University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 239/139 Spring 2018
Assignment 4

Due 12:30pm Monday, April 30, 2018

1. **Error correcting code brain-warmer.**

   (a) For the [7,4] Hamming code discussed in lecture, check that $Ht = 0$ where t is any codeword, and H is the given parity check matrix.

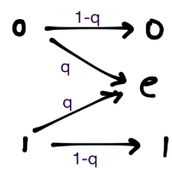
   (b) If you (B) are communicating with someone (A) through channel with $H(A|B) = 1/7$ using this code and you receive the string

   $$r = (0, 1, 0, 0, 0, 0, 1)^t,$$

   what is the most likely intended message string?

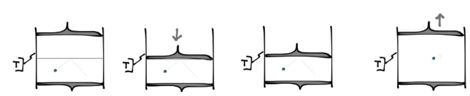
2. **Binary erasure channel.**

   Find the channel capacity of this channel:



3. **Mechanical engineering problem.** [optional]

In lecture I claimed that the expansion of an ideal gas against a piston, as in the figure at right, could be used to lift a weight.



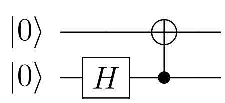
   Design a plausible system of strings and pulleys to make this happen.

4. **Test of Landauer's Principle.**

   Show that copying a known bit (say 0) onto an *unkown* bit by the method described in lecture with the double-well potential costs energy at least $k_B T \ln 2$.

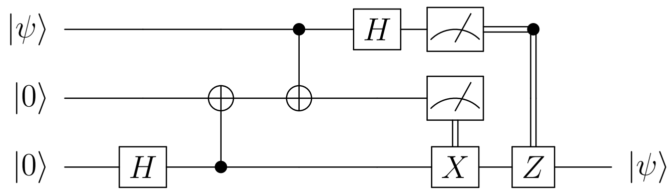
5. **Making a Bell pair from a product state.**

   Find the output of the following quantum circuit:



   Here $H = \frac{1}{\sqrt{2}}(X + Z)$ is a Hadamard gate, and the two-qbit gate is the CX gate as in lecture.

6. **Quantum Teleportation.** Convince yourself that it is possible to transmit an unknown state of a qbit by sending two classical bits to someone with whom you share a Bell pair, using the following circuit:



Time goes from left to right here; you should recognize the first two operations from the previous problem. Imagine that the register on the bottom line is separated in space from the top two after this point. The measurement boxes indicate measurements of Z; the double lines indicate that the outcomes of these measurements $s = 1, 0$ (this is the sending of the two classical bits) determine whether or not (respectively) to act with the indicated gate.

7. **Quantum Dense Coding.** Find a circuit which does the reverse of the previous: by sending an unknown qbit to someone with whom you share a Bell pair, transmit two classical bits. (Hint: basically just reverse everything in the previous problem.)