Transitions to neighboring phases:

A model of bosons $b = d_1 d_2$

fermionic partons

global $U(1)$: $b \to e^{i\alpha} b$.

A_μ

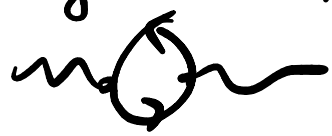
(or $b = S^+$)

[Kalmeyer-Laughlin chiral spin liquid.]

If $(c_1, c_2) = (1,1)$

$\longrightarrow$ Boson $\nu = 1/2$ Laughlin.

More generally



$$L_{\text{eff}} = \frac{c_1}{4\pi}(a_1 + q_1 A)\, d\, (a_1 + q_1 A)$$

$$+ \frac{c_2}{4\pi}(-a_1 + q_2 A)\, d\, (-a_1 + q_2 A)$$

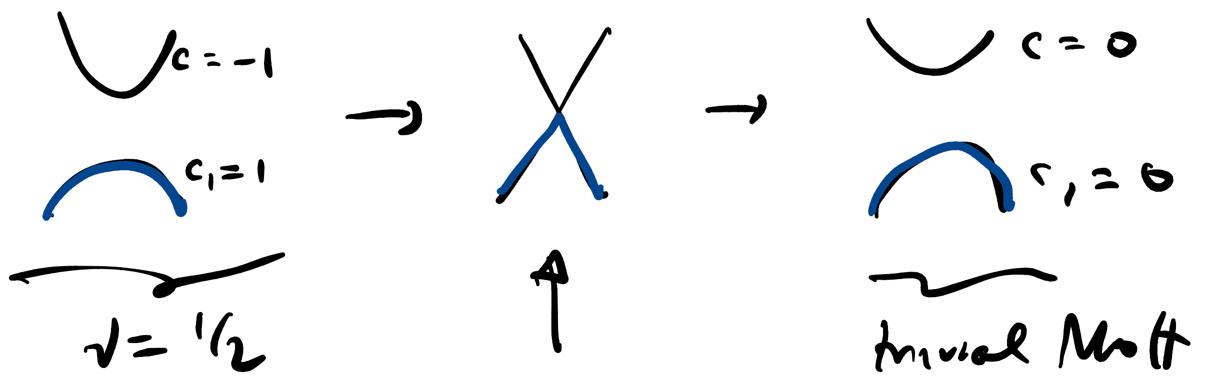
$$\xrightarrow[\text{out } a_1]{\text{integrate}} \boxed{\nu_b = \frac{c_1 c_2}{c_1 + c_2}}$$

$(q_1 + q_2 = 1.)$

If $c_2 = 1$. But $c_1 = 0$

$\rightarrow$ $\underline{\nu_b = 0}$. $\underline{\text{trivial Mott insulator of } b.}$

<u>Bandstructure of d_1</u> : (fix d_2)



If $(c_1, c_2) = (-1, 1)$ $\nu_b \overset{?}{=} 0$

$$\rightarrow L = \underbrace{\frac{1}{2\pi} a_1 dA}_{} + f_1^2$$

a_1 is NOT topologically massive!

$$= -\frac{1}{2\pi} A \wedge da_1$$

$\Rightarrow$ flux carries $U(1)_{global}$ charge

$\Rightarrow$ instantons are forbidden!

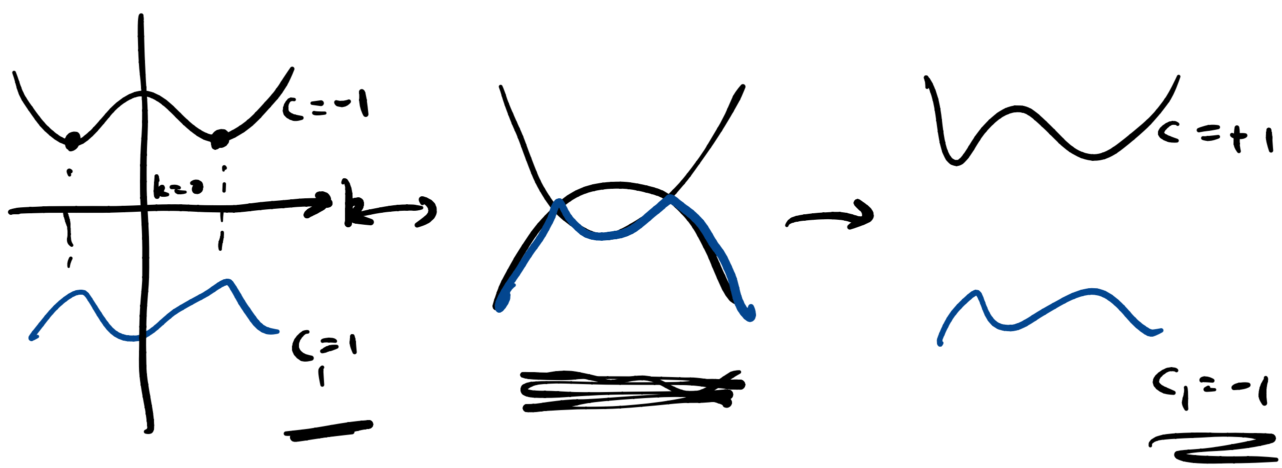
claim: this describes a superfluid phase of b. $U(1)_{global}$ is spontaneously broken

