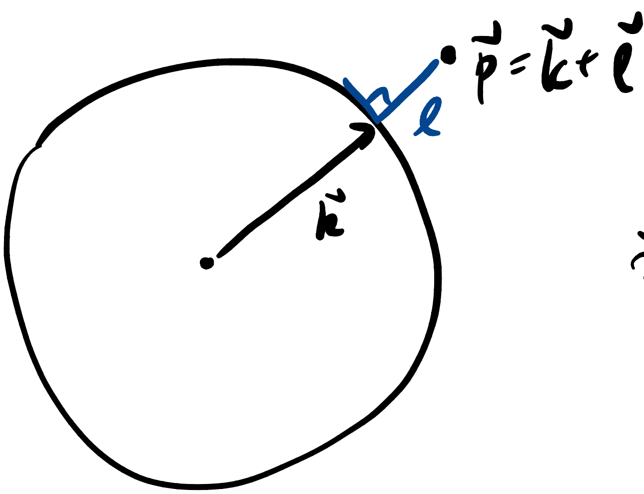
EFT of normal Metals, cont'd

$$S[\psi_\sigma] = \int dt\, d^d p \left(\psi_\sigma^\dagger(p)\, i\partial_t \psi_\sigma(p) - \underbrace{(\epsilon(p) - \epsilon_F)}_{\equiv \xi_p} \psi_\sigma^\dagger(p) \psi_\sigma(p) \right)$$

$\xi_p > 0$ above Fermi level

< 0 below.

$$+ \int dt \prod_{i=1}^{4} d^{d-1}k_i\, d\ell_i \; u(4321)\, \delta^d(p_1 + p_2 - p_3 - p_4)\, \psi_\sigma^\dagger(p_1)\psi_\sigma(p_3)\, \psi_{\sigma'}^\dagger(p_2)\psi_{\sigma'}(p_4)$$



LAST TIME:

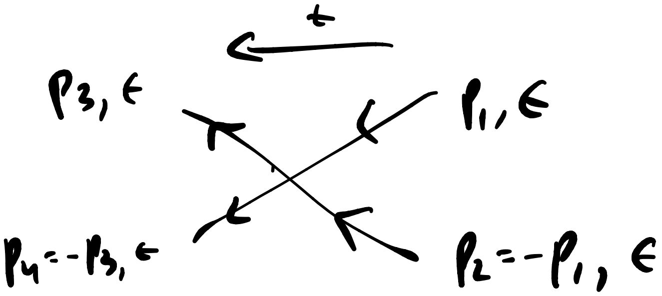
u is marginal

iff: (1) forward scattering

$\{k_1, k_2\} = \{k_3, k_4\}$

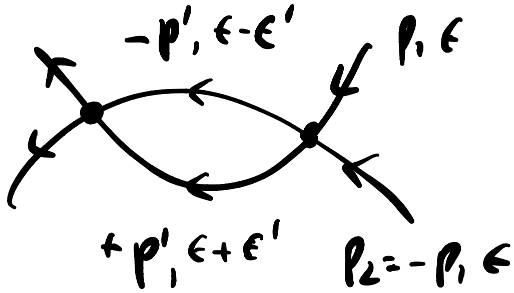
(2) nesting (BCS)

$k_2 = -k_1$



$= -iV$

$-i\delta^{(1)}V =$



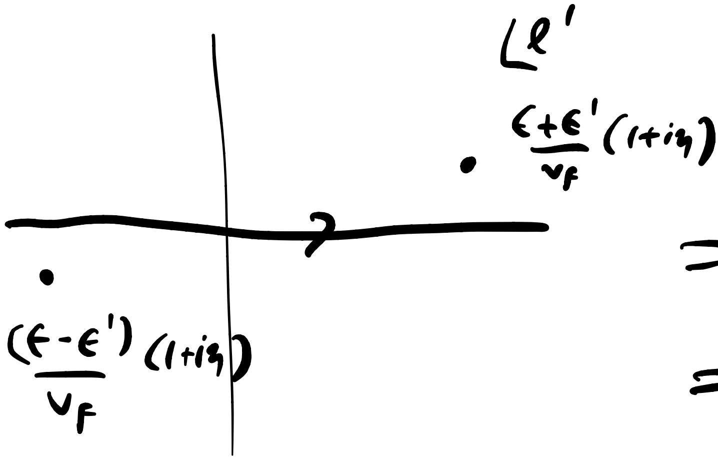
$$= -(-iV)^2 \int_{b\epsilon_0}^{\epsilon_0} d\epsilon'\, d^{d-1}k'\, dl' \left[\frac{i}{(\epsilon+\epsilon')(1+i\eta) - v_F(k')l'} \cdot \frac{i}{(\epsilon-\epsilon')(1+i\eta) - v_F(k')l'} \right]$$

