Recap of EFT of SF & SC

$$\vec{J} \propto \vec{\nabla}\varphi \qquad \Phi = |\Phi_0| e^{i\varphi}$$

Galilean boost : $\mathcal{L} = \Phi^* i\partial_t \Phi - \vec{\nabla}\Phi^* \cdot \vec{\nabla}\Phi - V(|\Phi|)$

$$\Phi(x,t) \longrightarrow \Phi'(x',t')$$

$$\Rightarrow \quad \underline{\underline{\Phi(x,t) = e^{-\frac{1}{2}imv^2t + imv_i x^i}\,\Phi'(x',t')}}$$

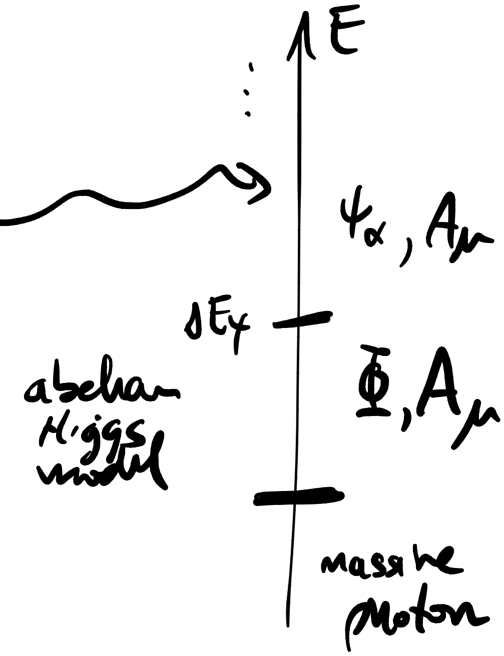
$$\begin{cases} \vec{x}' = \vec{x} - \vec{v}t \\ t' = t \end{cases}$$

at fixed time $(t=0)$

a boost acts by

$$\varphi(x) \longrightarrow \underline{\underline{\varphi(x) + mv_i x^i}}$$

EFT of METAL.



Metals are weird:

arbitrarily small $\vec{E}$

$\Longrightarrow$ nonzero $\vec{J} = \sigma \vec{E}$

$\Longrightarrow$ gapless d.o.f.

w/ energies $\ll$ $\underset{\sim\sim\sim}{\text{intrinsic scale}}$.

$\equiv$ Planck scale of solids

$$E_0 = \frac{1}{2}\frac{e^4 m}{\hbar^2} = \frac{e^2}{2a_0}$$

$\sim 13\,\text{eV}$.

$m \equiv m_e$

Rydberg

chemistry : solid state physics ::
melting of spacetime : high-energy physics

Another scale: $M \gg m$

$\sim$ proton mass

$\frac{m}{M}$ is a small parameter

Another quantity: $c \gg v_F$ treat $c \to \infty$.

$\frac{v_F}{c}$ suppresses spin-orbit couplings. $\underset{\sim\sim\sim}{SO(3)_{\text{rots}}} \times \underset{\sim\sim}{SU(2)_{\text{spin}}} \to SU(2)$

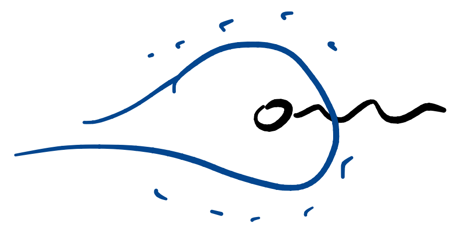
An EFT of this phenomenon? (Landau Fermi liquid theory)

What are the dofs?

Guess: basically, electrons.
i.e. fermions, spin $\frac{1}{2}$, electric charge 1

dressed electrons or quasi-electrons

(an example of quasiparticles)



to show: this guess is self-consistently robust.

