Gapping a Weyl Semimetal

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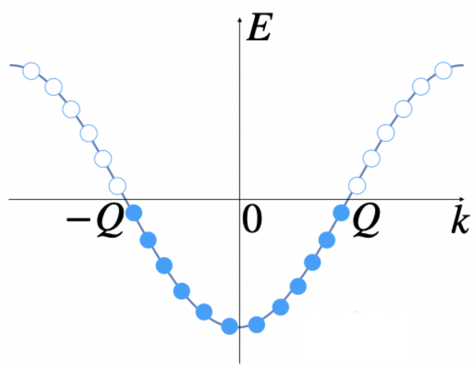
Weyl semimetals are gapless topological phase which have a chiral anomaly from gauging $U(1)$ charge conservation and translation symmetry. The gap opening is forbidden as long as we preserve the anomalies and symmetries. The system also has boundary fermi arc states and a quantized hall response for a Weyl fermion coupled to an external background. The magnitude of the Hall response is proportional to the position of the Weyl nodes. In this note we ask if it is possible to open a gap in the magnetic Weyl semimetal by adding strong interactions without breaking any of the symmetries, charge conservation and translation, and also preserving the quantized non-trivial topological responses following [1]. We learn that only when the location of the nodes is fine tuned can we open the gap while not breaking any of our demands. We also learn there exists an insulating fractional hall state in three dimensions that is not simply a coupled stack of two dimensional integer quantum hall states.

INTRODUCTION

We have spent this quarter studying topological insulators which have gapless edge modes, which cancel anomalies in the bulk. But what about gaplessness in the bulk. Spontaneously breaking a symmetry would lead to Goldstone modes. Suppose we assume that we do not want to break any symmetries. For odd filling fraction the bulk will be a metal. By adding strong Coulomb interactions the gap can be opened by forming a Mott insulator. When there are even electrons per unit cell the bulk can be gapless if there is an accidental band crossing with non-zero Chern number associated with it. The hamiltonian at the degeneracy is described by a massless Weyl fermion. By Nielsen-Ninomiya these Weyl fermions must always appear with opposite chiralities. We ask the following question - Is there a way to gap out the Weyl fermions in the bulk by adding interactions without breaking $U(1)$ and translational symmetry ?

CHIRAL ANOMALY

Suppose we have a magnetic Weyl semimetal with two nodes, located at $\mathbf{k} = \pm Q_z$.



The gauging obstructions are associated with symmetries $U(1)$ and translation in $\hat{z}$ (call this group $\mathbb{Z}^z$ and generator T_z).

The usual way to see the anomaly is gauging $U(1)\times\mathbb{Z}^z$ by adding gauge fields A_μ and $z \in H^1(M_4, \mathbb{Z})$. We also couple the theory to a background metric to probe the thermal hall response. The chiral anomaly can be understood of the 4d bulk can be understood as a boundary term of a 5d bulk action given by $S_{CA} = i2Q \int_{M_5} z \cup \left(\frac{1}{2} F \wedge F + \frac{1}{192\pi^2} R \wedge R\right)$ where R is Riemann curvature. The periodicity of the term in the parenthesis is similar to the periodicity of the $\Theta-$angle for charged 4d fermions. Thus the coefficient $2Q$ takes continuous value in $(0, 2\pi)$. Note that $2Q$ is not quantized and this is because we can always bring the two Weyl nodes together and get rid of them. Thus, another important assumption we make is that $2Q$ is fixed and we demand our interactions to be below the electron band-width.

Another way to see the chiral anomaly is to calculate the polarization associated with the extra Luttinger volume[2, 3] from the Weyl fermions, i.e., the hall conductivity is given by the Streda formula $\sigma_{xy} = \left(\frac{\partial n}{\partial B}\right)_\mu \propto \frac{\partial}{\partial B} \frac{Q}{2\pi l_B^2} \propto \frac{e^2}{h} \frac{Q}{2\pi}$. Tuning this separation $2Q$ between 0 and $2\pi\mathbb{Z}$ gives us a transition between a trivial insulator and an integer quantum hall state via a Weyl semimetal.

GAPPING A 2D DIRAC FERMION

Another place where we can see a plateau transition in conductivity is by tuning the mass of a 2D Dirac fermion, σ_{xy} jumps from 0 to e^2/h with $\sigma_{xy} = e^2/2h$ at $m = 0$. Let us first try to open the gap at this Dirac point while preserving the parity anomaly. Let us open a gap first by adding an s-wave pairing. We know from notes that the vortex solution for the Dirac equation carries a Majorana mode $\Phi = hc/2e = \pi$ in its core and we can condense pairs of Majorana modes. But, recall that because of the parity anomaly, $\mathcal{L} = \frac{1}{2}\sigma_{xy} a \wedge \partial a$ a 2π vortex carries a charge $e/2$, and are mutual semions, which means we cannot condense them. Thus the smallest such vortex that can be condensed is a 4π vortex. Condensing this vortex fractionalizes the charge into spin degrees of freedom and charge degrees of freedom, and we can write the fermion $c = b^2 f$ where b is a charge-1/2 boson and f is the spin degree of freedom. f describes the massive Dirac fermion which (half-)quantizes the thermal hall conductivity, while b forms a bosonic integer quantum hall state which (half-)quantizes the electrical hall conductivity.

