quantum in the $x\,y$-plane, then the compactified anomaly equation becomes

$$\partial_\mu \tilde{j}^\mu = \frac{m}{2\pi}(\partial_t A_\theta - \partial_\theta A_t).$$

The LU(1) anomaly can also be manifested through the central extension of its symmetry algebra, the $n(\theta)$. We take

$$[n(\theta), n(\theta')] = c(\theta, \theta') \in \mathbb{C}.$$

In order to determine $c(\theta, \theta')$, which should be Hamiltonian-independent, we choose a test Hamiltonian $H = \int d\theta\, A_t(\theta)\, n(\theta)$, where A_t can be arbitrary smooth function of θ. Then we have

$$\tilde{j}^\theta = \frac{\delta H}{\delta A_\theta} = 0, \ \tilde{j}^t = \frac{\delta H}{\delta A_t} = n(\theta).$$

If we further take the A_θ in the anomaly equation to be t-independent, then we have $\partial_t \tilde{j}^t = -\frac{m}{2\pi}\partial_\theta A_t$. Therefore, we have $\frac{d}{dt} n(\theta) = -\frac{m}{2\pi}\partial_\theta A_t$. On the other hand, from the Heisenberg equation of time evolution, we have $\frac{d}{dt} n(\theta) = i[H, n(\theta)] = i\int d\theta'\, A_t(\theta')[n(\theta'), n(\theta)] = i\int d\theta'\, A_t(\theta')\, c(\theta', \theta)$. Matching this with the previous expression, we eventually arrive at the Kac-Moody algebra

$$[n(\theta), n(\theta')] = -i\frac{m}{2\pi}\delta'(\theta - \theta').$$

Luttinger theorem: from LU(1) and its t' Hooft anomaly

FL satisfies Luttinger theorem. In this section, we will see how this theorem can be proved non-perturbatively using the LU(1) symmetry and its t' Hooft anomaly. In fact what we will be using is the Kac-Moody algebra, but it's really just a consequence of the t' Hooft anomaly.

Firstly consider the many-body lattice translations in the UV: T_α, $\alpha = x, y$. Then we thread a flux quantum uniformly through the whole 2d lattice (same conditioned used when proving the Kac-Moody algebra in the previous section). Then given the filling ν, the translation operators satisfy the following magnetic algebra

$$T_x T_y T_x^{-1} T_y^{-1} = e^{-i 2\pi\nu}.$$

In the IR, the $\vec{k}(\theta)$ points on the FS can be treated as lattice momentum, then the symmetry operators $e^{-i\int d\theta\, k_\alpha(\theta)\, n(\theta)}$ are just translations and can be identified with the lattice translations.

$$T_\alpha \sim e^{-i\int d\theta\, k_\alpha(\theta)\, n(\theta)}.$$

Then, using the BCH formula together with the Kac-Moody algebra we have

$$T_x T_y = e^{-i\int d\theta\, k_x(\theta)\, n(\theta)} e^{-i\int d\theta\, k_y(\theta)\, n(\theta)} = e^{-i\int d\theta\, k_x(\theta)\, n(\theta) - i\int d\theta\, k_y(\theta)\, n(\theta) - \frac{1}{2}\int d\theta d\theta'\, k_x(\theta)\, k_y(\theta')[n(\theta), n(\theta')]}$$

$$= e^{-i\int d\theta\, k_x(\theta)\, n(\theta) - i\int d\theta\, k_y(\theta)\, n(\theta) + i\frac{m}{4\pi}\int d\theta d\theta'\, k_x(\theta)\, k_y(\theta')\,\delta'(\theta-\theta')} = e^{-i\int d\theta\, k_x(\theta)\, n(\theta) - i\int d\theta\, k_y(\theta)\, n(\theta) - i\frac{m}{4\pi}\int d\theta\, k_x(\theta)\frac{d}{d\theta}k_y(\theta)}$$

$$= e^{-i\int d\theta\, k_x(\theta)\, n(\theta) - i\int d\theta\, k_y(\theta)\, n(\theta)}\, e^{-i\frac{m}{4\pi}V_{FS}},$$

$$T_y T_x = e^{-i\int d\theta\, k_y(\theta)\, n(\theta)} e^{-i\int d\theta\, k_x(\theta)\, n(\theta)} = e^{-i\int d\theta\, k_y(\theta)\, n(\theta) - i\int d\theta\, k_x(\theta)\, n(\theta)}\, e^{i\frac{m}{4\pi}V_{FS}}$$

$$= T_x T_y\, e^{i\frac{m}{2\pi}V_{FS}}.$$

Therefore, we have $T_x T_y T_x^{-1} T_y^{-1} = e^{-i\frac{m}{2\pi}V_{FS}}$. Comparing with $T_x T_y T_x^{-1} T_y^{-1} = e^{-i 2\pi\nu}$, we have

$$\nu = m\frac{V_{FS}}{(2\pi)^2} \bmod 1,$$

which is the Luttinger theorem of FL if we set $m = 1$. The value of m itself cannot be fixed by the arguments provided so far, but it can be shown semi-classically to be 1 for FL, presented in the original paper.

