

University of California at San Diego – Department of Physics – Prof. John McGreevy
Physics 211C (239) Phases of Quantum Matter,
Spring 2021
Assignment 4

Due 12:30pm Monday, May 3, 2021

Thanks for following the submission guidelines on [hw01](#). Please ask me by email if you have any trouble.

Abelian Chern-Simons problems.

1. Under a large U(1) gauge transformation,

$$a \rightarrow a - \mathbf{i}g^{-1}dg$$

find the variation of the U(1) Chern-Simons action on a closed 3-manifold M

$$S_0[a] = \int_M \frac{k}{4\pi} a \wedge da .$$

Conclude that in the absence of *other* interestingness (such as degenerate ground-states not coming from the dynamics of a), the level k must be an **even** integer.

2. For the abelian Chern-Simons theory with gauge group U(1) at level k ,

$$S[a, \mathcal{A}] = \int \left(\frac{k}{4\pi} a \wedge da + \mathcal{A} \wedge \frac{da}{2\pi} \right) .$$

do the (gaussian!) path integral over a to find the effective action for the back-ground field \mathcal{A} . Find the Hall conductivity.

3. Now do it for the general K matrix and general charge vector t^I , with

$$S[a^I, \mathcal{A}] = \int \left(\frac{K_{IJ}}{4\pi} a^I \wedge da^J + \mathcal{A} \wedge t_I \frac{da^I}{2\pi} \right) .$$

4. **Flux attachment.** Now consider

$$S_j[A] = \int \left(\frac{k}{4\pi} a \wedge da + a \wedge \star j \right) .$$

Find the equations of motion. Show that the Chern-Simons term *attaches k units of flux* to the particles: $F_{12} \propto \rho$.

5. **Anyons.** Show using the Bohm-Aharonov effect that the particles whose current density is j^μ have anyonic statistics with exchange angle $\frac{\pi}{k}$ (supposing they were bosons before we coupled them to A).

One way to do this is to consider a configuration of j which describes one particle adiabatically encircling another. Show that its wavefunction acquires a phase $e^{i2\pi/k}$. This is twice the phase obtained by going halfway around, which (when followed by an innocuous translation) would exchange the particles.

6. Describe the statistics of the anyonic quasiparticles in the case with general K matrix.