③ cutoff: E_0

② symmetries: a) particle # $\psi \to e^{i\theta}\psi$

b) spatial syms: - time translations

- either i) continuous rots & transl. (liquid ^3He)

ii) lattice symmetries

c) spin rot. sym. $SU(n)$ $\sigma = 1..n$

d) $\epsilon(p) = \epsilon(-p) \Longleftarrow$ parity

$$S_{free}[\psi] = \int dt\, d^dp \left(\psi^\dagger_\sigma(p)\, i\partial_t \psi_\sigma(p) - (\epsilon(p) - \epsilon_F)\psi^\dagger_\sigma(p)\psi_\sigma(p) \right)$$

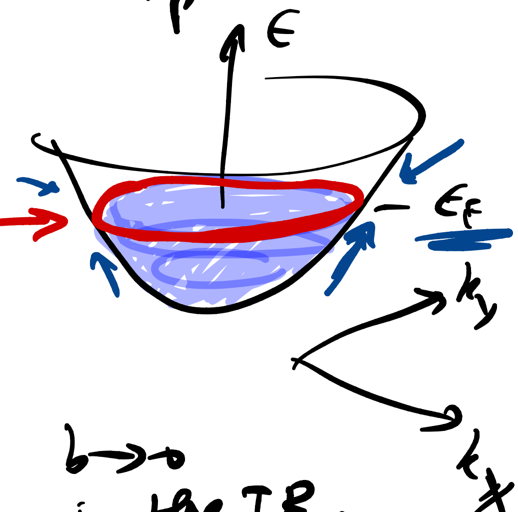
$$= \int dt\, d^dx \left(\psi^\dagger_\sigma(x)\, i\partial_t \psi_\sigma(x) - \psi^\dagger_\sigma \underline{(\epsilon(-i\vec{\nabla}) - \epsilon_F)}\psi_\sigma \right)$$

ϵ_F = fermi energy.

<u>eg:</u> $\epsilon(p) = \frac{p^2}{2m}$

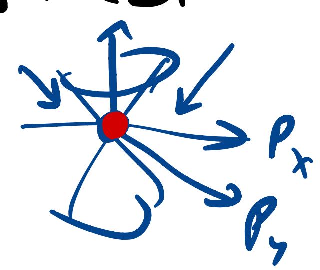
$$|gs\rangle = \prod_{p | \epsilon(p) < \epsilon_F} \psi^\dagger_p |0\rangle \qquad \psi_p|0\rangle = 0$$

$$FS \equiv \{ p \,|\, \epsilon(p) = \epsilon_F \}$$

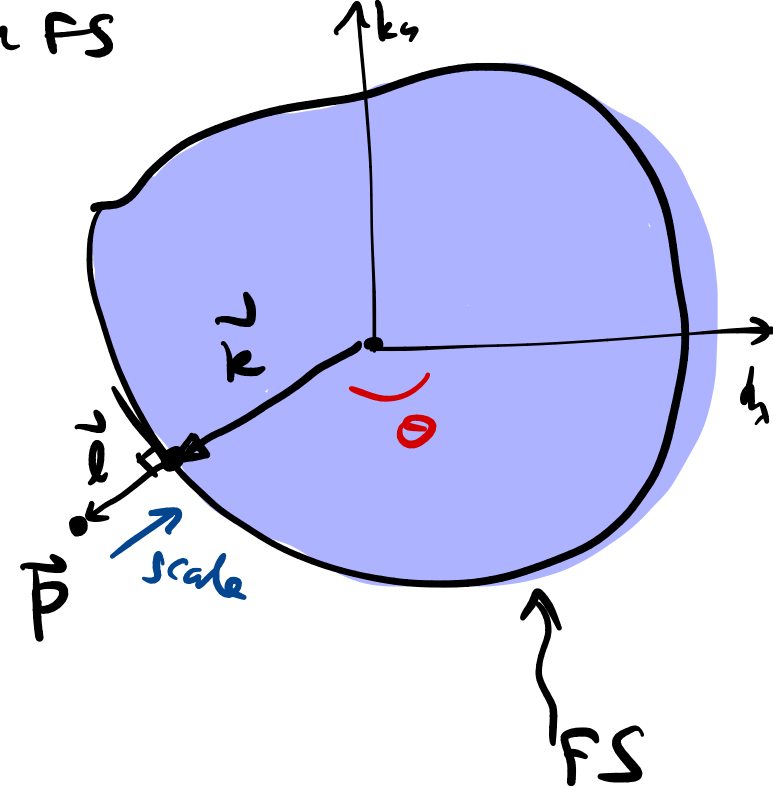


<u>Power Counting:</u> $E \to bE$, $b < 1$. $b \to 0$ is the IR.