$$\partial\sigma = \epsilon\partial a/2\pi$$

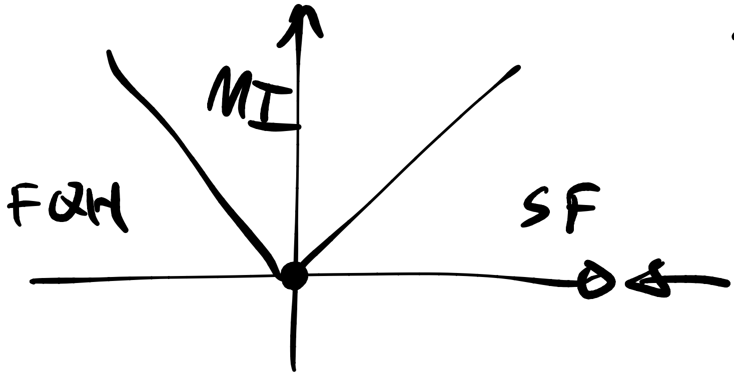
$$\Rightarrow L \ni A^\mu \partial_\mu \sigma$$

$$j_{b_\mu} = \partial_\mu \sigma \quad \text{io} \quad \sigma \to \sigma + \alpha \,!$$

dual photon is the goldstone. ($e^{i\sigma}$ is not symmetric)



Boson phase diagram:



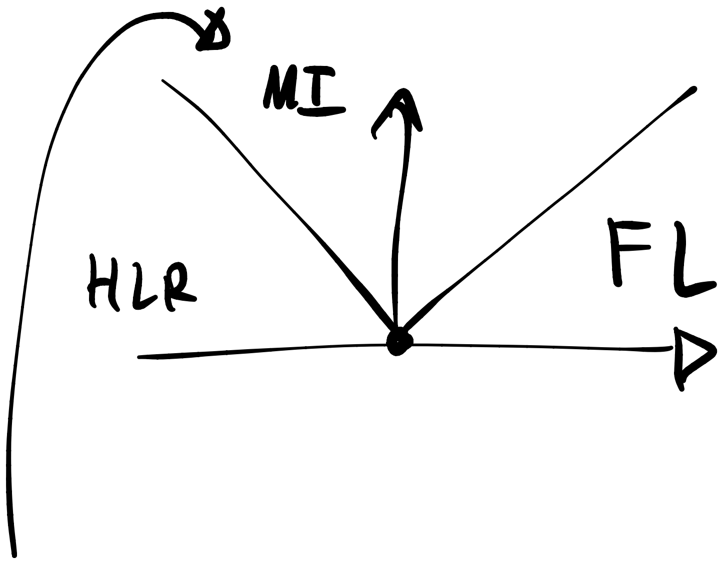
↑ $\mathbb{Z}_2$-breaking coupling

Electrons at $\nu = \frac{1}{2}$:

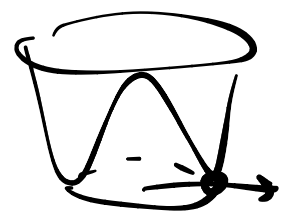
$$c = f\, b.$$

Fermi surface

	a_2
f	-1
b	$+1$



If $\langle b \rangle \neq 0$
it Higgses a_2
to nothing.



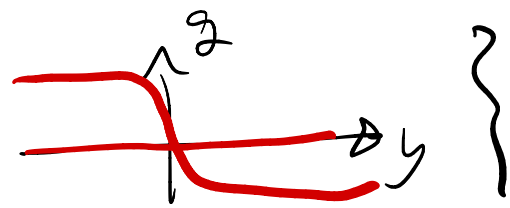
3. Symmetry-Protected Top. (SPT) Phase

What top. labels can we put on phases of matter?

