Plasma Mixing Effects in the Production of Dark Photons

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New particles beyond the standard model can be constrained by looking at their production in energetic environments like astrophysical plasmas. The production rate of such particles can be calculated using the imaginary part of the self-energy. However, as pointed out in [1], plasma effects may generate an effective in-medium mixing between the new field and plasma species. In this case the self-energy is not simply the sum of 1-particle-irreducible diagrams, but must take into account mixing effects. In this short note, the model of a dark photon is used to demonstrate this plasma-induced mixing and its impact on dark photon production rates.

INTRODUCTION

A good place to look for physics beyond the standard model (BSM) is in hot, energetic environments like plasmas. A typical way to do this is calculate the production rate of BSM particles, describe their impact on the relevant observables, and constrain the parameters of the model. Production rates may be obtained via the optical theorem by calculating the imaginary part of the self-energy. The self-energy must be calculated in the plasma environment, which can be done using the framework of thermal field theory. In some cases there is a further consideration: plasma effects may lead propagation eigenstates to be different than those in vacuum, mixing the BSM particle with species in the thermal bath. In this case the correct self-energy to calculate is not the sum of 1-particle-irreducible diagrams of the BSM field, but instead one containing mixed terms. This short note is based largely on the work of [1], which points out that (among other things) many calculations of dark photon production in stellar cores ignore these mixing effects, leading to incorrect constraints on BSM parameters. The point of this note is to briefly explain, using a specific model of massive dark photons, how this mixing arises in a plasma and its impact on dark photon production rates.

The rest of the paper is structured as follows. Section 2 introduces the dark photon lagrangian and describes the mixing between the two photon species. Section 3 shows how to use the mixed dark photon propagator to calculate a production rate. Section 4 gives a brief overview of the photon propagator in thermal field theory. Finally, Section 5 discusses the impact of mixing on the dark photon production rate in several regimes.

THE DARK PHOTON LAGRANGIAN AND PROPOGATION EIGENSTATES

Consider the lagrangian with a new vector field, the dark photon X:

$$\mathcal{L} \supset -\frac{1}{4}F^2 - \frac{1}{4}F_X^2 + \frac{m^2}{2}X^2 + eJ(A+\epsilon X) \qquad (1)$$

The dark photon has a Stueckelberg mass m, so there are no additional low-energy BSM states. Apart from this mass, the dark photon is identical to the QED photon with charge ϵe. The production and absorption rates in the plasma can be derived from the imaginary parts of the poles of the thermal propagator for X. The main point of reference [1] is that, because of the form of the lagrangian and plasma effects, the dark photon propagator cannot be computed simply by summing one-particle-irreducible diagrams to compute the self energy.

First, recall the situation for the QED photon in vaccuum. The propagator can be written as $D_F^{\mu\nu}(k) = \frac{-i\eta^{\mu\nu}}{k^2}$, and the photon self-energy $\Pi^{\mu\nu}(k^2)$ renormalizes the electric charge but does not provide the photon a mass (see section 1.7 of [2]). This self-energy is computed by summing all 1PI diagrams, like those shown in the upper left of Fig. 1. Furthermore, the tensor part of the self-energy can be factored out: $\Pi^{\mu\nu}(k^2) = \Pi(k^2)\Delta_T^{\mu\nu}$, where $\Delta_T^{\mu\nu}$ projects onto transverse modes. (We will see later that in a plasma the longitudinal mode is also important.) It will be convenient to work with only the scalar part of the self energy for now.

The dark photon thermal self-energy is more complicated because the fermion interactions $eJ(A+\epsilon X)$ mean that a single dark photon can turn into a single QED photon, at least in internal lines. For example, diagrams shown in the upper right of Fig. 1 are possible. These are not 1PI diagrams because they can be cut through the A line. This means that there is mixing between A and X: the propogation eigenstates are combinations of the A and X fields. A familiar example of this is the mismatch between mass and flavor eigenstates in the neutrino sector. The poles of the *propagation* eigenstates are what we want for computing production rates, and correspond to the zero-eigenvalue solutions ([1]) to

$$\begin{pmatrix} k^2 - \Pi^{AA} & -\Pi^{AX} \\ -\Pi^{XA} & k^2 - m^2 - \Pi^{XX} \end{pmatrix}$$