Lorentz sym $\vec{p} \to b\vec{p}$



Polchinski's labelling: ← For pts near FS



$$\vec{p} = \tilde{k} + \tilde{\ell}$$

closest point to $\vec{p}$ on the FS (d-1 coords)

$|\ell|$ = distance along the normal to FS at $\tilde{k}$ (1 coord)

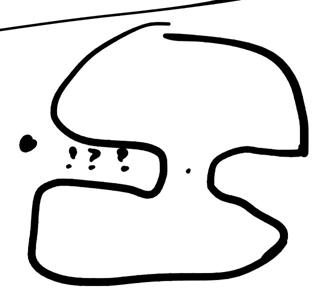
$$\vec{v}_F(k) \equiv \vec{\partial}_p \epsilon \big|_{p=k}$$

$$\epsilon(p) = \epsilon(k+\ell) = \epsilon(k) + \vec{\ell}\cdot\vec{v}_F(k) + O(\ell^2)$$
$$= \epsilon_F + \vec{\ell}\cdot\vec{v}_F(k) + O(\ell^2)$$

$$E \to bE \qquad \vec{k} \to \tilde{k} \qquad \tilde{\ell} \to b\,\vec{\ell}$$

stay on FS!

Don't worry about:



$$dt \to b^{-1} dt \qquad d^{d-1}\vec{k} \to d^{d-1} k \qquad d\vec{\ell} \to b\, d\vec{\ell}, \quad \partial_t \to b\, \partial_t$$

$$S_{free}[\psi] = \int \underbrace{dt\, d^{d-1}k\, d\ell}_{\sim b^0} \left(i\psi^\dagger \underbrace{\partial_t}_{\sim b^1} \psi - \underbrace{\vec{\ell}\vec{v}_F}_{\sim b^1}(k)\, \psi^\dagger(p)\psi(p) \right)$$

$\sim b^0$ if $\underline{\psi \to b^{-1/2} \psi}$.

$S[\psi] = S_{free}[\psi] +$ all possible terms.

$\psi \to e^{i\theta}\psi \Rightarrow \ldots \psi^\dagger\psi + \psi^\dagger\psi\psi^\dagger\psi + \ldots$

Possible Quadratic terms:

$$\int \underbrace{dt\, d^{d-1}k\, d\ell}_{b^0}\; \mu(k) \underbrace{\psi^\dagger_\sigma(p)\psi_\sigma(p)}_{b^1} \sim b^1$$

is Relevant!

Although any particular ~~ther~~ shape of FS is unnatural the EXISTENCE of a FS is natural.

$$\int dt\, d^{d-1}k\, dl \quad \mu(k)\, \psi^{+}_{\sigma}(p)\, \psi_{\sigma}(p)$$

∂_t or l $\longrightarrow b^{0}$ already there.

✓ ✓

more than one ∂_t or l $\longrightarrow b^{>0}$

Irrelevant

Quartic Terms:

$$S_4 = \int dt \underbrace{\prod_{i=1}^{4} \left(d^{d-1}k_i\, dl_i\right)}_{b^{-1+4}} \mu(4 \cdots 1)\, \overbrace{\psi^{+}_{\sigma}(p_1)\, \psi_{\sigma}(p_3)\, \psi^{+}_{\sigma'}(p_2)\, \psi_{\sigma'}(p_4)}^{b^{-1/2 \cdot 4}}\, .$$

$k_4 \cdots k_1$

$$\underbrace{\delta^{d}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)}_{?}$$

removes momentum p_3

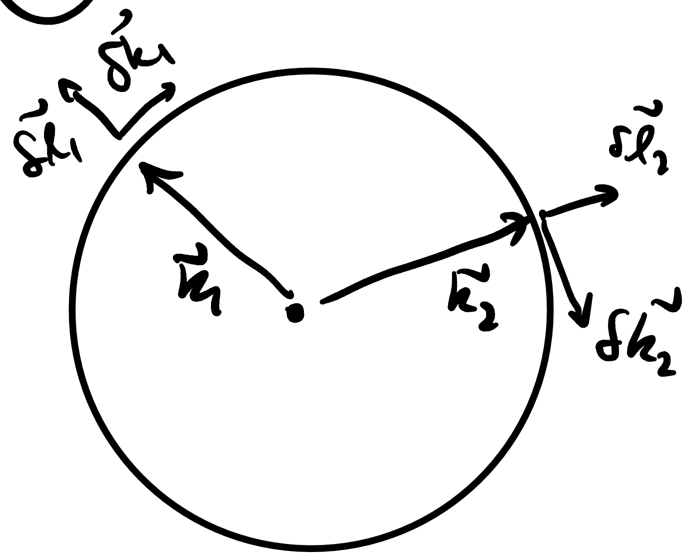
$$\delta(p_1 + p_2 - p_3 - p_4) = \delta(\underbrace{k_1 + k_2 - k_3 - k_4}_{?} + l_1 + l_2 - l_3 - l_4)$$

$k \gg l$ $\quad \overset{?}{\simeq} \delta(k_1 + k_2 - k_3 - k_4) \sim b^{0}$

IF SO: $\quad S_4 \sim b^{4-1-\frac{4}{2}} \sim b^{1}$ $\quad$ Irrelevant

BUT: ① $\exists$ phonons.