× 1) broken symmetries

× 2) (intrinsic) top. order : top. GSD & anyons.

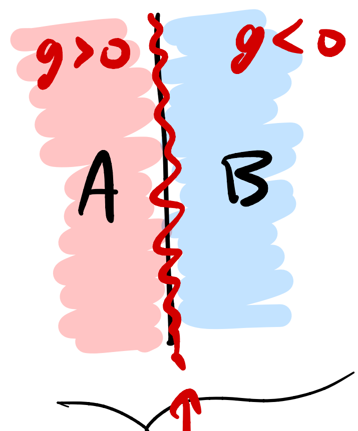
3) "edge modes"



varying g in time from H_A to H_B

$$H = H_1 + g(t) H_2$$

Requires closing the gap at $g(t)=0$.



$g(y)=0$.

$\overset{?}{\Rightarrow}$ maybe $H_1 + g(y) H_2$

also has interesting things at $g(y)=0$

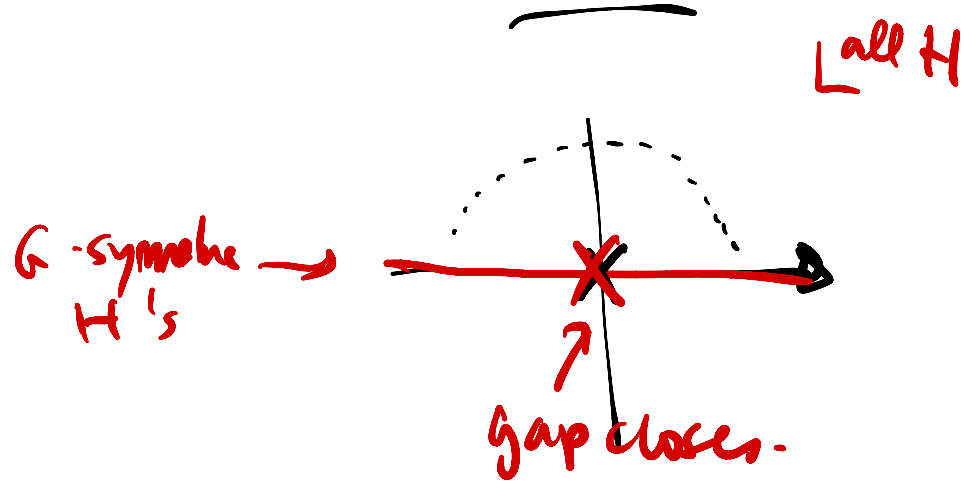
idea: Consider phases of matter CHARACTERIZED by their edge states.

The D-1 dim'l edge thy characteristic of a D dim'l bulk phase must be forbidden somehow.

(or else we could paint it on the surface of the trivial D-dim'l phase.)

(Preliminary) Def: A gapped phase w/ a G-symmetric LOCAL hamiltonian w/o T.O. Λ, distinct from any product state (in the space of G-symmetric LOCAL H) is an SPT w/ respect to G.

Possible:



$\mathrm{SPT}_D^G \equiv \{\text{SPT states for } G \text{ in } D \text{ dims}\}/\underset{\text{adiabatic deformation.}}{\simeq}$

