Last time: ① edge physics from CS action.

eg: CS w/ gauge group G

$$a = \sum_{A=1}^{\dim G} T^A a^A$$

$$S_{CS}[a] = k \int_M \mathrm{tr}\left(a \wedge da + \frac{2}{3} a \wedge a \wedge a\right)$$

Consider $M = \mathbb{R} \times \Sigma$ w/ $\partial \Sigma \neq 0$.

$a_0 = 0$ gauge $\quad 0 = \frac{\delta S}{\delta a^0} \propto f = da + a \wedge a$

$\tilde{d}$ ← spatial deriv

solved by $\quad a = i g^{-1} \tilde{d} g \qquad g_x \in G$

$$\underbrace{S_{CS}[a = i g^{-1} \tilde{d} g] + v \int_{\partial\Sigma \times \mathbb{R}} \mathrm{tr}\, a_x^2}$$

$$= \int_{\partial\Sigma \times \mathbb{R}} \Big[k\, \mathrm{tr}\, g^{-1} \partial_t g\, g^{-1} \partial_x g + \text{one more term.}$$

$$+ v\, \mathrm{tr}\, g^{-1} \partial_x g\, g^{-1} \partial_x g \Big]$$

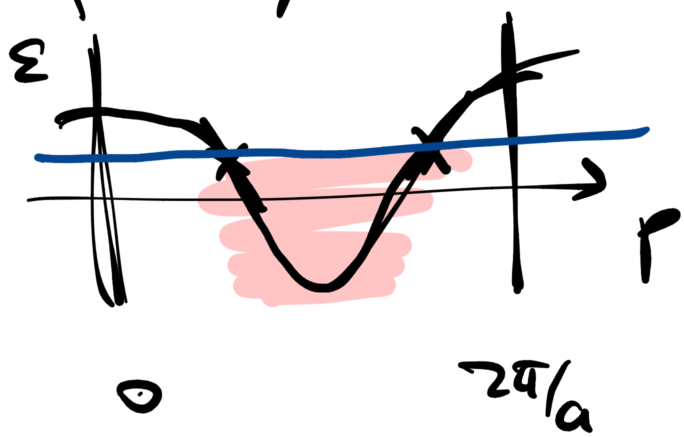
chiral WZW model. G_k.

eg: $G = U(1)$ chiral Luttinger liquid.

How to make a NON-CHIRAL Luttinger liquid:

fermions hopping in 1+1 d (add interactions)

partially fill the band.



$$v = \frac{\partial \varepsilon}{\partial p}$$

$$\varepsilon(p) = \varepsilon\left(p + \frac{2\pi}{a}\right)$$

$\to$ always non-chiral

(Nielsen-Ninomiya fermion-doubling thm)

(Nonperturbative argument: gravitational anomaly. (later))

② **parton construction**: "method to solve non-holonomic constraints" $\equiv$ inequalities

In $D = 0+0$.

Variable: $y \geq 0$.

$y \in \mathbb{R}$.

$$Z = \int_0^\infty dy\, e^{-S(y)}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} dx\, e^{-S(x(y)) + \log|x|}$$

gauge redundancy

"fluctuations of the gauge field".

eg: $H = H_{hop} + \sum_{\langle ij\rangle} V n_i n_j$

$V \gg t \Rightarrow$

constraint $n_i + n_j \leq 1$ if $\langle ij \rangle$.

