University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215C QFT Spring 2022
Assignment 3

Due 11:59pm Monday, April 18, 2022

Thanks in advance for following the guidelines on HW01. Please ask me by email if you have any trouble.

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1. **Right-handed neutrinos.**

   Consider adding a right-handed singlet (under all gauge groups) neutrino N_R to the Standard Model. It may have a majorana mass M; and it may have a coupling g_ν to leptons, so that all the dimension ≤ 4 operators are

   $$\mathcal{L}_N = \bar{N}_R \mathbf{i} \not{\partial} N_R - \frac{M}{2}\bar{N}_R^c N_R - \frac{M}{2}\bar{N}_R N_R^c + \left(g_\nu \bar{N}_R H_i^T L_j \epsilon^{ij} + h.c. \right)$$

   where $N_R^c = C\left(\bar{N}_R\right)^T$ is the the charge conjugate field, $C = \mathbf{i}\gamma_2\gamma_0$ (in the Dirac representation), H is the Higgs doublet, L is the left-handed lepton doublet, containing ν_L and e_L. Take the mass M to be large compared to the electroweak scale. Integrate out the right-handed neutrinos at tree level. [Hint: you may find it useful to work in terms of the Majorana field

   $$N \equiv N_R + N_R^c$$

   which satisfies $N = N^c$.]

   Show that the leading term in the expansion in $1/M$ is a dimension-5 operator made of Standard Model fields. Explain the consequences of this operator for neutrino physics, assuming a vacuum expectation value for the Higgs field.

   Place a bound on M assuming that the observed neutrinos have masses $m_\nu < 0.5$ eV.

2. **Gross-Neveu model.**

   Here's an example which illustrates the manipulations we did in describing the BCS phenomenon. Now that we've learned about fermionic path integrals, consider the partition function for an N-vector of fermionic spinor fields in D dimensions:

   $$Z = \int [d\psi d\bar{\psi}] e^{\mathbf{i}S[\psi]}, \quad S[\vec{\psi}] = \int \mathrm{d}^D x \left(\bar{\psi}^a \mathbf{i} \not{\partial} \psi^a - \frac{g}{N}\left(\bar{\psi}^a \psi^a\right)^2 \right).$$

(a) At the free fixed point, what is the dimension of the coupling g as a function of the number of spacetime dimensions D? Show that it is classically marginal in $D = 2$, so that this action is (classically) scale invariant.

(b) We will show that this model in $D = 2$ exhibits dimensional transmutation in the form of a dynamically generated mass gap. Here are the steps: first use the Hubbard-Stratonovich trick to replace ψ^4 by $\sigma\psi^2 + \sigma^2$ in the action, where σ is a scalar field. Then integrate out the ψ fields. Find the saddle point equation for σ; argue that the saddle point dominates the integral for large N. Find a translation invariant saddle point. Plug the saddle point configuration of σ back into the action for ψ and describe the resulting dynamics.