Physics 215C : QFT part three

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OH : after lecture ...

WORK : • weekly psets. 1st HW due Mon April 4 11:59 pm.

(less than 215 A, B).

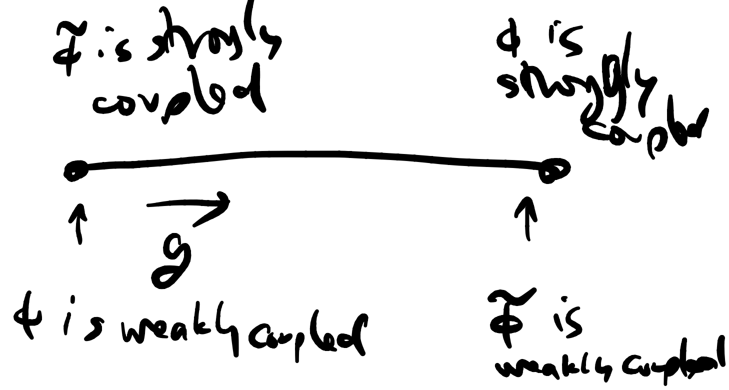
• SHORT paper. topic choice by wk. 8.

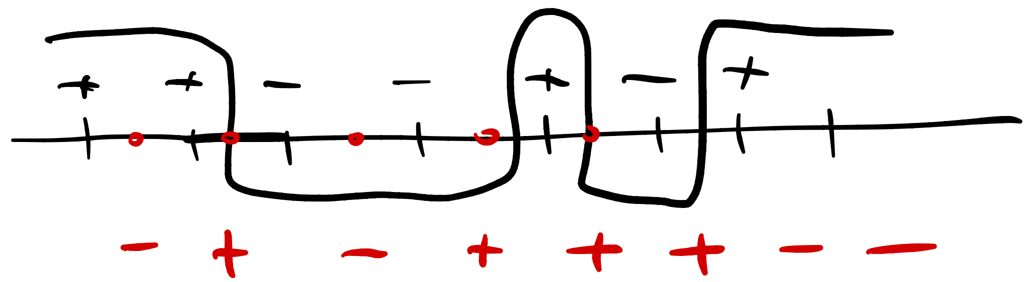
Introductory Remarks & Goals :

THEMES :

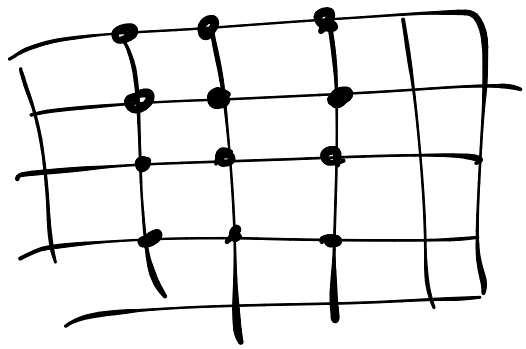
• Topology in QFT.

• Duality





- EMERGENCE of QFT.



— path integrals (from coherent states)

— large N

1. Anomalies

Suppose given $S[\text{fields}]$ in the continuum with some symmetry.

<u>Q</u>: $\exists$ a QFT with that sym. with a classical limit above

<u>A</u>: $$Z = \int [D\,\text{fields}]\, e^{iS[\text{fields}]}$$

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Anomaly $\equiv$ S is symmetric but measure is not.

$$D\phi = \prod_{\substack{x \in \text{discretization} \\ \text{of spacetime.}}} d\phi(x)$$

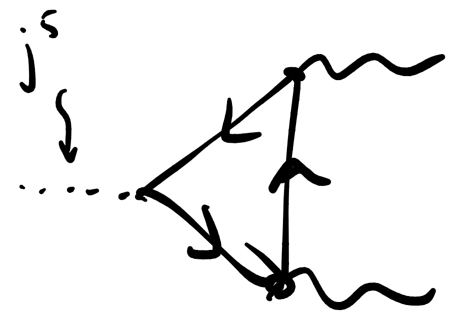
eg: chiral anomaly $\qquad D = \text{even}$

chiral (or axial) symmetry of $S[\bar{\psi}, \psi]$: $\left\{ \begin{array}{l} \psi_L \to e^{i\alpha} \psi_L \\ \psi_R \to e^{-i\alpha} \psi_R \end{array} \right\}$