Solve the constraint

$$y = x^2$$

price: $x \to -x$ is a "gauge redundancy"

Two steps of parton construction:

Kinematic step: $c = f^1 f^2 f^3$

$U(1)^2$ gauge invariance: $\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \to \begin{pmatrix} e^{i\alpha} & & \\ & e^{-i\alpha} & \\ & & 1 \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$ or $\begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{-i\alpha} \end{pmatrix}$

Dynamical step: pretend the partons are free
put in some (MF) groundstate

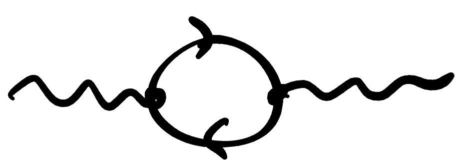
eg: $$H_{\text{partons}} = -\sum_{ij} t_{ij} f_i^{\dagger} e^{i a_{ij}} f_j + h.c.$$

then ask: what do the gauge dynamics do?

$$S_{\text{maxwell}} = \frac{1}{g^2} f^2$$

$$g(\text{lattice scale}) = \infty.$$

Microscopically: electrons.



$$g(IR) = ?$$

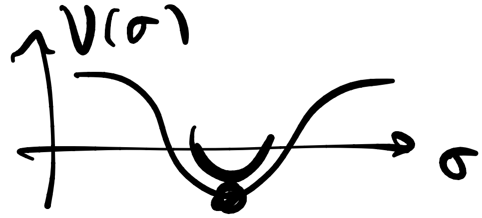
WARNING:

Compact U(1) gaugetheory in $D=2+1$ likes to confine:

$$\partial_\mu \sigma = \epsilon_{\mu\nu\rho} \partial_\nu a_\rho$$

$e^{i\sigma(x)}$ inserts 2π flux at x.

$$V_{eff}(\sigma) = \Lambda^3 e^{i\sigma} + h.c.$$
$$= \Lambda^3 \cos\sigma$$



$\Longrightarrow$ mass for σ

" for the photon. [Polyakov]

QCD !

E

free quarks & gluons

Λ_{QCD}

confinement

gauge inv't

baryons $\epsilon_{\alpha\beta\gamma} q^\alpha q^\beta q^\gamma$

mesons $\bar{q} q$.

anti-QCD ?

chemistry

electron

$S[c, A_{EM}]$

$\Lambda_{anti\,QCD}$

$S[a, f, A_{EM}]$

parton mass

$S[a, A_{EM}]$

(eg T.F.T)

gap

$S[A_{EM}]$

Reasons for deconfinement:

- In enough dims ($D \geq 3+1$) $\exists$ Coulomb phase
- partially higgs $G \to \mathbb{Z}_n$
- lots of charged gapless dofs
  eg: Fermi surface of partons
- In $D=2+1$: a CS term for a.
- other ways of assigning charge to the monopole.

Laughlin example: pile of e^- in a 2d area A w/ PBC. w/ uniform B.

Fill $\frac{1}{3}$ of LLL.

$$\frac{1}{3} = \nu_e = \frac{N_e}{N_\Phi(e)} = \frac{N_e}{eBA/hc}$$ degen. of LLL.

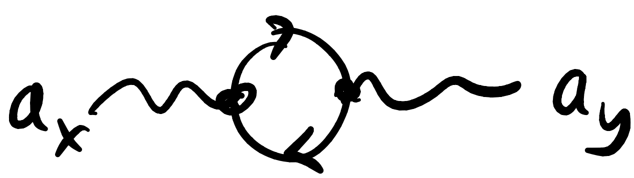
$c = f_1 f_2 f_3$ where f_α has elec. charge $\frac{1}{3}$.

each parton sees the same B.

$$\nu_{f_\alpha} = \frac{N_{f_\alpha}}{N_\Phi(e/3)} = \frac{N_e}{\frac{e}{3} BA/hc} = 3\nu_e = 1.$$

$\uparrow$

Filling parton LLL produces a <u>gapped state</u>



CS term for a encode the IQH response of partons.

