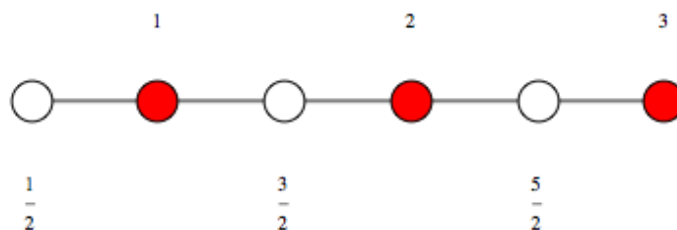


Physics 215C QFT Spring 2019 Assignment 8

Due 12:30pm **Wednesday, May 29, 2019**

1. **Brain-warmer on counting states.**

Consider the gauge theory description of the transverse field Ising model with $N = 3$ sites.



With open boundary conditions, find an explicit expression for the gauge transformation $\{s_j\}$ which eliminates the original \mathbf{X}_j variables (sets them to 1).

With closed boundary conditions, show that you cannot eliminate $\prod_j \mathbf{X}_j$. Find an expression for the gauge choice which eliminates all but one of the spins.

2. **Non-linear sigma models on more general spaces.** [Some knowledge of differential geometry is helpful here.]

In lecture we considered the 2d non-linear sigma model whose target space was a round 2-sphere, motivated by the low-energy physics of antiferromagnets. At weak coupling (large radius of sphere, which means large spin), we saw that the sphere wants to shrink in the IR.

Consider now a 2d non-linear sigma model (NLSM) whose target space is a more general manifold X with Riemannian metric $ds^2 = L^2 g_{ij}(x) dx^i dx^j$. Assume that the space is *big*, in the sense that we will treat the parameter L^{-1} as a small parameter, and *smooth* in the sense that we can Taylor expand around any point.

The NLSM is a field theory whose fields $x^i(\sigma)$ are maps from spacetime (here 2d flat space) to the *target space* X . The simplest action is

$$S[x(\sigma)] = \int d^2\sigma L^2 g_{ij}(x) \partial_{\sigma^\mu} x^i \partial_{\sigma^\nu} x^j \eta^{\mu\nu}$$

where $\eta^{\mu\nu}$ is the flat metric on the 2d spacetime ‘worldsheet’.

$D = 2$ is special because the free scalar field $x(\sigma)$ is dimensionless. As long as g_{ij} is nonsingular, in the limit $L \rightarrow \infty$, the local coordinate field becomes free.

Regard $g_{ij}(x)$ as a coupling *function*. What is the leading beta function (actually beta functional) for this set of couplings?

Hint: use the fact that the answer must be covariant under changes of coordinates on X plus dimensional analysis.

3. Haldane phase from the path integral.

Consider the $D = 1 + 1$ nonlinear sigma model with target space S^2 at $\theta = 2\pi$. Recall that this describes a spin-one antiferromagnetic chain. The θ term is a total derivative in the action, so it can manifest itself when we study the path integral on a spacetime with boundary.

- (a) Put this field theory on the half-line $x > 0$. Suppose that the boundary conditions respect the $\text{SO}(3)$ symmetry, so that the boundary values $\vec{n}(\tau, x = 0)$ are free to fluctuate. By remembering that the θ -term is a total derivative, and considering the strong-coupling (IR) limit, $g \rightarrow \infty$, show that there is a spin- $\frac{1}{2}$ at the boundary. (Hint: Recall the coherent state path integral for a spin- $\frac{1}{2}$.)
- (b) Now cut the path integral open at some fixed euclidean *time* $\tau = 0$. (Consider periodic boundary conditions in space.) Such a path integral computes the groundstate wavefunction, as a function of the boundary values of the fields, $\vec{S}(x, \tau = 0)$. Find the groundstate wavefunctional is $\Psi[\vec{n}(x, \tau = 0)]$ in the strong coupling limit $g \rightarrow \infty$ (where the gap is big).