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \to e^{i\alpha\gamma^5} \Psi$$

is generated by j^5_μ

$$\partial^\mu j^5_\mu = A(F) \overset{D=4}{\propto} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$



$\pi^0 \to \gamma\gamma$

virtues of method:

- one loop exact
- $\int_M A \in \mathbb{Z}$
- index thm.

D even Dirac rep of $SO(D-1,1)$ is Reducible

$$\gamma^5 \equiv a \prod_{\mu=0}^{D-1} \gamma^\mu \xRightarrow{\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}} \{\gamma^5,\gamma^\mu\}=0 \quad \forall \mu$$

choose a s.t.

$$(\gamma^5)^2 = 1$$

$$\Longrightarrow [\gamma^5, \Sigma^{\mu\nu}] = 0$$

$$\Sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu,\gamma^\nu]$$

generators of $SO(D-1,1)$.

$$P_{L/R} = \frac{1 \pm \gamma_5}{2}$$

eg: Weyl basis

$$\gamma^\mu = \begin{pmatrix} 0 & \bar\sigma^\mu \\ \sigma^\mu & 0 \end{pmatrix} \qquad \left.\begin{aligned} \sigma^\mu &= (\mathbb{1}, \vec\sigma)^\mu \\ \bar\sigma^\mu &= (\mathbb{1}, -\vec\sigma)^\mu \end{aligned}\right\} \Rightarrow \gamma^5 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\Sigma^{\mu\nu} = \left(\begin{array}{c|c} & 0 \\ \hline 0 & \end{array}\right)$$

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \psi_{L/R} = P_{L/R}\Psi$$

In $D=4k$ dims $\quad \psi_L^r \xrightarrow{CPT} i\gamma^0(\psi_L^r(-t,-x))^*$

in rep r of G $\qquad$ is right-handed in rep $\bar{r}$ of G

In $D=4k+2$ dims $\quad \psi_L^r \xrightarrow{CPT} \dots$

Left-handed in rep $\bar{r}$.

Anomalies in $D=4k$ dims can happen if the Weyl fermions transform in complex reps of G.

$$\delta\psi_a = i\epsilon^A (t_r^A)_{ab}\psi_b \qquad \text{is rep } r.$$

$t_r^A \quad A=1..\dim G$

assume $t=t^\dagger$, $\epsilon=\epsilon^*$

$a,b=1..\dim r.$

$$\delta\psi_a^{*T} = -i\epsilon^A (t_r^A)^T_{ab}\psi_b^{*T}$$

ie $\quad t_{\bar{r}}^A = -(t_r^A)^T.$

$R_1 \simeq R_2$ if $\exists\ U$ indep of A

s.t. $t^A_{R_1} = U\, t^A_{R_2}\, U^\dagger$.

r is complex $\iff$ $\bar{r} \not\simeq r$.

<u>simplest eg:</u> $G = U(1)$

$t_r \in \mathbb{Z}$ specifies the charge

$t_{\bar{r}} = -t_r$

if $t_r \neq 0$ r is complex.

$$e^{iS_{eff}[A]} \equiv \int [D\psi D\bar{\psi}]\, e^{iS[\psi,\bar{\psi},A]}$$

B.G. gauge field

$$S[\psi,\bar{\psi},A] = \int d^Dx\ i\bar{\psi} \not{D} \psi$$

singlet eg: $\not{D}\Psi = \gamma^\mu(\partial_\mu + iA_\mu)\Psi$

more generally
$$\bar{\Psi}\not{D}\Psi = \bar{\Psi}_a \gamma^\mu (\partial_\mu \delta_{ab} + i A^A_\mu T^A(R)_{ab})\Psi_b$$

$$S[\Psi, \bar{\Psi}, A] = \int d^D x \, (\psi_L^\dagger i \bar{\sigma}^\mu D_\mu \psi_L + \psi_R^\dagger i \sigma^\mu D_\mu \psi_R),$$