<u>Method 1</u>: to describe IQH: $j_\mu^{(\alpha)} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho}\partial_\nu b_\rho^{(\alpha)}$

$$\Delta L = \frac{1}{4\pi} b^\alpha \partial b^\alpha$$

$$\sum_\alpha q_\alpha = 1$$

$$4\pi L = \sum_\alpha b^\alpha d b^\alpha + 2A\sum_\alpha q_\alpha db^\alpha + 2a^1(db^1 - db^2) + 2a^2(db^2 - db^3)$$

$$0 = \frac{\delta S}{\delta a^1} \Rightarrow b^1 = b^2 \equiv b$$

$$0 = \frac{\delta S}{\delta a^2} \rightarrow b^2 = b^3 \equiv b$$

$$4\pi L = 3\, b\, db + 2 A\, db \underbrace{\sum_\alpha q_\alpha}_{=1}$$

$U(1)_3$ EFT of Laughlin state.

$\Big[\, 4\pi L = 2 a\, db$ is a trivial theory (det $K = 1$) $\Big]$

vs $4\pi L = 4 a\, db$ = T.C. (det $K = 4$)

<u>wave f'n</u>: $|\Phi_{mf}\rangle = \mathbb{P}\, |$ fill parton bands $\rangle$

$\uparrow$ projection onto gauge invt states.

$$\rightarrow \Psi(r) = \langle 0 | \prod_i^N c(r_i) | \Phi_{mf} \rangle$$

$$= \Big(\prod_{i<j}^{N} z_{ij}\, e^{-\sum_{i=1}^{N} |z_i|^2 / 4 l_B^2(\frac{e}{3})} \Big)^3$$

$$\underbrace{\prod_{i<j}^{N} z_{ij}\, e^{\cdots}}_{\nu=1 \text{ slater det of charge } e/3 \text{ particles}}$$

$z_{ij} \equiv z_i - z_j$

$z = x + iy.$

$l_B^2(e/3) = \frac{3\hbar}{eB} = 3 l_B^2(e)$

$$= \prod_{i<j} z_{ij}^3\, e^{-\sum |z_i|^2 / 4 l_B^2}$$

[Laughlin wave f'n.

$$\sigma^{xy} = \underbrace{\frac{(e/3)^2}{h}}_{\text{IQH of } e/3 \text{ particle}} \times 3 = \frac{1}{3}\frac{e^2}{h}. \quad \checkmark$$

Method 2: $L(f, a]$ ← SU(3) gauge field

$$\int Df\, e^{\int [f, A]} = e^{i\,CS[a] + \dots}$$

w/ gapped fermionic quasiparticles

SU(3), CS.

Level-Rank duality : (same anyons & GSD.)

$SU(3)_1 \longleftrightarrow U(1)_3$

w/ fermions gapped

Parton summary: useful also for spin systems bosons, Hubbard model...

good :
- new mean field ansatze
- candidate trial wavefns
- " EFTs
- transitions to nearby states

bad :
- contact w/ microphysics
- requires understanding gauge thy

2.4 Composite fermions & hierarchy states

start at $\nu = \frac{1}{m}$. fix ρ. vary B.

Cheapest way to change charge is to create f's. they can form an FQH state!

<u>composite fermions</u>: $\Psi = \tilde{\Psi}\, e^{-\sum |z_i|^2/4\ell_B^2}$

$$\tilde{\Psi}_{1/m} = \prod_{ij} \underbrace{z_{ij}}_{\text{IQH state of composite fs}} \; \underbrace{z_{ij}^{m-1}}_{\text{m-1 units of "flux" attached to the fs.}}$$

cf sees $B^* = B - \underline{(m-1)\rho\Phi_0}$

$= B - (m-1)\nu B$

$= B(1-(m-1)\nu)$

actual density of e^- = density of cfs

$$\rho = \frac{\nu B}{\Phi_0} = \frac{\nu^* B^*}{\Phi_0}.$$

if $B^* > 0$

$$\Longrightarrow \nu = \frac{\nu^*}{(m-1)\nu^* + 1}$$

If $\underline{\nu^* \in \mathbb{Z}}$: $m = 3$

$$\underline{\underline{\nu}} = \frac{\nu^*}{2\nu^* + 1} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9} \ldots$$

<u>trial wavefn</u>:

