University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 211C (239) Phases of Quantum Matter, Spring 2021
Assignment 2

Due 12:30pm Wednesday, April 14, 2021

Thanks for following the submission guidelines on hw01. Please ask me by email if you have any trouble.

---

1. $\pi_1(G)$ **is abelian.** [Optional] Complete the proof that for any Lie group (or, if you like, more generally any topological group) $\pi_1(G)$ is abelian by finding a homotopy between $f \star g$ and $g \star f$.

   One way to do it following the hint from lecture is to first show that $f \star g \simeq fg$ (where $(fg)(t) = f(t)g(t)$ is the pointwise multiplication of group elements), and then show $fg \simeq gf$.

2. [This problem is optional] Suppose we have a system with symmetry $G = \mathsf{SU}(2) \times U(1)_Y$ which is broken down to $\mathsf{U}(1)_Q$, where the unbroken subgroup is generated by

   $$Q = pT_3 + rY$$

   for some integers p, r with no common factor. Here $T_3^{(i)}$ means a generator of $\mathsf{SU}(2)$and we normalize T_3 and Y so that their smallest nonzero eigenvalue is one. Show that there are stable codimension-two defects which can disappear in clumps of r.

   Further bonus problem: Can you find an example of a linearly-transforming order parameter that produces this pattern of symmetry-breaking?

3. [This problem is also optional] Show that the Standard Model (SM) has no topologically-stable string or point-defect solutions.

   For purposes of this question[1], the SM can be regarded as an ordered medium that breaks $G = \mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)_Y$ down to $\mathsf{SU}(3) \times \mathsf{U}(1)_Q$, where

   $$Q = T_3 + Y$$

   with the same normalization of generators as in the previous problem.

[1] In the SM, these groups are actually gauge groups, and not symmetries. However, this only affects the energetics of the topological defects.