$= \text{SPT phases.}$

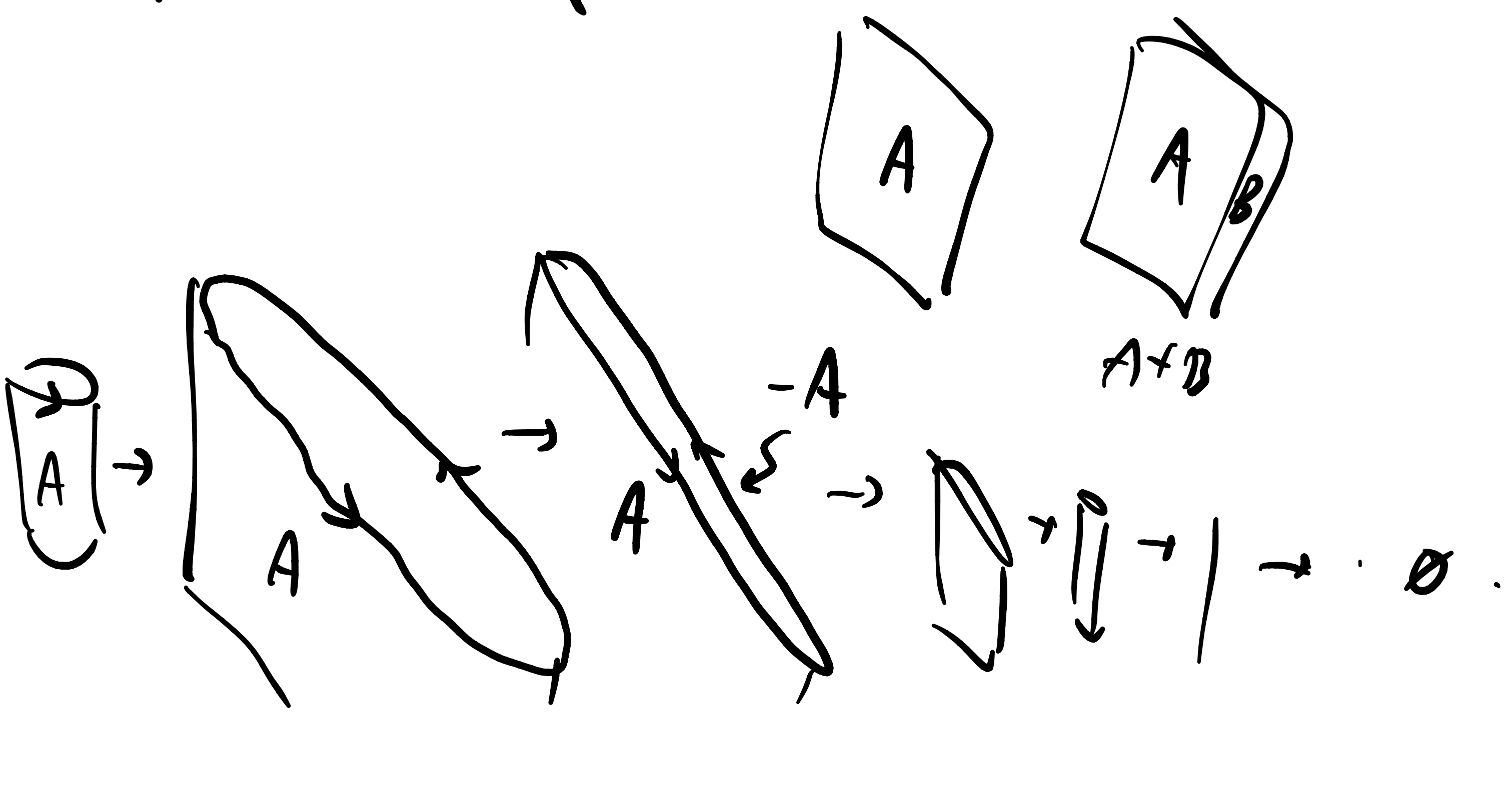
Is a <u>group</u>. under <u>stacking</u>.

stacking A & B means

A+B has $\mathcal{H}_{A+B} = \mathcal{H}_A \otimes \mathcal{H}_B$

$$H_{A+B} = H_A \otimes \mathbb{1} + \mathbb{1} \otimes H_B + \text{deformation}$$

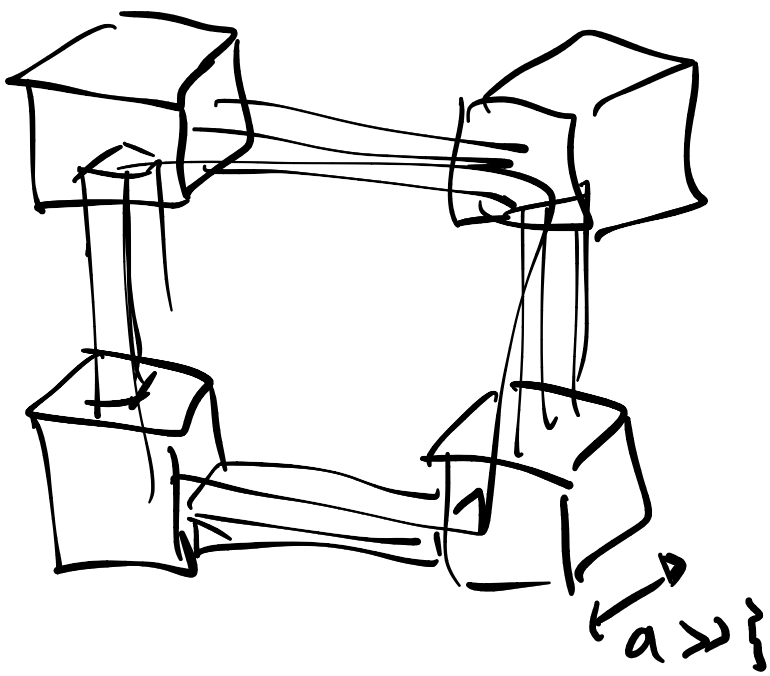
$\mathbb{1}$ wrt stacking is the trivial phase.