Here Π^{XY} represents the self-energy calculated from the sum of all 1PI diagrams connecting X and Y, and $k_\mu = (\omega, \vec{k})$. There are two zero-eigenvalue solutions

$$\omega_c^2 = |\vec{k}|^2 + \Pi^{AA} + \frac{(\Pi^{AX})^2}{\Pi^{AA} - m^2} + \mathcal{O}(\epsilon^4) \qquad (2)$$

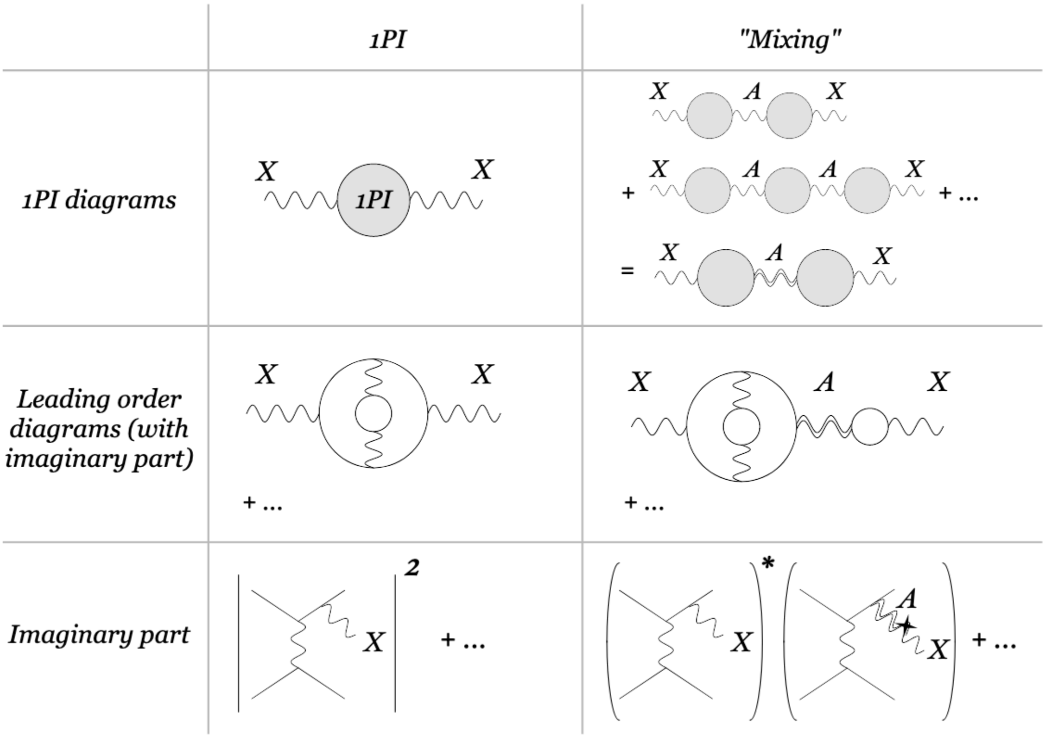

FIG. 1: Figure 1 from [1]. Diagrams relevant for the dark photon (X) self-energy calculation, including both 1PI diagrams (left) and mixing diagrams (right). *Row 1*: The left diagram shows the standard 1PI diagrams that would be exclusively used to calculate the dark photon self-energy in the absence of mixing. The right diagram shows diagrams that are not 1PI, but nevertheless contribute to the self energy through intermediate QED photon (A) thermal propagators. The double wavy line is the fully-resummed thermal propagator for A. *Row 2*: Leading order diagrams that contribute to the production and absorption of the dark photon. *Row 3*: The imaginary part of the diagrams in row 2. The star indicates the medium-induced mixing between A and X.

$$\omega_c^2 = |\vec{k}|^2 + m^2 + \Pi^{XX} - \frac{(\Pi^{AX})^2}{\Pi^{AA} - m^2} + \mathcal{O}(\epsilon^4) \quad (3)$$

These correspond to two (renormalized) propagation eigenstates, written in the (A, X) basis:

$$\sqrt{Z_A^{-1}} \begin{pmatrix} 1 \\ \frac{\Pi^{AX}}{\Pi^{AA}-m^2} \end{pmatrix} + \mathcal{O}(\epsilon^2) \quad (4)$$

$$\begin{pmatrix} -\frac{\Pi^{AX}}{\Pi^{AA}-m^2} \\ 1 \end{pmatrix} + \mathcal{O}(\epsilon^2) \quad (5)$$

Since Π^{AX} is $\mathcal{O}(\epsilon)$, if ϵ is small then so is the mixing. The first solution/eigenvector corresponds to the mostly-A state, and the second to the mostly-X state. We are interested in the latter for computing the dark photon production rate. Eq. 3 is the dispersion relation for the mostly-dark-photon state, with mass m, 1PI self-energy Π^{XX}, and a mixing term proportional to $(\Pi^{AX})^2$.