Electric response

In a uniform electric field, say in x-direction, the lattice momenta will be shifted according to $k_x \to k_x + q\,E\,t$. As a consequence, the densities $n(\theta)$ will no longer be conserved at each point on FS. For the small segment on $[\theta, \theta + \mathrm{d}\theta]$, the new charge density is given by $\frac{d\,(k_x + q\,E\,t)\,d\,k_y}{(2\,\pi/L_x)\,(2\,\pi/L_y)} = \frac{V}{(2\,\pi)^2}\left(d\,k_x\,d\,k_y + q\,E\,d\,t\,d\,k_y\right) = (n(\theta) + d\,n(\theta))\,\mathrm{d}\theta$. Therefore, we have the change in density given by

$$\frac{d}{\mathrm{dt}}\,n(\theta) = \frac{q\,V}{(2\,\pi)^2}\,E\,\frac{d\,k_y}{\mathrm{d}\theta}.$$

This equation can also be derived using t' Hooft anomaly, only that the result will contain the level m of the CS term: $\frac{q\,m\,V}{(2\,\pi)^2}\,E\,\frac{d\,k_y}{\mathrm{d}\theta}$.

Beyond FL: Ersatz Fermi liquid with LU(1)

Defining EFL: beyond-Landau quasiparticles

We can in fact go beyond FL using the framework of LU(1) symmetry developed above. We define any compressible IR theory at generic microscopic filling that has LU(1) symmetry as the ersatz FL (EFL). In this more general setting, we can still take the generic symmetry operators to be of the form $e^{-i\int \mathrm{d}\theta\,f(\theta)\,n(\theta)}$. However, in order to include non-FL systems where there are no Landau quasiparticles, the $n(\theta)$ can no longer to assumed to be the conserved Landau quasiparticle density. Instead, we directly start from the symmetry consideration and take $n(\theta)$ to be the conserved charge of LU(1), i.e. some physical quantity that is conserved at each point on the FS.

In this language, a localized (on FS) excitation that carries $n(\theta)$ is given by

$$n(\theta') \sim \delta(\theta' - \theta).$$

The LU(1) symmetry says $\int \mathrm{d}\theta'\,n(\theta') \in \mathbb{Z}$, therefore we can put $n(\theta') = N\,\delta(\theta' - \theta)$, $N \in \mathbb{Z}$. These excitations, termed FS quanta, are the generalization of Landau quasiparticles.

Now let's look for the $U(1)$ subgroup. Microscopic $U(1)$ gives a conserved total electron charge quantum number $Q \in \mathbb{Z}$. Then embedding of the microscopic $U(1)$ into LU(1) says $Q \sim N$. If we put $Q = q\,N$, then the q can be identified with the electric charge carried by each of the FS quantum. This argument doesn't seem to exclude the case where q is fractional. Then it seems that we could have exotic fractional excitations near the FS in a gapless system.

Spinful FL: not quite LU(2)

If we include the spin degree of freedom of the electrons, then in the UV we will also have the SU(2) spin rotation symmetry. In total, we have $U(1) \times \mathrm{SU}(2)$. If we further mod out the $\mathbb{Z}_2$ of spin index interchange, then we eventually have $\frac{U(1) \times \mathrm{SU}(2)}{\mathbb{Z}_2} = U(2) = G_{\mathrm{UV}}$.

In the IR, the quantum number $n(\theta)$ is still conserved at each point on the FS, which again gives us LU(1). However, locally on the FS spin is not conserved due to Landau interactions in the spin channel, i.e. $n_\uparrow(\theta)$ and $n_\downarrow(\theta)$ are not separately conserved even though $n(\theta) = n_\uparrow(\theta) + n_\downarrow(\theta)$ is. Therefore, we do not have a naive generalization from LU(1) to LU(2).

