University of California at San Diego – Department of Physics – Prof. John McGreevy

# Physics 215A QFT Fall 2021
# Assignment 8

**Due 11:59pm Thursday, November 18, 2021**

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1. **Brain warmer.** Mandelstam variables are Lorentz-invariant combinations of the kinematic variables in $2 \leftarrow 2$ scattering

$$s = (p_1 + p_2)^2, t = (p_1 - p_3)^2, u = (p_1 - p_4)^2$$

where $p_i^\mu$ are on-shell four-vectors $p_i^2 = m_i^2$ satisfying overall momentum conservation $p_1 + p_2 = p_3 + p_4$. Show that they are not independent but rather satisfy the relation

$$s + t + u = \sum_i m_i^2$$

where $i$ runs over the four external particles.

2. **Particle creation by an external source, continued.**

Consider again the Hamiltonian

$$H = H_0 + \int d^3x \left(-j(t, \vec{x})\phi(x)\right)$$

where $H_0$ is the free Klein-Gordon Hamiltonian, $\phi$ is the Klein-Gordon field, and $j$ is a c-number scalar function.

(a) Compute the probability amplitude for the source to create one particle of momentum $k$. Perform this computation first to $\mathcal{O}(j)$, and then to all orders, using the trick from HW07 to sum the series.

Compute the analogous amplitude for $n$ particles of definite momenta.

(b) Show that the probability of producing $n$ particles (of any momenta) is given by the Poisson distribution,

$$P(n) = \frac{1}{n!}\lambda^n e^{-\lambda}.$$

[Note that for $n > 1$, this requires the measure for the final state phase space, which we'll discuss in lecture on Tuesday, November 16.]

(c) Prove the following facts about the Poisson distribution:

$$\sum_{n=0}^{\infty} P(n) = 1, \quad \langle N \rangle \equiv \sum_{n=0}^{\infty} nP(n) = \lambda,$$

that is, $P(n)$ is a probability distribution, and $\langle N \rangle = \lambda$ as predicted. Compute the fluctuations in the number of particles produced $\langle (N - \langle N \rangle)^2 \rangle$.