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

is invt under
$$\begin{cases} \Psi \to e^{i\alpha\gamma^5}\Psi \\ \bar{\Psi} = \Psi^\dagger \gamma^0 \to \Psi^\dagger e^{-i\alpha\gamma^5}\gamma^0 \\ \qquad = \bar{\Psi} e^{+i\alpha\gamma^5}. \end{cases}$$

$$L_m = \bar{\Psi}(\mathrm{Re}\, m + \mathrm{Im}\, m \gamma^5)\Psi$$
$$= m \psi_L^\dagger \psi_R + \text{h.c.}$$

explicitly breaks this sym.

$$j^5_\mu = \bar{\Psi}\gamma^5\gamma_\mu\Psi \underset{\not{\partial}\Psi=0}{\Longrightarrow} \partial^\mu j^5_\mu \overset{?}{=} 0.$$

$$j^\mu = \bar{\Psi}\gamma^\mu\Psi \qquad \mathcal{L} \ni A_\mu j^\mu$$

$$\partial_\mu j^\mu \overset{!}{=} 0$$

Pf:

$$S_{eff}[A_\mu] \overset{!}{=} S_{eff}[A_\mu + \partial_\mu\lambda]$$

$$\overset{IBP}{=} S_{eff}[A_\mu] - i\log\langle e^{i\int\lambda(x)\partial_\mu j^\mu}\rangle$$

$$\forall\,\lambda(x)$$

Noether method: $\Psi'(x) \equiv e^{i\alpha(x)\gamma^5}\Psi(x)$

$$S[\Psi', \bar{\Psi}', A] = \int d^Dx\, \bar{\Psi} e^{+i\alpha\gamma^5} i\not{D} e^{i\alpha\gamma^5}\Psi$$

$\alpha \ll 1$:

$$= \int d^Dx\left(\bar{\Psi} i\not{D}\Psi + \bar{\Psi} i\gamma^5\not{\partial}\alpha\Psi + O(\alpha^2)\right]$$

$$\overset{IBP}{=} S[\Psi, \bar{\Psi}, A] - i\int d^Dx\,\alpha(x)\partial^\mu j^5_\mu.$$

If $[D\Psi'] \stackrel{?}{=} [D\Psi] \xrightarrow{?} \partial^\mu j_\mu^5 \stackrel{?}{=} 0.$

Claim:

$$e^{iS_{eff}[A]} = \int [D\Psi' D\bar{\Psi}'] e^{iS[\Psi', \bar{\Psi}', A]}$$

$$= \int [D\Psi D\bar{\Psi}] e^{iS[\Psi, \bar{\Psi}, A]} \times$$

$$\exp i\int d^Dx\, \alpha(x) \underbrace{\left(\partial_\mu j^\mu_5 - A(x)\right)}_{=0}$$

$\forall \alpha(x)$

$$[D\Psi' D\bar{\Psi}'] = [D\Psi D\bar{\Psi}] \det\left(e^{i\alpha\gamma^5}\right)$$

$$= [D\Psi D\bar{\Psi}]\, e^{-i\int d^Dx\, \alpha(x) A(x)}$$

$$e^{-i\int \alpha A} = \det e^{i\alpha\gamma^5} = e^{\mathrm{Tr}\log e^{i\alpha\gamma^5}} = e^{\mathrm{Tr}\, i\alpha\gamma^5}$$

$$\Rightarrow A(x) = \mathrm{tr}\,\gamma^5 = \sum_n \mathrm{tr}\, \bar{\xi}_n(x) \gamma^5 \xi_n(x)$$

Conclusion : $\underbrace{\partial^\mu j_\mu^5}$ $= \mathcal{A}(x)$

formally $\mathcal{A}(x) =$ " $\mathrm{tr}\, \gamma^5$ " .