PRODUCTION OF DARK PHOTONS

The imaginary part of ω_c is related to the mostly-X state production and absorption rates via the optical theorem (see section 2.2 of [2] for a reminder):

$$\Gamma_{abs}(\omega) - \Gamma_{prod}(\omega) \equiv \Gamma_{damping}(\omega) \quad (6)$$

$$= -\frac{1}{\omega}\mathrm{Im}\left(\Pi^{XX} - \frac{(\Pi^{AX})^2}{\Pi^{AA} - m^2}\right) + \mathcal{O}(\epsilon^4) \quad (7)$$

The total production rate per unit volume is

$$\frac{dN_{prod}}{dV dt} \int đ^3 k \Gamma_{prod} \quad (8)$$

The middle two panels of Fig. 1 show the leading order contributions to the pole in the propagator. The left-middle panel is the usual Π^{XX} diagram (note that a diagram with a single fermion loop is not shown because kinematics forbid a single photon decaying into two fermions). The right-middle panel shows the mixing term related to Π^{AX}.

Detailed balance requires $\Gamma_{prod} = e^{-\frac{\omega}{T}}\Gamma_{abs}$, so $\Gamma_{prod} = \frac{1}{e^{\frac{\omega}{T}}-1}\Gamma_{damping} \equiv n_X\Gamma_{damping}$, with occupation number n_X. Since the QED photon and dark photon interaction terms $e(A+\epsilon X)J$ differ only by ϵ, the 1PI self energies can fortunately all be expressed in terms of the QED photon: $\Pi \equiv \Pi^{AA} = \epsilon\Pi^{AX} = \epsilon^2\Pi^{AX}$. Writing $\Pi = \Pi_r + i\Pi_i$, the production rate is:

$$\Gamma_{prod}(\omega) = n_x \frac{\epsilon^2 m^4}{\omega}\frac{\Pi_i}{\Pi_i^2 + (\Pi_r - m^2)^2} + \mathcal{O}(\epsilon^4) \qquad (9)$$

To progress any further requires taking a look at the self-energy of the photon in a plasma.

THE THERMAL PHOTON SELF-ENERGY

A derivation of the photon self-energy at finite temperature can be found in e.g. [3]; the result will just be quoted here. In an isotropic plasma, the photon self-energy is

$$\Pi_{\mu\nu}(k) = (\epsilon_\mu^+\epsilon_\nu^{+*} + \epsilon_\mu^-\epsilon_\nu^{-*})\Pi_T + \epsilon_\mu^L\epsilon_\nu^L\Pi_L \qquad (10)$$

ϵ^+ and ϵ^- are the transverse photon polarization vectors, and ϵ^L is the vector for the longitudinal mode that has appeared in-medium. We'll focus on the transverse mode to illustrate the effects of mixing. For a non-relativistic plasma (ie $\frac{T}{m_e} << 1$):

$$\mathrm{Re}(\Pi_T)(k) = \omega_p^2(1 + \frac{|\vec{k}|^2 T}{\omega^2 m_e}) \qquad (11)$$

$$\equiv m_p(k)^2 \qquad (12)$$

$$\omega_p^2 = \frac{e^2 n_e}{m_e}(1 - \frac{5T}{2m_e}) \qquad (13)$$

In a medium the transverse photon modes have gained an effective mass $m_p(k)$ approximately equal to the plasma frequency ω_p, with small modifications due to the plasma temperature. This mass is depends on the electron density, which generally varies throughout an astrophysical environment such as a star. So for many relevant calculations, the plasma mass is spatially dependent.

EFFECT OF MIXING ON PRODUCTION RATES

Consider the production rate for transverse polarization modes. Define $\Gamma_T(k) \equiv \frac{\mathrm{Im}(\Pi_T)}{\omega}$ for convenience. If mixing effects were neglected, we would keep only the Π^{XX} term in eq. 3, and using eq. 12 the production rate of the transverse state would be

$$\Gamma_{prod,T} = \epsilon^2 n_x \Gamma_T \qquad (14)$$

With mixing, however, the production rate is given by eq. 9 as:

$$\Gamma_{prod,T} = n_x\epsilon^2 m^4 \frac{\Gamma_T}{\omega^2\Gamma_T^2 + (m_p^2 - m^2)^2} \qquad (15)$$

There are 3 relevant limits:

1. $(m_p^2 - m^2)^2 >> \omega^2\Gamma_T^2$ and $m >> m_p$: Dark photon mass above the plasma mass. The production rate is the same as without mixing - the plasma mass is effectively ignored.