Require: $\mathcal{U} \, | \Psi_A \rangle \otimes | \Psi_{-A} \rangle =$ product state

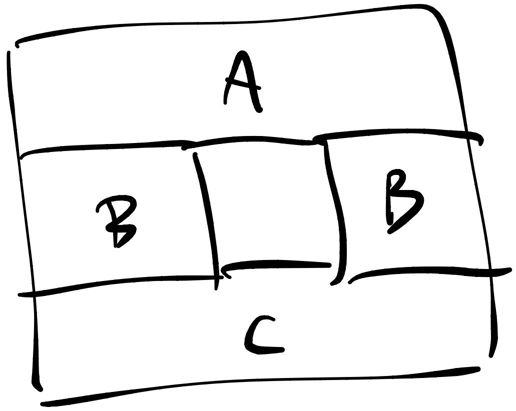
finite depth circuit.

Implicit definition: TO $\equiv$ an non-edge mode obstruction to $\exists$ of finite depth circuit constructing the g.s.



$$I(p\text{-cells} : (p+1)\text{-cells} \mid \leq p-1 \text{ cells}) = \text{TEE}.$$

$D = 2+1$:



$TEE \equiv I(A:C|B)$

How to label?

① If $G \supset U(1)$

couple to bg. gauge field A_μ.

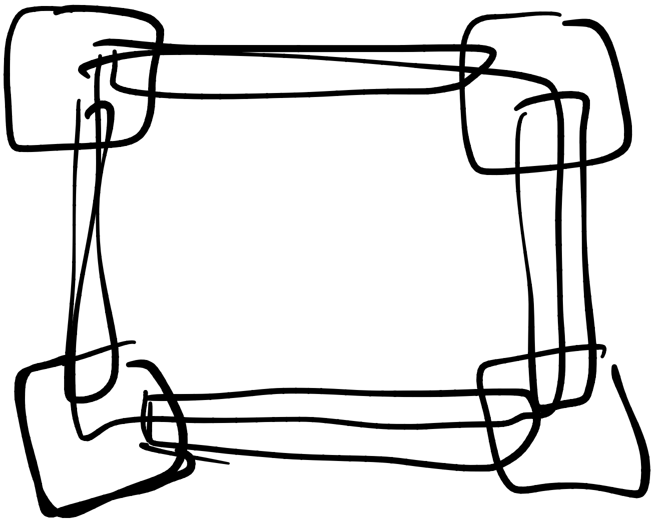
$\longrightarrow S_{eff}[A_\mu]$

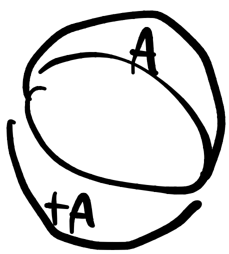
② What happens if we gauge G?

$\longrightarrow$ excitations of this gauge theory are labels on the SPT.

③ A surface anomaly (for G)

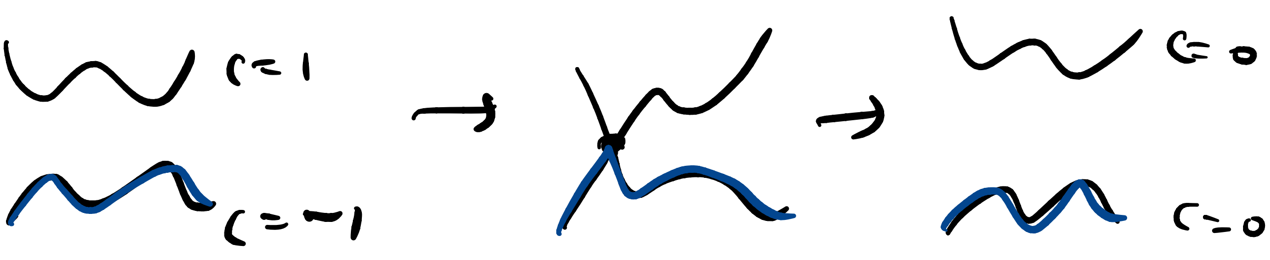
($\equiv$ obstruction to gauging G.)





U^{-1} (product state) $= |\psi_A\rangle \otimes |\psi_{\bar{A}}\rangle$

Q: (1,0) vs (0,0) Mott insulator?



unnecessary