② kinematic subtlety



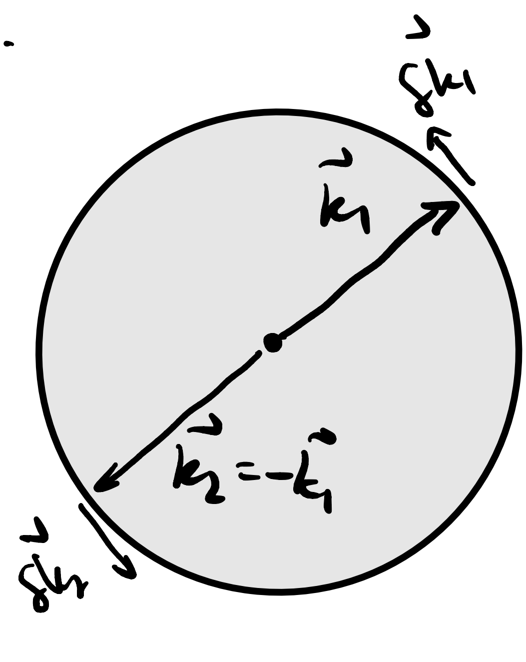
$$\vec p_3 = \vec p_1 + \vec{\delta k}_1 + \vec{\delta l}_1$$
$$\vec p_4 = \vec p_2 + \vec{\delta k}_2 + \vec{\delta l}_2$$

$$\delta^d(p_1 + p_2 - p_3 - p_4)$$
$$= \delta^d(\vec{\delta k}_1 + \vec{\delta l}_1 + \vec{\delta k}_2 + \vec{\delta l}_2)$$

generic kinematics $(\vec k_1, \vec k_2)$: $\vec{\delta k}_1, \vec{\delta k}_2$ are linearly indep.

$$\Rightarrow \delta^d(\delta k + \delta k + \delta l + \delta l)$$
$$\sim \delta^d(\delta k + \delta k) \ \checkmark.$$

Special k_1, k_2 (nested): $\vec{\delta k}_1$ & $\vec{\delta k}_2$ are linearly dependent.



$\Rightarrow$ one component of $\vec{\delta k}_1 + \vec{\delta k}_2 = 0$

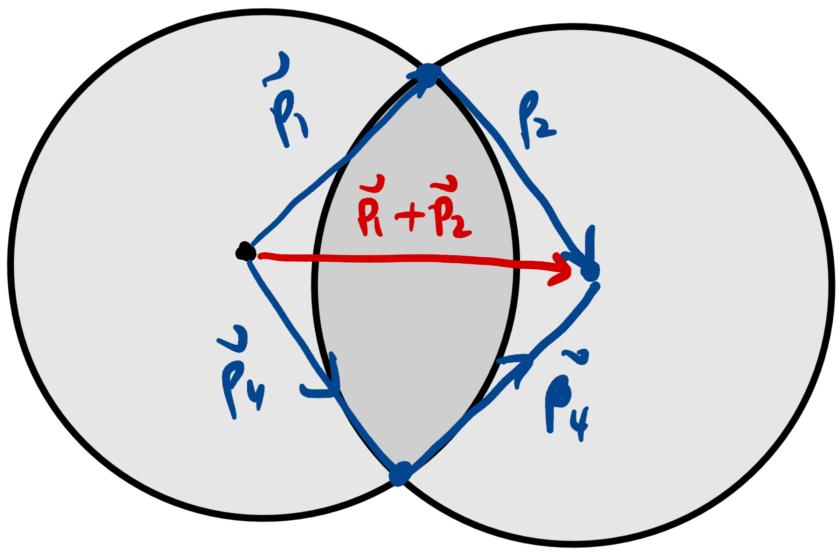
$$\Rightarrow \delta^d(\ ) \sim b^{-1}.$$

