

Physics 220 Symmetries Fall 2024 Assignment 1

Due 3:30pm Thursday, October 3, 2024

- Homework will be handed in electronically. Please do not hand in photographs of hand-written work. The preferred option is to typeset your homework. It is easy to do and you need to do it anyway as a practicing scientist. A LaTeX template file with some relevant examples is provided [here](#), and the texfile with the problems below is on the homework page. If you need help getting set up or have any other questions please email me.
- To hand in your homework, please submit a pdf file through the course's Canvas website, under the assignment labelled hw01.

Thanks in advance for following these guidelines. Please ask me by email if you have any trouble.

1. **Brain-warmer.** Consider the group made from the set $\{2, 4, 6, 8\}$ under multiplication modulo 10.
 - (a) Write the multiplication table and check the sudoku rule. What is the identity?
 - (b) Is it equivalent to $\{1, 2, 3, 4\}$ under multiplication modulo 5? If not, why not?
2. **A more economical definition of group.** Show that we can weaken two of the conditions defining a group without changing the outcome, as follows:
 - (2') A left identity $\mathbb{1}$ exists, such that for any element g , $\mathbb{1}g = g$.
 - (3') For any element g , a left inverse f exists: $fg = \mathbb{1}$.Show that these two conditions imply that the left identity is also the right identity and the left inverse is also the right inverse.
3. **The group with three elements.** Write down the multiplication table for the group with three elements; show that it is uniquely fixed by the definition. Is it abelian?
4. **Groups with four elements.** Write down the multiplication table for all groups with four elements. Are they all abelian?