fermion loop

$$G(t,p) = \langle gs |\, T\, c_p(t)\, c_p^\dagger(0)\, | gs \rangle$$

$$\langle gs |\, c_p^\dagger c_p \,| gs \rangle = \begin{cases} 1 & \xi_p < 0 \\ 0 & \xi_p > 0 \end{cases}$$

$$[H_{\text{free}}, c_p(0)] = i\xi_p\, c_p(0)$$



ϵ' near cutoff

$\epsilon \sim 0$

$\Rightarrow \epsilon' > \epsilon$

$\Rightarrow \mathrm{sign}(\epsilon+\epsilon') = -\,\mathrm{sign}(\epsilon-\epsilon')$

$$\int dx \; \frac{1}{x+i} \; \frac{1}{x-i}$$

$$= 2\pi i \;\mathrm{Res}\,(\text{either pole})$$

$$\int dx \; \frac{1}{(x+i)^2} = 0.$$

$$-i\delta^{(1)}V = -V^2 \int d\epsilon' \, d^{d-1}k' \; \frac{2\pi i}{v_F(k')} \; \frac{1}{\underbrace{\epsilon-\epsilon'-(\epsilon+\epsilon')}_{=-2\epsilon'}}$$

$$= +iV^2 \underbrace{\int_{b\epsilon_0}^{\epsilon_0} d\epsilon'}_{\log 1/b} \; \underbrace{\frac{1}{2}\int \frac{d^{d-1}k'}{v_F(k')}}_{\equiv N}$$

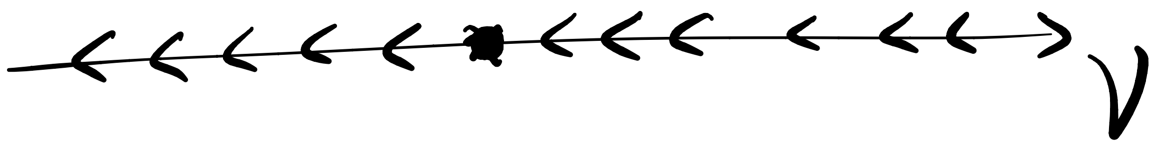
d.o.s. at the FS.

$$V(b) = V - V^2 N \log \frac{1}{b} + O(V^3)$$

$$\Rightarrow -b \frac{d}{db} V(b) = \beta_V = -N V^2(b)$$

($b \to 0$ is the IR)

$$\Rightarrow V(E) = \frac{V_1}{1 + N V_1 \log \frac{E_1}{E}} \to \begin{cases} 0 & \text{in IR if } V_1 > 0 \\ -\infty & \text{in IR if } V_1 < 0 \ \ \text{(negative)} \end{cases}$$



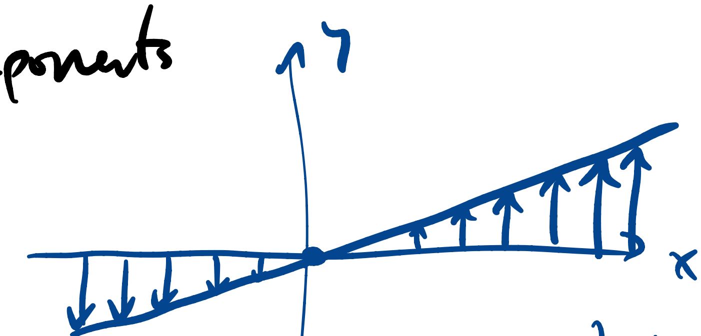
$V_1 \equiv V(E_1)$.

Why is there E_1 s.t. $V_1 < 0$?