$$\Rightarrow S_4\Big|_{k_1=-k_2} \sim b^1 \cdot b^{-1} \sim b^0$$

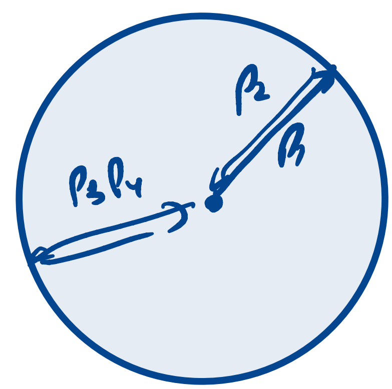
(classical) **MARGINAL!**

Cover of AGD:

generic p_1, p_2:

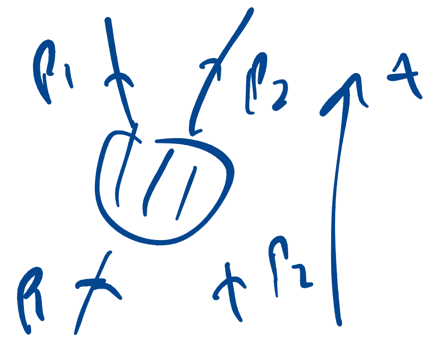


But: if $\vec p_1 + \vec p_2 = 0$



Special k_1, k_2: $k_1 = k_3$. $(\Rightarrow k_2 = k_4)$

FORWARD SCATTERING.



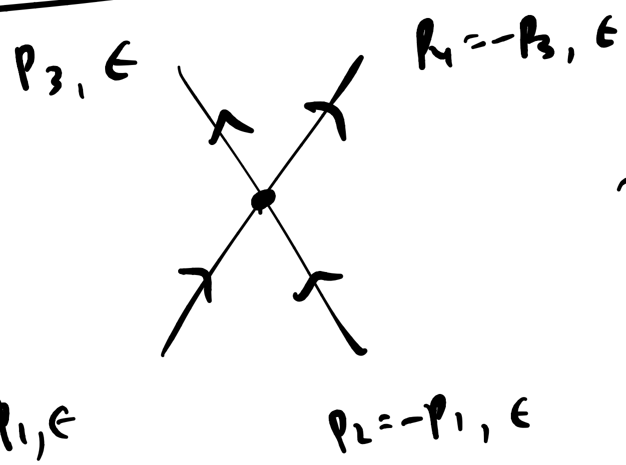
family of $\overset{\text{CLASSICALLY}}{\text{marginal}}$ perturbations:

In 2d: $F(\theta_1, \theta_2) = u(\theta_4 = \theta_2, \theta_3 = \theta_1, \theta_2, \theta_1)$

forward $\rightarrow$

nesting (BCS):

$$V(\theta_1, \theta_3) = u(\theta_4 = -\theta_3, \theta_3, \theta_2 = -\theta_1, \theta_1)$$



$= -iV(\text{momenta})$

$\overset{\text{simplicity}}{=} -iV$ const.

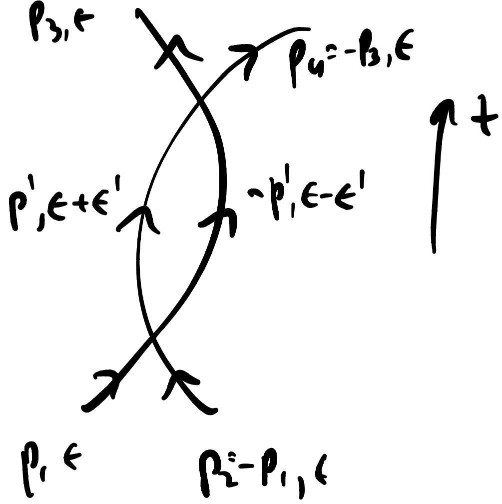
w/ Rot. sym: $V(\theta_3, \theta_1) = V(\theta_3 - \theta_1)$. $V_\ell \equiv \int d\theta\, e^{i\ell\theta} V(\theta)$.

Q: how is V renormalized?

$$G(\epsilon, p = k + \ell) = \frac{i}{\epsilon(1 + i\eta) - v_F(k)\cdot\ell + \mathcal{O}(\ell^2)}$$

$\eta = 0^+$.