GAPPING THE 3D WEYL FERMION

Now, let's ask the same question for the Weyl fermions, can we gap them while preserving the chiral anomalies, i.e. the half-quantized electrical and thermal hall responses.

The first step is to add a superconducting pairing between the chiral fermions. BCS pairing $\propto \psi_{k,R} i\sigma^y \psi_{-k,L} + h.c$ does not open a gap due to particle-hole symmetry. A CDW pairing, $\propto \psi_{k,\uparrow}\psi_{-k,\downarrow} + h.c$ opens a gap but breaks translational symmetry because the condensate $\Delta(Q) \propto \left\langle \psi^\dagger_{-Q}\psi^\dagger_{Q}\right\rangle$ carries momentum $2Q$. To preserve gauge invariance under chiral rotations, $T_z^\dagger c^\dagger_{\pm\mathbf{Q}} T_z = e^{\mp iQ} c^\dagger_{\pm\mathbf{Q}}$, we should condense $\varrho(Q) = \Delta(Q)\Delta^*(-Q)$ which carries momentum $4Q$. In general, this breaks translational symmetry except if $Q = G/4$, where G is the smallest nonzero reciprocal lattice vector.

The next step is to condense the vortices. Once again, there are chiral Majorana modes each carrying flux $\Phi = \pi$ in the vortex cores, and for an even number $2n$ of such Majorana's can be paired upto into a n 1D Weyl Fermions and gapped about by adding a BCS pairing. For such a case when the vortex can be gapped without breaking translational symmetry we want to make sure we match the chiral anomaly $\sigma_{xy} = 1/4\pi$ when we condense vortices, which are loops in 3D. We would like the loops to be trivial under translation. One way to understand the loops under translation, we link the loop to a lattice dislocation along some Burger vector, say along $\hat{z}$ which inserts a half-xy plane at the dislocation line. The braid statistics of two such loops for even Majorana modes per layer, each carrying a 2π flux will have statistics, $\theta = \pi\sigma_{xy}/(1/2\pi) = \pi/2$. Each time a loop links a dislocation it traps a semion, and thus the 3-loop braiding process describing the braiding of the two-vortex loops is semionic. This means that condensing pairs of Majoranas is not allowed. Once again we come to the conclusion that we can only condense the vortices in a four-fold manner. The insulator produced by such a condensation preserves the translation symmetries and has $U(1)$ charge conservation. It has an electrical hall conductivity of $\sigma_{xy} = 1/4\pi$ and thermal hall conductivity $\kappa_{xy} = \sigma_{xy}\left(\frac{\pi^2 k_B^2 T}{3}\right)$.

Once again, by spin charge separation we can write a parton ansatz for the fermion $c = b^2 f$, where b, f are the chargon and spinon degrees of freedom respectively, invariant under a $\mathbb{Z}_4$ gauge transformation, $b \to (i)^n b$, $f \to (-1)^n f$, $n \in \mathbb{Z}_4$. The insulator has $\mathbb{Z}_4$ topological order and the $\mathbb{Z}_4$ gauge flux loops are the uncondensed one, two and three fold vortices. This state is the analogue of the Moore-Read Pfaffian-antisemion state in three dimensions.

The fermion f is described by a band structure of the Weyl semimetal with $2Q = \pi$ and the CDW pairing term, that does not violate the translational symmetry. The boundary Fermi arc states lead to a half-quantized thermal hall conductivity. The $e/2-$charged bosons b form a layered bosonic integer quantum hall state, with half-quantized electrical hall conductivity per layer. The insulating phase produced has the right chiral and gravitational anomalies while preserving gauge invariance and translation.

The above state is also a way to realize the fractional quantum hall effect in 3D, which provides a intermediate phase between a trivial insulator with $\sigma_{xy} = 0$ and an integer quantum hall insulator with $\sigma_{xy} = 1/2\pi$. Similar to the 2D Dirac fermion we can tune between the two phases by tuning a TR-breaking parameter, say the magnetization m. This is also like tuning the filling fraction by applying an external magnetic field in the case of 2D quantum hall state.

CONCLUSION

In conclusion, in this note we found a way to open a gap in a magnetic Weyl semimetal while preserving the electrical and thermal hall responses proportional to the Weyl node separation $2Q$. When $2Q \neq \mathbb{Z}\pi$ the anomalies prohibit opening a gap. We have demonstrated that such gap opening is possible, but only when the $2Q = \pi$, and at the cost of introducing a non-trivial topological order $\mathbb{Z}_4$ which can be understood as a generalization of Fractional Quantum Hall insulator in three dimensions.

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