2. $(m_p^2 - m^2)^2 >> \omega^2\Gamma_T^2$ and $m << m_p$: Below the plasma mass. The rate is suppressed by 4 powers of $\frac{m}{m_p}$ - the plasma mass prevents the formation of dark photons. $\Gamma_{prod,T} = \frac{m^4}{m_p^4}\epsilon^2 n_x\Gamma_T$.

3. $|m_p^2 - m^2|^2 << \omega^2\Gamma_T^2$: Resonant production (see note [4]). When the thermal and dark photon masses are similar, the production rate may exceed that without mixing. $\Gamma_{prod,T} = \frac{m^4}{\omega^2\Gamma_T}\epsilon^2 n_x$.

So the effect of mixing is to introduce three distinct production regimes for the transverse dark photon based on the relationship between the dark photon mass and the plasma mass. In environments with a variable plasma density, the plasma frequency will vary as well, and all three may potentially apply. Fig. 2, taken from [1], shows how the mixing effects modify constraints on ϵ from SN1987A observations. The suppression due to the plasma mass is particularly important, changing the bound by four orders of magnitude over the specified dark photon mass range.

It is also worth noting that the mixing is only relevant due to the plasma effects: as the plasma mass becomes arbitrarily small, we will generally be in the limit $m >> m_p$ where the 'naive' production rate calculated using only the 1PI Π^{XX} self-energy will give the correct result. The longitudinal mode was not discussed here and has different rates, but the general idea that mixing effects alter the production rates also applies.

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[1] E. Hardy and R. Lasenby, Journal of High Energy Physics **2017** (2017), URL `https://doi.org/10.1007%2Fjhep02%282017%29033`.

[2] J. McGreevy, *Physics 215b: Quantum field theory, part 2* (2022).

[3] G. G. Raffelt, *Stars as laboratories for fundamental physics: The astrophysics of neutrinos, axions, and other*

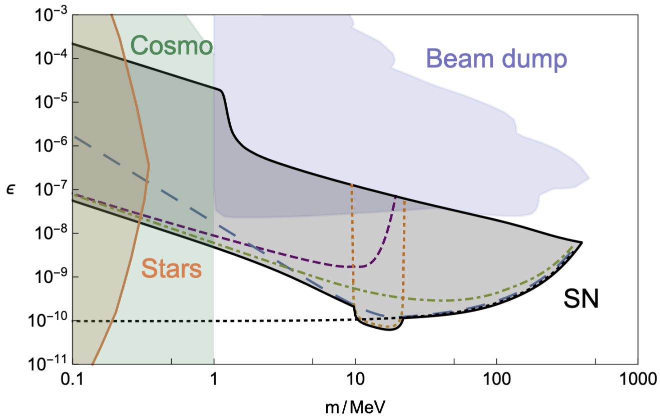

FIG. 2: Fig. 3 from [1]. Constraints on the dark photon coupling parameter ϵ from the lack of anomalous energy loss due to dark photon production in SN1987A. The dashed black line shows the constraints ignoring mixing effects. The solid black line shows the proper constraints, and all three regimes of production. Resonant production occurs for dark photon masses $\approx 10-20MeV$. Above this range, mixing effects are negligible. Below, the production is suppressed, leading to worse constraints by a factor of $\epsilon \propto \frac{m^2}{m_p^2}$. Subcomponents of the total emission are indicated by dashed line colors; dotted (orange), resonant transverse emission, long-dashed (blue), continuum transverse emission, dot-dashed (green), continuum longitudinal emission, dashed (purple), resonant longitudinal emission. The shaded regions show constraints from cosmological observations, stellar cooling, and beam dump experiments.

weakly interacting particles (1996), ISBN 978-0-226-70272-8.

[4] In this case you may worry about the expansion in eq. 3, which is valid when $\epsilon << $ any other scale. On resonance the requirement becomes more stringent, $\epsilon << \frac{\omega\Gamma_T}{m^2}$; if the reverse is true then the dark photon production rate is approximately half that of the SM photon.