Phonons: Goldstones for SSB spacetime syms

$\vec{D}(r)$ $\quad d$ components

why no extra goldstones for rotations?



ROT = x-dependent translation.

$\vec{D}(\tilde{r}) \propto \sqrt{M}$ (displacement $\delta\tilde{r}$ of the ion closest to $\tilde{r}$ from its eqbm position.)

Microscopically:

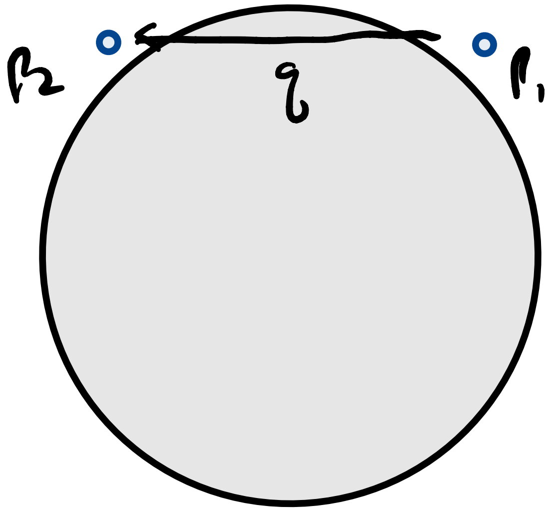
$$L_{\text{ions}} \sim \frac{1}{2} \underline{\underline{M}} \sum_{\substack{\text{ions} \\ I}} \dot{\delta r_I^i}^2 - \underbrace{K_{ij}^{IJ} \delta r_I^i \delta r_J^j}_{} + \ldots$$

indep of M.

$\sim 10^3 m_e$

$$S[D \sim \sqrt{M}\delta r] = \frac{1}{2}\int dt\, d^dq \Big(\partial_t D_i(q)\, \partial_t D(-q) - \omega_{ij}^2(q)\, D_i(q) D_j(-q) \Big) + S_{int}$$

- $\omega^2 \propto M^{-1}$
- $\omega^2(q) \overset{\text{small } q}{\sim} |q|^2 \Leftarrow$ Goldstone



such a q that takes p_1 to p_2 both on FS

$$\underline{q \sim b^0}.$$

<u>Recall</u>: $\begin{cases} \vec{k} \to \vec{k} \\ \vec{\ell} \to b\,\vec{\ell} \end{cases}$

If we demand $\int dt\, d^dq\, (\partial_t D)^2 \sim b^0$

$$\sim b^{1+2[D]} \implies D \sim b^{-1/2}.$$

$$\implies \int dt\, d^dq\, D^2 \text{ is } \underline{\underline{\text{relevant}}}$$

dominates for $E < E_D \equiv \sqrt{\frac{m}{M}}\, E_0$

Debye energy.

$$S_{int}[D,\psi) = \int dt\, d^d q\, d^{d-1}k_1\, \underline{d\ell_1}\, d^{d-1}k_2\, \underline{d\ell_2} \cdot$$

$$M^{-1/2}\, g_i(q,k_1,k_2)\, \underline{D_i}(q)\, \psi^\dagger_\sigma(p_1)\, \psi_\sigma(p_2)$$