$-i\,\delta^{(1)}V =$



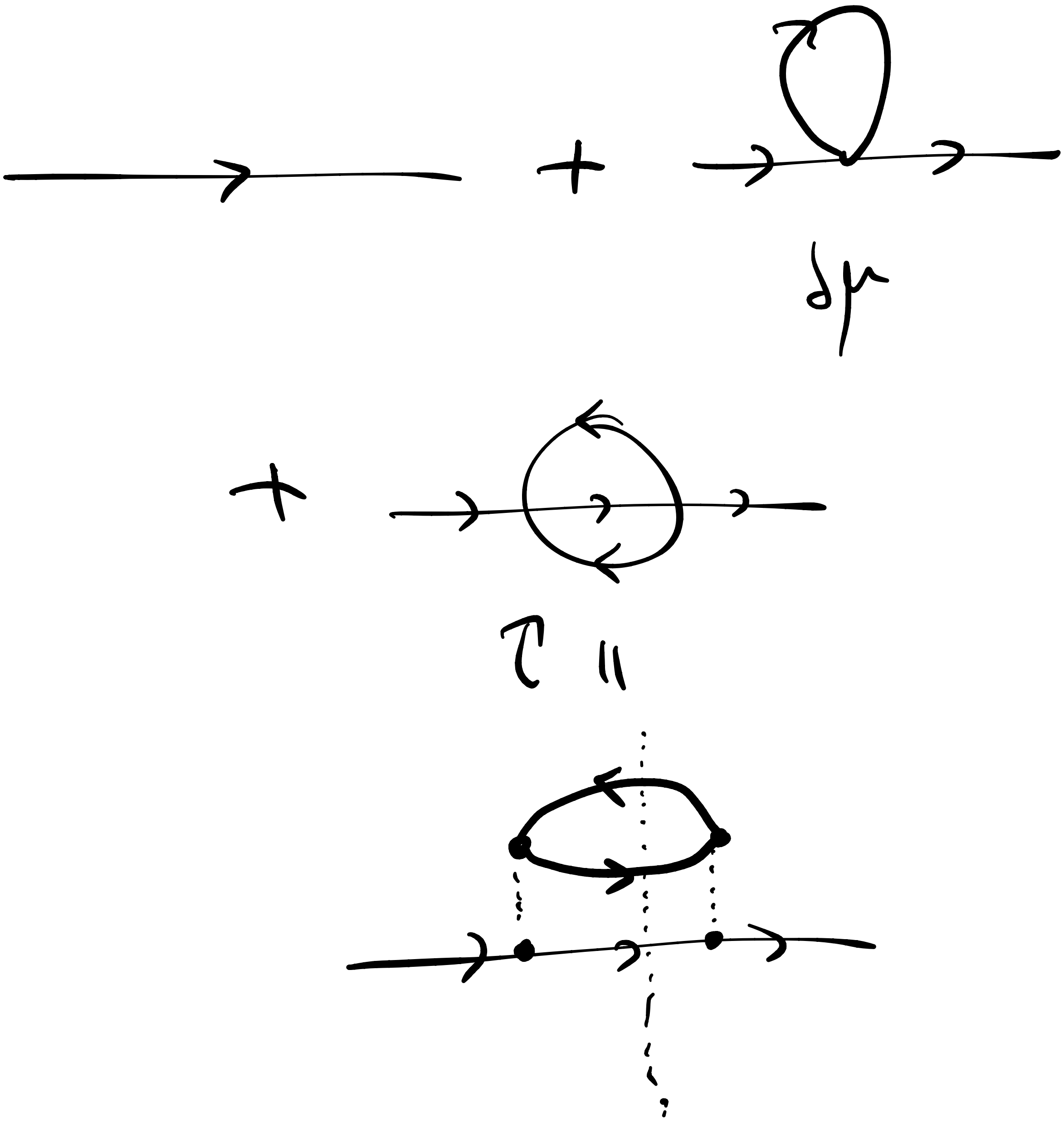
$$= -(-iV)^2 \int_{b\epsilon_0}^{\epsilon_0} d\epsilon'\, d^{d-1}k'\, d\ell \quad \times$$

$$\frac{i}{(\epsilon+\epsilon')(1+i\eta)-v_F(k')\ell'} \cdot \frac{i}{(\epsilon-\epsilon')(1+i\eta)-v_F(k')\ell'}$$

...

<u>preview:</u> $V(b) = V - V^2 N \log(1/b) + O(V^3)$

$\rightarrow$ V is marginally relevant if $V<0$
irrelevant if $V>0$


δμ