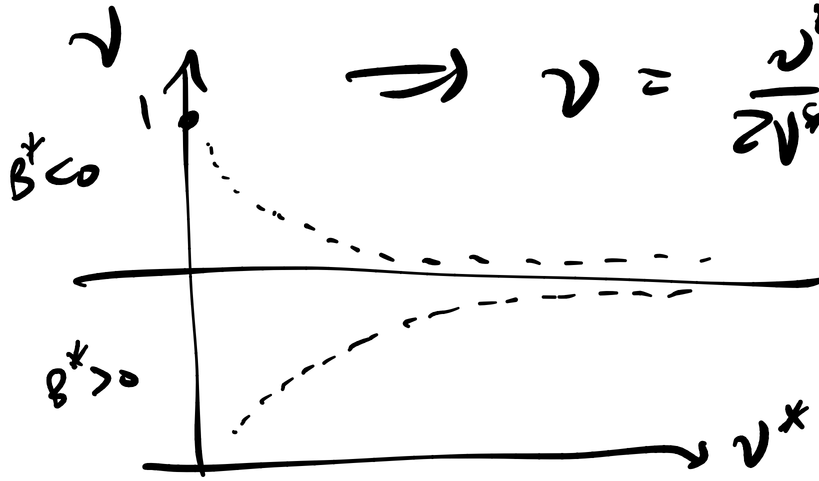
$$\tilde{\Psi}_\nu(z) = P_{LLL} \prod_{i<j} \underline{\underline{z_{ij}^2}} \, \tilde{\Psi}_{\nu^*}(z, \bar{z})$$

$$\uparrow$$

$$\bar{z}_i \longmapsto 2 \ell_B^2 \frac{\partial}{\partial z_i}$$

If $B^* < 0$ $\quad \rho = \frac{\nu B}{\Phi_0} = -\frac{\nu^* B^*}{\Phi_0}$

$$\Longrightarrow \nu = \frac{\nu^*}{2\nu^* - 1} = 1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}.$$



cfs are partons :

Hint :

$$c = \underset{\substack{\uparrow \\ \nu = \frac{1}{2} \text{ Boson FQHE.}}}{\overset{\substack{\text{cf} \\ \downarrow}}{f}\, b}$$

$$\underbrace{\tilde{\Psi}_{\nu=\frac{1}{2}} = \prod_{i<j} z_{ij}^2}$$

$c_i^\dagger c_i + c_j^\dagger c_j$ can still be 2 ?

$$\vec{S}_x = f_x^\dagger \vec{\sigma} f_x$$

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eg : $b^\dagger = f_1^\dagger f_2^\dagger \Rightarrow n_b \leq 1.$

quasihole:

$$\langle 0| \prod c(z) \left(f_y \, |\text{filled LL}\rangle \right)$$

$$= \prod_i^N (z_i - y) \prod^{N-1} z_{ij})$$

quasiparticle

$$f_y^\dagger \, |\text{filled LL}\rangle$$

$$\longrightarrow$$

$$P_{LL}$$

$$\bar{y} \to \frac{\partial}{\partial y}$$

Gurarie-Nayak.

$$S_{CS}[c] = \int_{M_7 = \Sigma_6 \times \mathbb{R}} c^I \wedge dc^J K_{IJ}$$

$$\underline{c^I = db^I} .$$

① $\sigma^z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ ② $\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\langle v | w \rangle = v_i \, \sigma_{ij} \, w_j$$

$$\| v \|^2 = v_i \, \sigma_{ij} \, v_j$$

① $v_1^2 - v_2^2$ can be odd

② $2 v_1 v_2$ $v_i \in \mathbb{Z}$.

$K_{IJ} \, a^I \, d a^J$ $a^I \to W^I{}_J \, a^J$

$$\begin{cases} K \to W K W^T \\ t \to W t. \end{cases}$$

$W \in GL(n, \mathbb{Z})$

$(\det W = \pm 1.)$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$