$$\underbrace{\delta^d(p_1-p_2-q)}_{\sim b^0}$$

$$\sim b^{-1+1+1-3/2} \sim b^{-1/2}$$

$\uparrow$ if $(\partial_t D)^2$ dominates at low E

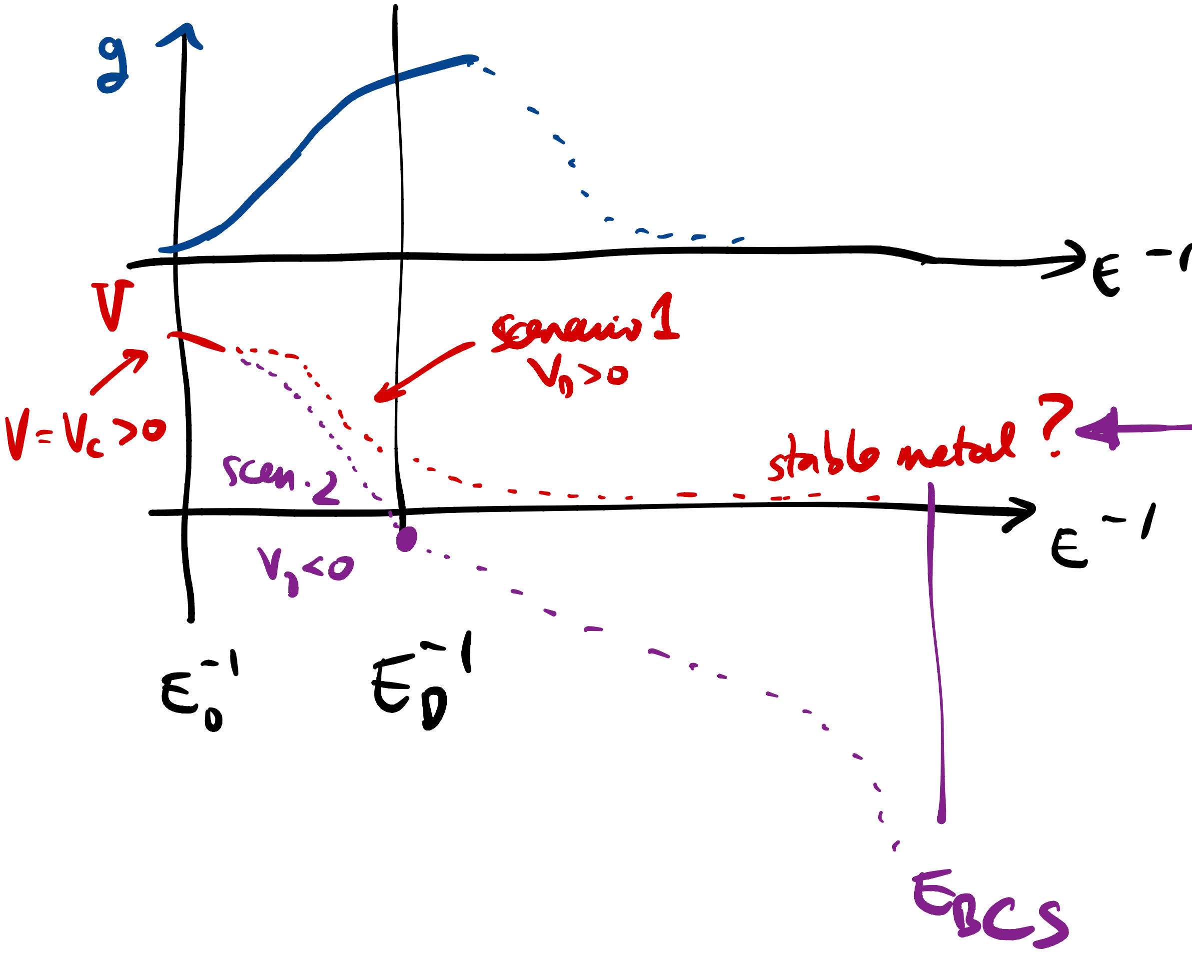
Relevant $\leftarrow$

irrelevant. (for generic kinematics)

where does V become strong? $\underline{V(E_{BCS}) = 1}$.

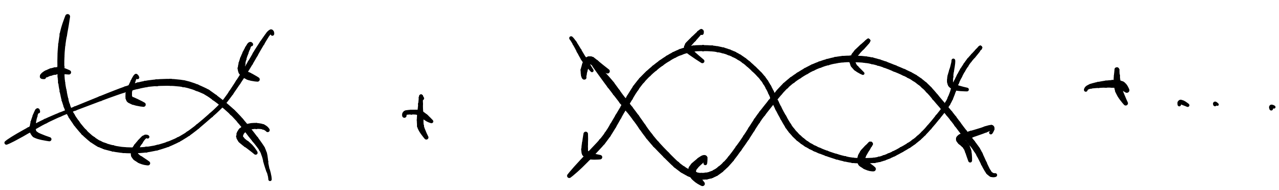
$$E_{BCS} \sim E_D\, e^{-\frac{1}{NV_D}} \qquad V_D = V(E_D).$$

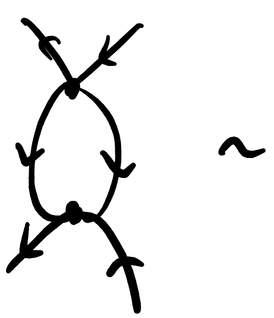
$\uparrow$ $E_D \sim \frac{1}{\sqrt{M}} \Rightarrow$ isotope effect.



g = e - phonon coupling

In fact there's always some $V_\ell = \int e^{i\ell\theta} V(\theta) d\theta$ which is attractive.