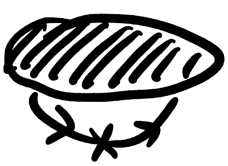
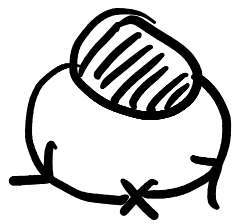
WHO IS $\mathcal{A}$?

$$\langle \partial_\mu j_5^\mu \rangle = \partial_\mu \langle \bar{\Psi}(x) \gamma^\mu \gamma^5 \Psi(x) \rangle$$

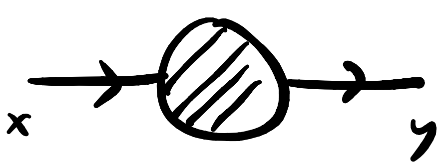
$$J_\mu^5(x) \equiv \langle \bar{\Psi}(x) \gamma^\mu \gamma^5 \Psi(x) \rangle$$

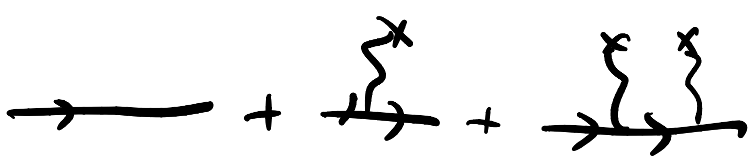
euclidean.

$$= Z^{-1}[A] \int [D\psi D\bar{\psi}]\, e^{-S[\psi,\bar{\psi},A]}\, j_\mu^5(x)$$

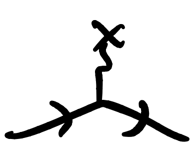
$=$  $=$  $= - \mathrm{tr}\, \gamma^\mu \gamma^5 G^{[A]}(x,x)$

spinors.

 $\equiv$ fermion propagator in the background gauge field.

$=$  $+ \ldots$

$= \left(\frac{1}{\not{D}^{[A]}} \right)_{xy} = G^{[A]}(x,y)$.

$\bar{\Psi}\, i A_\mu \psi \rightarrow$ 

to invert H :

$$H|n\rangle = \epsilon_n |n\rangle$$

$$\mathbb{1} = \sum_n |n\rangle\langle n|$$

$$H = \sum_n \frac{1}{\epsilon_n} |n\rangle\langle n| .$$

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to invert $i\slashed{D}^{[A]}$.

$$\left\{ \begin{array}{l} i\slashed{D}\, \xi_n(x) = \epsilon_n \xi_n(x) \\ \bar{\xi}_n(x)\, i\gamma^\mu\left(-\overleftarrow{\partial}_\mu + iA_\mu\right) = \epsilon_n \bar{\xi}_n \end{array} \right.$$

$$\Longrightarrow \quad G^{(A)}(x,x') = \left(\frac{1}{i\slashed{D}}\right)_{xx'}$$

$$= \sum_n \frac{1}{\epsilon_n} \xi_n(x)\, \bar{\xi}_n(x') .$$

$$\mathrm{tr}(\gamma \ldots) =$$

tr ↑ spin space

$$\mathrm{Tr}_{\mathcal{H}}(\ldots) = \underline{\mathrm{tr}} \overbrace{\int d^D x} \langle x| \ldots |x\rangle$$

$$= \mathrm{tr} \int d^D p \langle p| \ldots |p\rangle$$

$\mathcal{H}$ is the space on which $\not{D}$ acts

$$= \sum_n \bar{\xi}_n (\ldots) \xi_n$$

$$\partial^\mu j^5_\mu(x) = \underline{\mathcal{A}}(x) \mathbb{1}$$

$$\underbrace{\langle \partial^\mu j^5_\mu \rangle_{\psi,\bar{\psi}}} = \underbrace{\langle \mathcal{A}(x) \mathbb{1} \rangle = \mathcal{A}(x)}$$