Now consider the group LU(1)$\times U(2)$, this group is apparently bigger than G_{IR} because both LU(1) and $U(2)$ contain a $U(1)$ subgroup that means the same thing. The subgroup $U(1)^{(1)} \subset$ LU(1) has been discussed in previous sections and it means total particle number conservation. The subgroup $U(1)^{(2)} \subset U(2)$ also means total number $Q = Q_\uparrow + Q_\downarrow$ is conserved. Therefore, for the IR symmetry group we have

$$G_{\text{IR}} = \text{LU}(1) \times U(2)|_{U(1)^{(1)} \sim U(1)^{(2)}},$$

meaning identifying the two $U(1)$.

Strange metal as ersatz Fermi liquid

Central dogmas of strange metal

For the mysterious strange metal phase, we can list down three assumptions based on available experimental data.

- Assumption 1: the essential physics is clean.

There is ample experimental evidence that shows adding disorder only shifts the ρ-T curve without changing the slope of the T-linear part of the DC resistivity ρ. Therefore, it's reasonable to assume that disorder does not play an essential role in strange metal physics. As a result, we can just focus on systems with lattice translation symmetry.

- Assumption 2: universal scaling of conductivity $\sigma(\omega, T) = T^{-1}\,\Sigma(\omega / T)$.

This scaling behavior is observed in many experiments of cuprates and heavy fermion systems. In the DC limit, $\sigma_{\text{DC}}(T) = \lim_{\omega \to 0} \sigma(\omega, T) = T^{-1}\,\Sigma(0)$. Therefore, $\Sigma(0)$ needs to be a finite quantity in order to have T-linear DC resistivity.

- Assumption 3: compressibility.

This is a natural assumption because we are focusing on metallic phase and the filling factor can be continuously tuned.

Intrinsic resistivity

Intrinsic resistivity is defined to be the non-zero DC resistivity at finite temperature at the RG fixed point, i.e. it's the resistivity that remains when all the irrelevant perturbations vanish. Here we can argue that the universal scaling assumption of the conductivity $\sigma(\omega, T)$ implies that strange metals, as defined above, have non-zero intrinsic resistivity. This fact will be useful later.

Consider the RG flow near the IR fixed point perturbed by irrelevant couplings, the conductivity can be written as a scaling function dependent on the irrelevant couplings u: $\sigma(\omega, T) = F(\omega, T, u)$. Then with s being the scaling parameter, we have

$$F(T, \omega, u) = s^{1+\delta}\, F(s\, T, s\, \omega, u(s)),$$

where δ is some scaling exponent and $u(s) \to 0$ as $s \to \infty$. Now we take some finite reference temperature T_0 and take the RG scale to be $s = T_0 / T$, then we will have
$F(T, \omega, u) = (T_0 / T)^{1+\delta}\, F(T_0, \omega\, T_0 / T, u(T_0 / T))$. At low temperature and low frequency $T \to 0$, $\omega \to 0$ with $\omega / T \to 0$, we have

$$F(T, \omega, u) = T^{-1-\delta}\, T_0^{1+\delta}\, F(T_0, \omega\, T_0 / T, 0) \equiv T^{-1-\delta}\, \Sigma(\omega / T),$$

which, after comparing with the scaling in Assumption 2, says $\delta = 0$. Also, we have $\Sigma(\omega / T \to 0) = T_0 F(T_0, 0, 0)$, which is some finite constant that gives rise to T-linear DC resistivity. Thus, we conclude that strange metals have intrinsic resistivity.

Divergent susceptibility from intrinsic resistivity and LU(1) symmetry

We again focus on the 2d case. The idea here is to make use of the conserved quantities $n(\theta)$ from the LU(1) symmetry to calculate the current $\vec{J}$ in linear response, and then obtain from that an expression of the σ_{DC}. Then combining with the previous conclusion of non-zero intrinsic resistivity, we can arrive at the requirement of divergent susceptibility with respect to the conserved quantities for strange metals described by EFL.