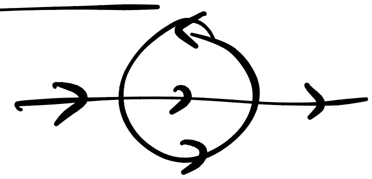
 $\sim$

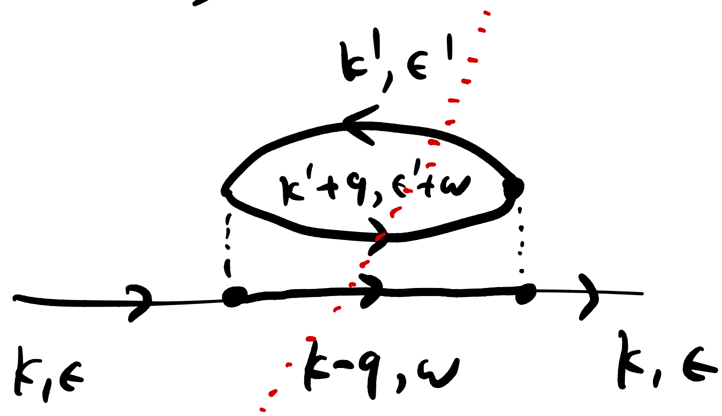
$$\int \cdots d\ell' \frac{1}{(\quad)} \frac{1}{(\quad)} = 0$$

<u>claim</u>: both poles are in the same half-plane.

Wave f'n Renormalization

leading contrib. to $\frac{\partial}{\partial p}\Sigma$.

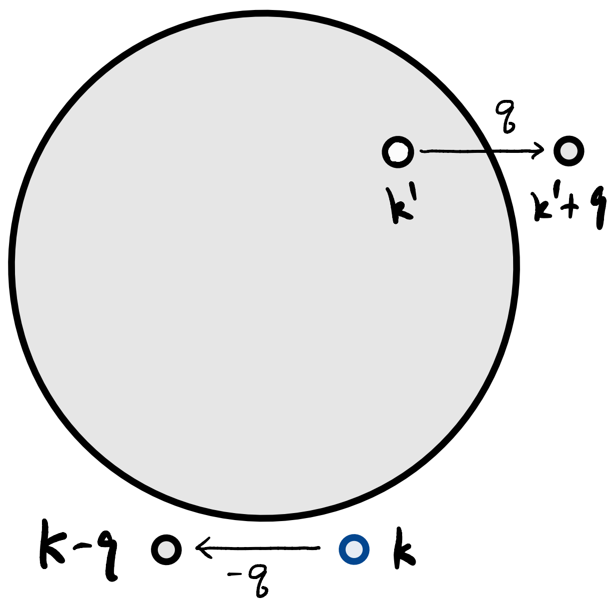
$\Sigma_1 =$ 

$\equiv$ 

$\mathrm{Im}\,\Sigma_1(k,\epsilon_k) = \tau^{-1}$ (exc. w/ momentum k)

$$= \pi \int d^{d+1}q \, d^{d+1}k' \, \delta(D) \, |u_q|^2$$

$$f(-\epsilon_{k'}) f(\epsilon_{k'+q}) f(\epsilon_{k-q})$$



$$f(\epsilon) = \lim_{T\to 0} \frac{1}{e^{\frac{\epsilon-\epsilon_F}{T}}+1} = \Theta(\epsilon<\epsilon_F)$$

$\Rightarrow$ Both $\epsilon_{k'+q}$ and $\epsilon_{k'}$ lie in a shell of radius ϵ about the FS

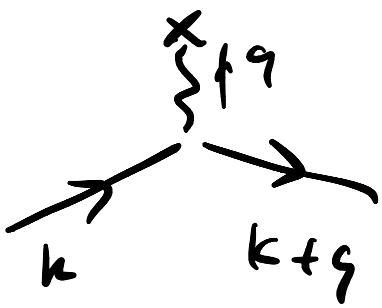
$$\tau^{-1} \propto \left(\frac{\epsilon}{\epsilon_F}\right)^2 << \epsilon.$$

sharp resonance.

easy observable : $\rho \sim$ rate at which charge carriers lose momentum.

