University of California at San Diego – Department of Physics – Prof. John McGreevy

Group Theory (220) Fall 2024

End-of-Term Project

Deadlines: Topic choices are due before Thanksgiving.
Papers are due on Thursday, December 12, 2024.

The end of term project for Physics 220 will be a *short* paper explaining a nugget of truthy goodness about symmetry in physics.

Your goal in deciding what to say should be to try to save the rest of us from having to read the papers. Give some context, say what the crucial point is, say what the implications are.

I plan to post all the papers on the course webpage, so we can all read them. Reading the other papers is part of the assignment.

The paper should be approximately 2 pages in a TeX format for which I'll provide a template. You may include arbitrarily many figures, which need not count toward the page limit. The page limit is not sharp, but keep in mind that I will post all the papers to the course webpage and everyone should read everyone else's paper: you don't want to torture your classmates.

I would prefer a level of detail and technical sophistication comparable to that of my lecture notes. Anything we've covered may be assumed known (though a reference to a specific section of the notes might be helpful). Your paper may contain as much detail as you like, but complicated technical details must be put in a box and labelled, so that the reader may read only the label on the box without losing the narrative thread. Examples of boxes into which you may put details are footnotes, appendices and actual boxes. There is no page limit on appendices.

Please tell me (by email) what topic you plan to study as soon as possible, but not later than Thanksgiving. Below are some topic suggestions, involving wildly varying levels of difficulty. The list is certainly not in any sense exhaustive. Creative topics are encouraged.

Submission instructions:

I will post an assignment on Canvas by which you can submit your paper. It would help me if you name the file in the following format:

`2024F-220-YourLastName-YourFirstName.pdf`

Some topic suggestions:

Anything in blue below is a link to the literature.

Lists of and links to references below are intended as entry points to the literature, and not as complete citations of all good work on the subject. For each paper you should of course always also read all papers which cite it[1][2], as well as all of the papers to which it refers. The order below is not meaningful, though I've tried to group related topics together. I will keep adding to this file.

1. Bootstrap for matrix quantum mechanics. [here] [Suggested and claimed by Yu-Hsueh Chen]

2. Bootstrap for the BFSS matrix model. Around equation (20) is a simple but crucial application of representation theory. [here] [Claimed by Jade Chismar]

3. Bootstrap for representation theory of finite groups. [I don't know a reference for the approach we discussed in lecture, but it seems fun to try it. A different (maybe less fun and less scalable) approach is to search for the fusion coefficients; this would be a special case of Parsa Bonderson's thesis.]

4. (Classical) ferromagnets for general finite groups: put an element g_i of G at each site i of a graph, and consider the energy functional

$$E[g] = \sum_{\langle ij \rangle} \sum_a c_a \chi_a(g_i g_j^{-1}),$$

   where c_a are a collection of coupling constants. For which groups and which c_a is there a finite-temperature phase transition? What is the nature of the coarsening dynamics? [I don't know a reference yet, but come talk to me about it.]

5. FFT for non-abelian groups [Diaconis and Rockmore] Start here. You may also want to think about quantum implementations of this transform.

6. Coset construction of effective field theories for Goldstone bosons. Starting here [Suggested and claimed by Lynn Lin]

7. McKay correspondence and its applications to string theory. [Suggested and claimed by Eric Yang]

[1] Am I exaggerating?

[2] For some of the papers listed below, a relatively complete citation list can be found using Spires: `http://inspirehep.net`.

8. Continuous symmetry versus quantum error correction. [V Albert, ... Salton] here and here. These papers allege that the Knill-Eastin theorem is involved. Explain how. [Claimed by Shuhan Zhang]

9. Affine Lie algebras.

10. Counting invariant operators of given dimension. [start here]

11. Moonshine.

12. Haar-random states and their entanglement properties. [Page]

13. Quantum error correction and large N. [Milekhin, https://arxiv.org/abs/2008.12869] [Claimed by Lingyuan Lyu]

14. There are more symmetric unitaries than can be made by evolution by symmetric Hamiltonians. [here] [Claimed by Yaotian Ji]

15. Kondo effect for other spin symmetry groups [here] [Claimed by Tyler Wannamaker]

16. Group cohomology description of classically-impossible quantum tasks, aka [contextuality] [Claimed by Deepak Aryal]

17. Steady-state bifurcation with icosahedral symmetry [here] [Suggested and claimed by Yi-Chieh Lai]

18. Majorana neutrino and its violation of lepton number conservation [ch 7 of Zee] [Suggested and claimed by Haiwen Xu]

19. Anything else we don't get to in lecture that's on the syllabus or any other topic related to group theory.