Consider a FS with rotational symmetry, we can represent the conserved charges $n(\theta)$ in angular momentum basis as the following:

$$n_l = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta\, e^{-i l \theta}\, n(\theta).$$

For the new set of conserved charges $\{n_l\}$, we can introduce the conjugate fields $\{\lambda_l\}$ that couples to the charges, just like electric field couples to electric current. Then we have a Gibbs ensemble given by $\rho_{\mathrm{Gibbs}} = \frac{1}{\mathcal{Z}} e^{-\beta(H - \lambda_l n_l)}$, with H being the Hamiltonian of the system and β is inverse temperature. Then we have the following susceptibility matrix between conserved charges in different angular momentum channels:

$$\chi_{l l'} \equiv \chi_{n_l n_{l'}} = \frac{\partial^2}{\partial \lambda_l \partial \lambda_{l'}} \log \mathcal{Z}.$$

This susceptibility matrix basically gives the response of the conserved charges to the applied fields, more precisely $\delta \langle n_l \rangle = \chi_{l l'} \delta\lambda_{l'}$. Quite similarly, it's also possible for the fields $\{\lambda_l\}$ to induce currents in the system through a susceptibility matrix due to some couplings

$$\delta J^i = \chi_{J^i l}\, \delta\lambda_l = \chi_{J^i l} (\chi^{-1})_{l l'}\, \delta \langle n_{l'} \rangle,$$

where we have inverted $\delta \langle n_l \rangle = \chi_{l l'} \delta\lambda_{l'}$ so that we can directly relate the induced current to the conserved charges. It can also be shown that under electric field, we have $\delta \langle n_l \rangle = \chi_{J^i l} E_i$. Therefore, we have $\delta J^i = \chi_{J^i l}\, \delta\lambda_l = \chi_{J^i l} (\chi^{-1})_{l l'}\, \chi_{J^j l'} E_j$, implying:

$$\sigma^{i j} = \frac{1}{V} \chi_{J^i l} (\chi^{-1})_{l l'}\, \chi_{J^j l'}.$$

Recall the previously derived electric response of $n(\theta)$:

$$\left\langle \frac{d}{\mathrm{dt}} n(\theta) \right\rangle_E = \frac{V m q}{4\pi^2} \epsilon^{i j} E_i \frac{d k_j(\theta)}{\mathrm{d}\theta},$$

which says

$$\chi_{J^x n(\theta)} = \frac{V m q}{4\pi^2} \frac{d k_y(\theta)}{\mathrm{d}\theta} = \frac{V m q}{4\pi^2} k_F \cos\theta = \frac{V m q}{8\pi^2} k_F \left(e^{i\theta} + e^{-i\theta}\right).$$

On the other hand, in the angular momentum basis $n(\theta) = \sum_l e^{-i l \theta} n_l$,

$$\sum_l e^{-i l \theta} \left\langle \frac{d}{\mathrm{dt}} n_l \right\rangle_{E_x} = E_x\, \chi_{J^x l} \sum_l e^{-i l \theta},$$

i.e. $\chi_{J^x l} \sum_l e^{-i l \theta} = \chi_{J^x n(\theta)}$. Therefore we have $\chi_{J^x l} = \frac{V m q}{8\pi^2} k_F \left(\delta_{l,1} + \delta_{l,-1}\right)$.

Eventually we have

$$\sigma^{xx} = \frac{1}{V}\chi_{J^x l}(\chi^{-1})_{l l'}\chi_{J^x l'}$$
$$= \frac{V m^2 q^2}{64\pi^4} k_F^2\left[(\chi^{-1})_{1,1} + (\chi^{-1})_{1,-1} + (\chi^{-1})_{-1,1} + (\chi^{-1})_{-1,-1}\right]$$
$$\sim \frac{V\nu^2}{\chi_{1,1}},$$

where ν is the filling factor in the Luttinger theorem proved in previous sections and we have used the fact that $\chi_{l l'}$ is diagonal in systems with rotational symmetry.

The final stretch is to notice that the above linear response calculations are in time domain. We are looking at what happens from $t = -\infty$ to $t = +\infty$ when perturbations are applied at some finite time interval in between. The long time conductivity $\sigma^{xx}(t \to \infty)$ is basically the weight factor of $\delta(\omega)$ in $\sigma^{xx}(\omega)$:

$$\sigma^{xx}(\omega) \sim \frac{V Q^2}{\chi_{1,1}}\delta(\omega) + \text{regular part.}$$

In order to conform to the regularity of the intrinsic DC conductivity at $\omega \to 0$, the only possibility here is to have a divergent $\chi_{1,1}$.

Therefore, strange metals satisfying the three assumptions above and are described by EFL theory should generally exhibit divergence in the susceptibility of emergent conserved quantities of LU(1). This could be a signature of continuous phase transition that can have possible experimental implications, further discussed in the original paper.

Main references:

- D.V. Else, R. Thorngren, T. Senthil “Non-Fermi liquid as ersatz Fermi liquids: general constraints on compressible metals”
- D.V. Else, T. Senthil “Strange metals as ersatz Fermi liquids”