$\sim$ rate at which $*$ qps decay.

if qp decay is b/c ee interactions $\sim T^2$.



static impurities

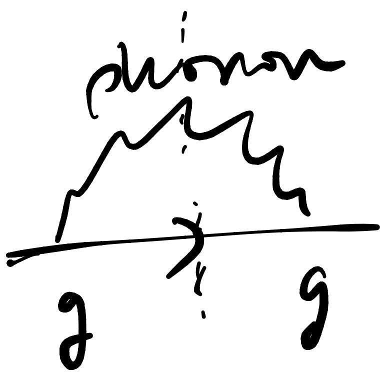
$\rho \sim$ const

phonons $g \propto q$

$$\rho \sim T^{3+2} \qquad T < T_D$$

$$\sim T \qquad T > T_D$$

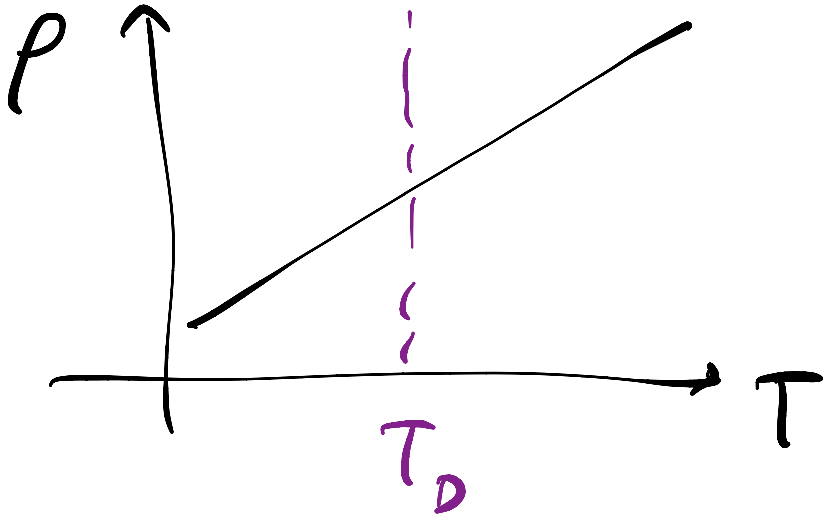
phonon



$g \sim |q| \sim |\omega|$

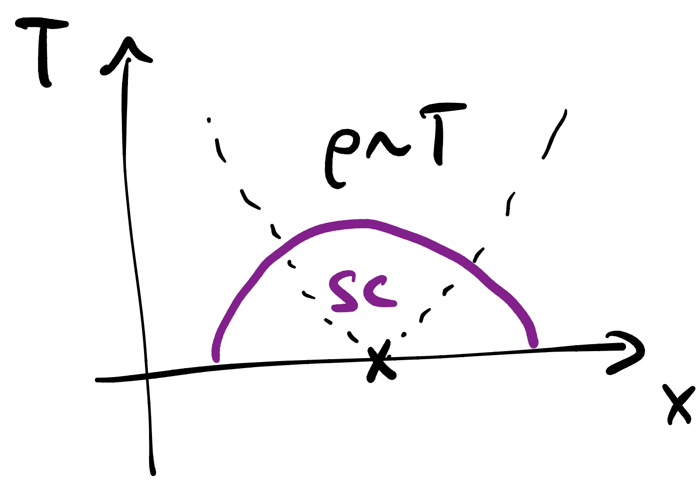
vs: high T_c superconductor (near optimal doping)

$\rho \sim$ cst + T through T_D.



Q: where did we go wrong?

Loophole: other light dofs.



3 Geometric & Topological terms in QFT actions

$$\text{operator or Hamiltonian} \underset{\text{resolution of } \mathbb{1}.}{\overset{\text{canonical quantization 215A}}{\rightleftarrows}} \text{path integral or Lagrangian}$$

- non-unique: each choice of basis for $\mathcal{H}_x$ $\longleftrightarrow$ different path integral.